A GOAL PROGRAMMING METHOD FOR GENERATING PRIORITY WEIGHTS BASED ON INTERVAL-VALUED INTUITIONISTIC PREFERENCE RELATIONS

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Abstract:
Interval-valued intuitionistic preference relations are a powerful means to expressing a decision maker’s uncertainty and hesitation about its preference over criteria in the process of multi-criteria decision making. In this paper, we define the notion of consistent interval-valued intuitionistic preference relations. Goal programming models are established for generating priority interval weights based on interval-valued intuitionistic preference relations. Two illustrative numerical examples are furnished to demonstrate how to apply the approach.

Keywords:
Interval-valued intuitionistic preference relation; Priority interval weight; Consistency; Goal programming

1. Introduction

As an important extension of fuzzy logic, intuitionistic fuzzy sets (IFSs) [1] allow a decision-maker (DM) to express both the degree of belonging of an element to a particular set (membership function) and the degree of non-belonging to the set (nonmembership function). In an IFS, the membership and nonmembership functions are real-valued. To further enhance the capability of handling uncertainty in the membership and nonmembership functions, Atanassov and Gargov [2] have extended the notion of IFSs to interval-valued intuitionistic fuzzy sets (IVIFs) by allowing the membership and nonmembership functions to assume interval values. Current research has been focusing on the basic theory of IVIFSs such as basic relations and operations of IVIFSs [3], the correlation and correlation coefficients of IVIFSs [4-7], the topology of IVIFSs [8], relationships between IFSs, L-fuzzy sets, interval-valued fuzzy sets and IVIFSs [9-11], and the entropy and subhedseth of IVIFSs [12]. Recent efforts have been directed to apply IVIFSs to multiattribute decision analysis. For instance, Xu [13] proposes some aggregation operators for multiattribute decision analysis with interval-valued intuitionistic fuzzy information. Xu and Yager [14] further investigate dynamic intuitionistic fuzzy aggregation operators and devise two procedures for dynamic intuitionistic fuzzy multiattribute decision making with intuitionistic fuzzy numbers (IFNs) or interval-valued intuitionistic fuzzy numbers (IVIFNs). Wang et al. [15] develop a framework for handling multiattribute decision making with IVIFS assessments and incomplete attribute weights.

In the process of multicriteria decision making, a DM is expected to provide its preference over criteria, which may be conveniently characterized by fuzzy preference relations [16-18] when the DM has only vague knowledge about its preference of one criterion over another. In this case, it is more appropriate to express the DM’s preference in an interval number rather than an exact numerical value. Xu and Chen [19] define some new concepts such as additive and multiplicative consistent interval fuzzy preference relations, and develop some linear programming models to derive interval weights based on various interval fuzzy preference relations. Szmidt and Kacprzyk [20] investigate how to reach consensus with intuitionistic fuzzy preference relations in group decision making. Xu [21] introduces consistent, incomplete, and acceptable preference relations and develops another approach to group decision making under the intuitionistic fuzzy environment. In this article, it is assumed that the DM provides its preference over multiple criteria in terms of interval-valued intuitionistic fuzzy numbers (IVIFNs). We shall develop a method for generating priority weights based on these interval-valued intuitionistic fuzzy preference relations.

The remainder of this paper is organized as follows: Section 2 reviews some basic concepts related to IFSs and IVIFSs. Section 3 introduces the concept of consistent interval-valued intuitionistic fuzzy preference relations. In
Section 4, we develop a goal programming approach to deriving criteria weights based on interval-valued intuitionistic preference relations, and finally, two numerical examples are furnished to demonstrate how the approach can be applied.

2. Preliminaries

Some basic concepts on IFSs and IVIFSs are reviewed below to provide a basis for future discussions.

Definition 2.1 (Atanassov [1]). Let a set $X$ be fixed, an intuitionistic fuzzy set (IFS) $\tilde{A}$ in $X$ is defined as:

$$A = \{<x, \mu_{A}(x), \nu_{A}(x) | x \in X \}$$

where the functions $\mu_{A} : X \rightarrow [0, 1]$ and $\nu_{A} : X \rightarrow [0, 1]$ satisfy the condition:

$$0 \leq \mu_{A}(x) + \nu_{A}(x) \leq 1, \quad \forall x \in X.$$  

(2)

$\mu_{A}(x)$ and $\nu_{A}(x)$ denote the degrees of membership and nonmembership of element $x \in X$ to set $A$, respectively. $\pi_{A}(x) = 1 - \mu_{A}(x) - \nu_{A}(x)$ is usually called the intuitionistic fuzzy index of $x \in A$, representing the degree of indeterminacy or hesitation of $x$ to $A$. It is obvious that $0 \leq \pi_{A}(x) \leq 1$ for every $x \in X$.

Definition 2.2 (Atanassov and Gargov [2]). Let $X$ be a non-empty set of the universe, and $D[0, 1]$ be the set of all closed subintervals of $[0, 1]$, an interval-valued intuitionistic fuzzy set (IVIFS) $\tilde{A}$ in $X$ is defined by

$$\tilde{A} = \{< x, \tilde{\mu}_{A}(x), \tilde{\nu}_{A}(x) | x \in X \}$$

where $\tilde{\mu}_{A} : X \rightarrow D[0, 1]$ and $\tilde{\nu}_{A} : X \rightarrow D[0, 1]$ satisfy the condition:

$$0 \leq \sup(\tilde{\mu}_{A}(x)) + \sup(\tilde{\nu}_{A}(x)) \leq 1 \text{ for any } x \in X. $$

(4)

Similarly, the intervals $\tilde{\mu}_{A}(x)$ and $\tilde{\nu}_{A}(x)$ denote, respectively, the degree of membership and nonmembership of $x \in X$ to $A$. But, by definition, for each $x \in X$, $\tilde{\mu}_{A}(x)$ and $\tilde{\nu}_{A}(x)$ are closed intervals rather than real numbers and their lower and upper boundaries are denoted by $\mu_{L}(x), \mu_{U}(x), \nu_{L}(x), \nu_{U}(x)$, respectively. Therefore, an equivalent way to express an IVIFS $\tilde{A}$ is

$$\tilde{A} = \{< x, [\mu_{L}(x), \mu_{U}(x)], [\nu_{L}(x), \nu_{U}(x)] | x \in X \}$$

where

$$0 \leq \mu_{L}(x) + \nu_{L}(x) \leq 1, \quad 0 \leq \mu_{U}(x) \leq \mu_{L}(x) \leq 1, \text{ and } 0 \leq \nu_{U}(x) \leq \nu_{L}(x) \leq 1.$$  

(5)

Similar to IFSs, for each element $x \in X$ we can compute its hesitation interval relative to $\tilde{A}$ as:

$$\tilde{\pi}_{A}(x) = [\tilde{\pi}_{L}(x), \tilde{\pi}_{U}(x)] = [1 - \mu_{U}(x) - \nu_{U}(x), 1 - \mu_{L}(x) - \nu_{L}(x)]$$

If each of the intervals $\mu_{L}(x)$ and $\nu_{L}(x)$ contains only one real value, i.e., for every $x \in X$, $\mu_{A}(x) = \tilde{\mu}_{A}(x) = \mu_{U}(x)$ and $\nu_{A}(x) = \tilde{\nu}_{A}(x) = \nu_{U}(x)$, then the given IVIFS $\tilde{A}$ is reduced to an ordinary IFS.

For any given $x$, the pair $(\tilde{\mu}_{A}(x), \tilde{\nu}_{A}(x))$ is called an interval-valued intuitionistic fuzzy number (IVFN) [14]. Without causing confusion, the pair $(\bar{\mu}_{A}(x), \bar{\nu}_{A}(x))$ is often denoted by $([\bar{\mu}^{L}, \bar{\mu}^{U}],[\bar{\nu}^{L}, \bar{\nu}^{U}])$, where $[\bar{\mu}^{L}, \bar{\mu}^{U}] \in D[0, 1], [\bar{\nu}^{L}, \bar{\nu}^{U}] \in D[0, 1]$ and $\bar{\mu}^{L} + \bar{\nu}^{L} \leq 1.$

3. Consistent Interval-Valued Intuitionistic Fuzzy Preference Relations

Definition 3.1 (Xu [17, 21]). An intuitionistic preference relation $R$ on the set $X$ is represented by a matrix $R = (r_{ij})_{n \times n} \in X \times X$ with $r_{ij} = \langle x_{i}, x_{j} \rangle, (\mu(x_{i}, x_{j}), \nu(x_{i}, x_{j}))$, $\forall (i, j) = 1, 2, \cdots, n$ for convenience, let $r_{ij} = (\mu_{ij}, \nu_{ij})$, for all $i, j = 1, 2, \cdots, n$, where $r_{ij}$ is an IFN, indicating the degree of $\mu_{ij}$ to which $x_{i}$ is preferred to $x_{j}$ and the degree $\nu_{ij}$ to which $x_{j}$ is not preferred to $x_{i}$. Furthermore, $\mu_{ij}$ and $\nu_{ij}$ satisfy the following characteristics:

$$0 \leq \mu_{ij} + \nu_{ij} \leq 1, \quad \mu_{ij} = \nu_{ij} = 0.5, \quad \forall i, j = 1, 2, \cdots, n.$$  

(6)

Note that an intuitionistic preference relation $R = (r_{ij})_{n \times n}$ is equivalent to an interval fuzzy preference relation $\tilde{R}(\tilde{r}_{ij})_{n \times n}$, where $\tilde{r}_{ij} = (\mu_{ij}, 1 - \nu_{ij})$, for all $i, j = 1, 2, \cdots, n$.

Let $\omega = (\omega_{1}, \omega_{2}, \cdots, \omega_{n})^{T}$ be the vector of priority weights, where $\omega_{i}$ reflects the importance degree of criterion $x_{i}$, and $\omega_{i}, i = 1, 2, \cdots, n, \sum_{i=1}^{n} \omega_{i} = 1$, then an intuitionistic preference relation $R = (r_{ij})_{n \times n}$ is called a consistent intuitionistic preference relation if the following condition [19] is satisfied:

$$\pi_{ij} \leq 0.5(\omega_{i} - \omega_{j} + 1) \leq 1 - \nu_{ij}, i = l, 2, \cdots, n; j = l + 1, l + 2, \cdots, n.$$  

(7)

Definition 3.2 (Xu [17]). An interval-valued intuitionistic fuzzy preference relation $\tilde{R}$ on $X$ is represented by a
matrix $\hat{R}=(\hat{r}_{ij})_{n\times n}$ in $X \times X$ with $\hat{r}_{ij}< (x_i, x_j, \mu(x_i, x_j), \nu(x_i, x_j))$ for all $i, j=1, 2, \ldots, n$. Similarly, let $\hat{r}_{ij}=(\mu_{ij}, \nu_{ij})$, for all $i, j=1, 2, \ldots, n$, where $\hat{r}_{ij}$ is an IVIFN, consisting of the interval degree $\mu_{ij}$ to which $x_i$ is preferred to $x_j$, and the interval degree $\nu_{ij}$ to which $x_i$ is not preferred to $x_j$, and $\hat{r}_{ij}(x)=[\hat{\mu}_{ij}, \hat{\nu}_{ij}]=\{1-\hat{\mu}_{ij}^U, 1-\hat{\nu}_{ij}^U\}$ is interpreted as the hesitation interval degree to which $x_i$ is preferred to $x_j$. Furthermore, $\hat{\mu}_{ij}=[\mu_{ij}^U, \mu_{ij}^L]$ and $\hat{\nu}_{ij}=[\nu_{ij}^U, \nu_{ij}^L]$ possess the following characteristics:

$$0 \leq \hat{\mu}_{ij}(x)+\hat{\nu}_{ij}(x) \leq 1,$$

$$\hat{\mu}_{ij}=\nu_{ij}, \hat{\nu}_{ij}=\mu_{ij},$$

$$\mu_{ij}=[0, 0.5], \nu_{ij}=[0.5, 1]$$

for all $i, j=1, 2, \ldots, n$. (8)

Definition 3.3 (24)]. Let $\overline{\omega}=(\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n)^T$ with $\overline{\omega}_i=[\alpha_i, \beta_i]$ satisfies $0 \leq \alpha_i \leq \beta_i \leq 1$, for all $i=1, 2, \ldots, n$, then $\overline{\omega}$ is called a normalized interval weight vector, if the following condition is satisfied

$$\sum_{i=1}^n \alpha_i + \beta_i \leq 1,$$

$$\alpha_i + \sum_{i=1}^n \alpha_i \geq 1, \text{ } j=1, 2, \ldots, n.$$ (9)

Definition 3.2 indicates that each element $\hat{r}_{ij}$ in the interval-valued intuitionistic preference relation $\hat{R}$ consists of an IVIFN $(\mu_{ij}, \nu_{ij})$. Given that each IVIFN $(\mu_{ij}, \nu_{ij})$ must satisfy the condition:

$$0 \leq \hat{\mu}_{ij}(x)+\hat{\nu}_{ij}(x) \leq 1,$$

then $\hat{\mu}_{ij} \leq \hat{\nu}_{ij} \leq 1-\hat{\nu}_{ij} \leq 1-\hat{\nu}_{ij}$. Based on Definition 3.3 and motivated by the models for deriving priority weights based on interval fuzzy preference relations in [19], we define consistent interval-valued intuitionistic preference relations as follows.

Definition 3.4. Let $\hat{R}=(\hat{r}_{ij})_{n\times n}$ be an interval-valued intuitionistic preference relation, if there exists a normalized interval weight vector $\overline{\omega}=(\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n)^T$, such that

$$\hat{\mu}_{ij} \leq 0.5(\alpha_i - \alpha_j + 1) \leq 1-\hat{\nu}_{ij},$$

$$\hat{\nu}_{ij} \leq 0.5(\alpha_i - \alpha_j + 1) \leq 1-\hat{\nu}_{ij},$$

$$i=1, 2, \ldots, n-1; \text{ } j=1, 2, \ldots, n$$

where $\overline{\omega}_i=[\alpha_i, \beta_i]$, $0 \leq \alpha_i \leq \beta_i \leq 1$, $i=1, 2, \ldots, n$, $\sum_{i=1}^n \alpha_i + \beta_i \leq 1$, $\alpha_i + \sum_{i=1}^n \alpha_i \geq 1$, $j=1, 2, \ldots, n$, then $\hat{R}$ is a consistent interval-valued intuitionistic preference relation. Otherwise, $\hat{R}$ is called an inconsistent interval-valued intuitionistic preference relation.

4. A Method for Generating Priority Weights

Priority weights of criteria obviously play an important role in a multiple criteria decision making process. Next, we shall develop some simple and practical goal programming models for deriving priority weights based on both consistent and inconsistent interval-valued intuitionistic preference relations:

(1) If $\hat{R}=(\hat{r}_{ij})_{n\times n}$ is a consistent interval-valued intuitionistic preference relation, then the interval weight vector $\overline{\omega}=(\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n)^T$ derived from $\hat{R}$ should satisfy (9) and (10). In general, the interval weight vector satisfying these two conditions is not unique, but each interval weight $\overline{\omega}_i, i=1, 2, \ldots, n$, should belong to an interval-valued range. As such, based on conditions (9) and (10), we establish the following four linear programming models:

$$(M-1) \quad \alpha_i^+ = \min \alpha_i, \alpha_i^- = \max \alpha_i, \alpha_i^- = \min \alpha_i \text{ and } \alpha_i^+ = \max \alpha_i, \text{ st } 0.5(\alpha_i - \alpha_j + 1) \leq \hat{\mu}_{ij}$$

$$i=1, 2, \ldots, n-1; \text{ } j=1, 2, \ldots, n$$

$$0.5(\alpha_i - \alpha_j + 1) \leq \hat{\nu}_{ij}$$

$$i=1, 2, \ldots, n-1; \text{ } j=1, 2, \ldots, n$$

$$0 \leq \alpha_i \leq 1$$

$$i=1, 2, \ldots, n$$

$$\sum_{i=1}^n \alpha_i + \beta_i \leq 1$$

$$j=1, 2, \ldots, n$$

$$\alpha_i + \sum_{i=1}^n \alpha_i \geq 1$$

$$j=1, 2, \ldots, n.$$ (9)

Solving model (M–1), we can obtain the bounds of the interval-valued weights as follows:

$$\theta_i=\{\overline{\omega}=(\overline{\omega}_1, \overline{\omega}_2, \ldots, \overline{\omega}_n)^T \text{ } | \text{ } \overline{\omega}=[\alpha_i, \beta_i], 0 \leq \alpha_i \leq \beta_i \leq 1, \text{ } \alpha_i \leq \alpha_i^+ \leq \alpha_i^-, \text{ } i=1, 2, \ldots, n \}$$

(10)

(2) If $\hat{R}=(\hat{r}_{ij})_{n\times n}$ is an inconsistent interval-valued intuitionistic preference relation, then (10) does not always hold. In this case, we relax (10) by introducing the deviation variables $d_{ij}, d_{ij}^+, e_{ij}^-$ and $e_{ij}^+$, for all $i=1, 2, \ldots, n-1; \text{ } j=1, 2, \ldots, n$:

$$\hat{\mu}_{ij} - d_{ij} \leq 0.5(\alpha_i - \alpha_j + 1) \leq 1-\hat{\nu}_{ij} + d_{ij}^+$$

$$i=1, 2, \ldots, n-1; \text{ } j=1, 2, \ldots, n$$

$$\hat{\nu}_{ij} - d_{ij} \leq 0.5(\alpha_i - \alpha_j + 1) \leq 1-\hat{\nu}_{ij} + e_{ij}^-$$

$$i=1, 2, \ldots, n-1; \text{ } j=1, 2, \ldots, n.$$ (11)

where $d_{ij}, d_{ij}^+, e_{ij}^-$ and $e_{ij}^+$ are nonnegative real numbers. Obviously, the smaller the deviation variables $d_{ij}, d_{ij}^+, e_{ij}^-$
and \( a^*_i \), the closer \( \tilde{R} \) is to a consistent interval-valued intuitionistic preference relation. Hence, we establish the following optimization model:

\[
J = \min_{i,j} \sum_{i,j} (d_{ij}^+ + d_{ij}^- + e_{ij}^+ + e_{ij}^-)
\]

\[s.t. \quad 0.5(a^-_i - a^-_j + 1) + d_{ij}^+ + e_{ij}^+ \geq \bar{\mu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0.5(a^-_i - a^-_j + 1) - d_{ij}^- \leq 1 - \bar{\mu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0.5(a^-_i - a^-_j + 1) + e_{ij}^+ \geq \bar{\nu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0.5(a^-_i - a^-_j + 1) - e_{ij}^- \leq 1 - \bar{\nu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0 \leq a^-_i \leq a^+_i \leq 1, \quad i = 1, 2, ..., n; \]

\[\sum_{i,j} a^-_i + a^+_i \leq 1, \quad j = 1, 2, ..., n; \]

\[a^-_i + \sum_{i,j} a^-_i \geq 1, \quad j = 1, 2, ..., n.\]

Solving this model, we get the optimal deviation values \( d_{ij}^+, d_{ij}^-, e_{ij}^+ \), \( e_{ij}^- \) and \( a^*_i \), we further establish the following optimization model:

\[(M - 3) \quad \omega_i = \min \omega_i, \quad \omega_i^- = \max \omega_i, \quad \omega_i^- = \min \omega_i^- \]

\[s.t. \quad 0.5(a^-_i - a^-_j + 1) + d_{ij}^+ \geq \bar{\mu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0.5(a^-_i - a^-_j + 1) - d_{ij}^- \leq 1 - \bar{\mu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0.5(a^-_i - a^-_j + 1) + e_{ij}^+ \geq \bar{\nu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0.5(a^-_i - a^-_j + 1) - e_{ij}^- \leq 1 - \bar{\nu}_{ij}, \quad i, j = 1, 2, ..., n; \]

\[0 \leq a^-_i \leq a^+_i \leq 1, \quad i = 1, 2, ..., n; \]

\[\sum_{i,j} a^-_i + a^+_i \leq 1, \quad j = 1, 2, ..., n; \]

\[\omega^-_i + \sum_{i,j} \omega^-_i \geq 1, \quad j = 1, 2, ..., n.\]

Solving the model \((M - 3)\), we can similarly obtain the bounds of interval-valued weights for an inconsistent interval-valued intuitionistic preference relation as follows:

\[\varnothing = \{\theta = (\bar{\mu}, \bar{\nu}, ..., \bar{\mu}) | \bar{\mu} = [\omega_i^-, \omega_i^+] \} \]

\[\omega_i^- \leq \omega_i^+ \leq \omega_i^- \leq \omega_i^+, i = 1, 2, ..., n \} \quad (13)\]

These analyses reveal that the weights derived from an interval-valued intuitionistic preference relation are in the form of bounded interval numbers, which can be expressed as IVIFNs:

\[\bar{\omega} = ([a^-_i, a^+_i], [1 - a^-_i, 1 - a^+_i]), i = 1, 2, ..., n. \quad (14)\]

After priority weights are derived as IVIFNs in \((14)\), one may use an appropriate approach such as that developed by Xu [13] or Wang et al. [15] to rank \( \theta_i (i = 1, 2, ..., n) \). Next two illustrative examples are developed to demonstrate how to apply the proposed approach to determine whether a given interval-valued intuitionistic preference relation is consistent and then, derive priority weights accordingly by using model \((M - 1)\) or \((M - 3)\) accordingly.

**Example 1:** Assume that a multiple criteria decision making problem consists of four criteria \( x_i (i = 1, 2, 3, 4) \). A DM provides its pairwise comparison of two criteria \( x_i \) and \( x_j \) as interval-valued intuitionistic fuzzy preference value \( r_{ij} = (\bar{\mu}_{ij}, \bar{\nu}_{ij}) = ([\bar{\mu}_{ij}^1, \bar{\mu}_{ij}^2], [\bar{\nu}_{ij}^1, \bar{\nu}_{ij}^2]) \), \( (i, j = 1, 2, 3, 4) \), where \( \bar{\mu}_{ij} = [\bar{\mu}_{ij}^1, \bar{\mu}_{ij}^2] \) provides an interval degree to which \( x_i \) is preferred to \( x_j \) and \( \bar{\nu}_{ij} = [\bar{\nu}_{ij}^1, \bar{\nu}_{ij}^2] \) gives an interval degree to which \( x_j \) is not preferred to \( x_i \). Assume further that the DM’s pairwise comparisons are given in the following interval-valued intuitionistic preference relation:

\[\tilde{R} = \{\bar{\omega}_{ij} \} = \]

\[\{ [0.50, 0.50], [0.25, 0.35], [0.45, 0.55], [0.35, 0.45], [0.50, 0.90], [0.55, 0.65], [0.25, 0.35], [0.50, 0.50], [0.15, 0.25], [0.25, 0.35], [0.15, 0.25], [0.50, 0.90], [0.45, 0.55], [0.35, 0.45], [0.15, 0.25], [0.50, 0.90] \}\]

Due to the space limit, each element in \( \tilde{R} \) has to be split into two rows. Therefore, \( \tilde{r}_{ij} = ([0.5, 0.5], [0.5, 0.5]) \), indicating that the DM is indifferent when criterion \( x_i \) is compared to itself. As a matter of fact, all diagonal elements assume the same interval-valued intuitionistic fuzzy numbers, \( \tilde{r}_{ii} = r_{ii} = r_{jj} = ([0.5, 0.5], [0.5, 0.5]) \). Similarly, \( \tilde{r}_{12} = ([0.25, 0.35], [0.55, 0.65]) \) implies that the DM has a [0.25, 0.35] margin of preferring \( x_1 \) to \( x_2 \) and a [0.55, 0.65] margin of not preferring \( x_1 \) to \( x_2 \). Other elements in \( \tilde{R} \) can be interpreted in a similar fashion.

Solving model \((M - 2)\), we have \( J = 0 \), indicating that there exists an optimal solution such that all deviation variables, \( d_{ij}^+, d_{ij}^-, e_{ij}^+ \) and \( e_{ij}^- \) \( (i = 1, 2, 3; j = i + 1, ..., 4) \) are.
equal to zero. Therefore, we can tell that $\tilde{R}$ is a consistent interval-valued intuitionistic preference relation. Then by the model (M-1) and (14), we have
\[
\tilde{\delta} = (([0.0, 0.1], [0.5, 0.75]), ([0.3, 0.4], [0.2, 0.525]),
([0.0, 0.15], [0.6, 0.85]), ([0.2, 0.325], [0.05, 0.775]))^T
\]

Example 2: Suppose that the DM provides its preference information over the four criteria $x_i$ ($i = 1, 2, 3, 4$) as shown in the following interval-valued intuitionistic preference relation:
\[
\tilde{R}(\tilde{c})_{ij} = 
\begin{bmatrix}
([0.50, 0.50], ([0.35, 0.45], ([0.45, 0.55], ([0.35, 0.45], ([0.35, 0.45], ([0.15, 0.25], ([0.50, 0.50], ([0.15, 0.25], ([0.50, 0.50])
\end{bmatrix}
\]

Solving the model (M-2), we have $J = 0.25$. Therefore, there does not exist any optimal solution such that Eq. (10) is satisfied without any positive deviation. By Definition 3.4, $\tilde{R}$ is an inconsistent interval-valued intuitionistic preference relation, and the corresponding optimal deviation values are as follows:
\[
\begin{align*}
\tilde{d}_{i1} &= \tilde{d}_{i2} = \tilde{d}_{i3} = \tilde{d}_{i4} = \tilde{d}_{i5} = 0, \\
\tilde{d}_{51} &= 0.15, \tilde{d}_{52} = \tilde{d}_{53} = \tilde{d}_{54} = \tilde{d}_{55} = 0, \\
\tilde{e}_{54} &= 0.05, \tilde{e}_{53} = \tilde{e}_{52} = \tilde{e}_{51} = \tilde{e}_{55} = 0, \\
\tilde{e}_{55} &= 0.05, \tilde{e}_{54} = \tilde{e}_{53} = \tilde{e}_{52} = \tilde{e}_{51} = 0.
\end{align*}
\]

Plugging these optimal deviation values into model (M-3), we can derive the priority weights as follows:
\[
\tilde{\delta} = (([0.0, 0.025], [0.7, 0.8]), ([0.4, 0.425], [0.3, 0.4]),
([0.0, 0.1], [0.8, 0.9]), ([0.3, 0.325], [0.6, 0.7]))^T
\]

5. Conclusions

This article first introduces the concept of consistent interval-valued intuitionistic preference relations. When the interval-valued intuitionistic preference relation provided by a DM is consistent, a linear programming based approach is established to determine priority weights for all criteria. When the interval-valued intuitionistic preference relation is inconsistent, a goal programming based method is designed to derive priority weights.

References


