State Estimation in Mobile Edge Computing with Unreliable Communications

Weisong Tian and Guodong Wang*

Abstract—**In this letter, we investigate the state estimation problem in Mobile Edge Computing with unreliable communications. A finite-time convergent estimator is proposed, aiming to restore the unmeasurable states within a finite time length. We apply the proposed estimator in two practical scenarios to evaluate its performance. The numerical results show that the proposed estimator is able to quickly make accurate estimations once the interrupted connection resumes. The results also suggest that the effect of short-term data loss is easier to remedy than that of long-term data loss, which shed light on relay deployment and future investigations in this field.**

Index Terms—**Mobile Edge Computing, Unreliable Communications, Wireless Communications, Nonlinear Systems, State Estimation.**

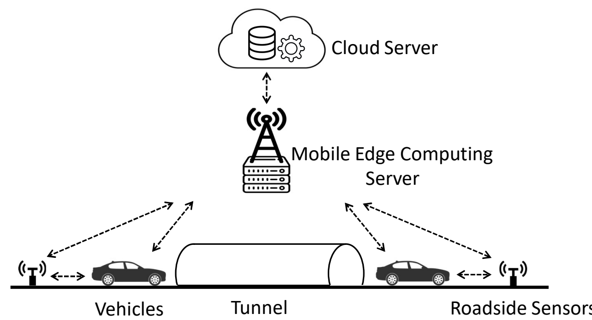

Fig. 1. Connected vehicles leveraged by mobile edge computing

I. Introduction

Driven by the visions of Internet of Things and 5G communications, Mobile Edge Computing (MEC) brings the capability of computing from cloud to the edge of networks, which resolves several vital challenges, such as bandwidth saturation, low latency transmission, and data security and privacy. MEC promises to significantly reduce latency and mobile energy consumption, and to meet the key challenges of achieving the 5G vision. It is estimated that MEC service scenarios such as connected vehicles, augmented reality, cloud gaming, and real-time drone detection will generate tens of billions of edge devices in the near future [1] [2].

To illustrate the advantages of MEC-enabled applications, we take the connected vehicle system as an example. As depicted in Fig. 1, a typical MEC-enabled connected vehicle system consists of cloud server, MEC servers, roadside sensors and vehicles. In this system, MEC server, roadside sensors and vehicles exchange critical safety and operational data in order to increase the safety, efficiency, and convenience of the transportation system. For example, the MEC server is able to inform adjacent MEC servers to propagate hazard warnings to vehicles that are close to the affected area. The communication between MEC servers and vehicles can also be used to provide valuable services, such as car finder, parking location and entertainment services, e.g., video distribution. In addition, the MEC servers are also able to send local information to the cloud server for further centralized processing and reporting.

Dr. Weisong Tian is with the Department of Electrical Engineering, Widener University, Chester, PA, 19013, USA.

Dr. Guodong Wang is with the Computer Science Department, Massachusetts College of Liberal Arts, North Adams, MA, 01247, USA. Corresponding author: Guodong Wang (email: guodong.wang@mcla.edu).

This work is supported in part by Widener University Provost's Grant and Massachusetts College of Liberal Arts' Grant.

MEC applications highly rely on the quality of wireless communications. However, it remains a challenge to achieve high reliable and error tolerance wireless communications due to the inherent instability and best-effort nature of wireless network [3]. Researchers have investigated the unreliable communications from various perspectives. Elbamby et al. [4] discussed the significance of control in MEC and the challenges of enabling ultra-reliable and low latency edge computing services for mission-critical applications. Lian and Xu [5] adopted finite horizon optimal control to achieve stabilization for networked control systems over unreliable communication channel. Ouyang et al. [6] investigated a decentralized control problem for linear systems with unreliable inter-controller communications. They identified the critical thresholds for the link failure probabilities and the optimal decentralized control strategies when all link failure probabilities are below their thresholds. Li et al. [7] investigated the challenges of ensuring vehicle safety under unreliable wireless network with limited bandwidth. Their strategy was to use a two-state Markov chain to model the channel fading in vehicular networks and develop an event triggered strategy so that the vehicle platooning system achieves stochastic L2 string stability while ensuring an efficient use of the limited communication bandwidth.

MEC-enabled vehicle systems are also subject to channel fading in metropolitan areas due to tall buildings and traffic tunnels (as shown in Fig. 1), which result in unreliable communications (more specifically, intermittent communications). In particular, travelling among tall buildings may result in short-term data loss, while the traffic tunnels may lead to long-term data loss. The unreliable communications will inevitably affect the control messages exchanged among MEC servers, vehicles and roadside sensors. Therefore, connected vehicle systems often call for quick state estimation recovery from an intermittently lost of feedback data. In other words, quick recovery/convergence of the estimated states is highly demanded in unreliable communications of MEC-enabled vehicle systems. In order to achieve a quick convergence, we propose to apply a finite-time convergent estimator to address the unreliable communications in MEC applications.

Our major contributions are summarized as follows: First, we formulate the problem and derive the system model. Second, we propose a finite-time convergent estimator that can accurately estimate all the system states by using limited output data. Third, we study a class of nonlinear systems in this letter, which, comparing with linear systems, are more general and practical, and also are more difficult to be stabilized. Last but not the least, numerical results show that the proposed finite-time convergent estimator is able to significantly reduce the convergence time in unreliable communications.

The rest of this letter is organized as follows: Section II introduces the system model; Section III illustrates the finite-time convergent estimator; Numerous results are presented in Section IV; Conclusions are made in Section V.

II. System Model

In this work, we consider the following class of nonlinear systems, which has the representative structure of chaotic synchronization systems. Such systems have been proposed to achieve security communications in MEC [8][9].

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1) \\ &\vdots \\ \dot{x}_{n-1} &= x_n + f_{n-1}(x_1, x_2, \cdots, x_{n-1}) \\ \dot{x}_n &= f_n(x_1, x_2, \cdots, x_n) \\ y &= x_1(t_k), \end{aligned} \tag{1}$$

where $f_i(x_1, \cdots, x_i)$ for $i = 1, 2, \cdots, n$, are continuously differentiable functions, and $X = (x_1, \cdots, x_n)^T$ is the system state. In System (1), among all the n states only the state x_1 is measured and delivered over the network. For instance, in order to locate one specific vehicle, we only need to measure and transmit its displacement to the MEC server. Other states, e.g., velocity and acceleration, can be estimated on the server. Such control structure can reduce unnecessary data transmission, so energy consumption will be reduced and valuable bandwidth will be saved in MEC-enabled systems.

For MEC-enabled applications, control messages as well as data packages are sampled and delivered through wireless networks. Since most of the physical control systems are operated in continuous-time domain, MEC-enabled control systems have the continuous-discrete hybrid structure. Therefore, in order to reflect the hybrid structure in System (1), we assume that the time stamp t_k is the beginning of a sampling period with a length of T, i.e. $t_{k+1} - t_k = T$. In this sampling period $t \in [t_k, t_{k+1})$, the system output y remains as a constant $x_1(t_k)$. At the beginning of the next sampling period, i.e. $t = t_{k+1}$, if the data is received, the output is updated as $y = x_1(t_{k+1})$ in the next sampling period $t \in [t_{k+1}, t_{k+2})$.

In addition, from the perspective of control systems, a purely random data package loss can be viewed as enlarging the sampling period. For MEC-enabled control systems with discrete-continuous time hybrid structure, a longer sampling period may decrease the control performance, but a more lethal challenge is the consecutive data loss. Consecutive data loss is dangerous for many control systems, especially nonlinear systems, because the outdated feedback signal may drive the nonlinear systems towards finite escape. In this work, we are mainly focused on the scenario when data packages are not received due to communication interruptions, or the delay is greater than the sampling period.

III. Finite-Time Convergent Estimator

In this section, we propose to apply the finite-time convergent estimator to the MEC-enabled systems over unreliable communications. The proposed estimator should be able to solve the unmeasurable states by using the system output. More importantly, under the threat of consecutive data loss, the proposed estimator must rapidly reduce the estimation error by using limited available output data.

Before we start our design of the estimator, we list the following lemmas that will be used in the mathematical proof.

Lemma III.1 ([10]). *Given a dilation weight $R = (r_1, r_2, \cdots, r_n)$, suppose $V_1(x)$ and $V_2(x)$ are respectively homogeneous functions of degree τ_1 and τ_2. Then $V_1(x)V_2(x)$ is also homogeneous with respect to the same dilation weight R with degree of $\tau_1 + \tau_2$.*

Lemma III.2 ([10]). *Assume that $V\colon \mathbb{R}^n \to \mathbb{R}$ is a homogeneous function of degree τ with respect to the dilation weight R. Then the following holds:*

- *$\frac{\partial V}{\partial x_i}$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.*
- *For a positive definite function $W(x)$, which is homogeneous of degree τ_1 with respect to the dilation weight R, there is a constant $\overline{c}$ such that $V(x) \le \overline{c} W^{\frac{\tau}{\tau_1}}(x)$.*

Next, we construct an estimator to estimate the system states by using the output data.

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + f_1(\hat{x}_1) + \lambda_k c_1 (y - \hat{x}_1)^{r_2} \\ &\vdots \\ \dot{\hat{x}}_{n-1} &= \hat{x}_n + f_{n-1}(\hat{x}_1, \cdots, \hat{x}_{n-1}) + \lambda_k c_{n-1} (y - \hat{x}_1)^{r_n} \\ \dot{\hat{x}}_n &= f_n(\hat{x}_1, \cdots, \hat{x}_n) + \lambda_k c_n (y - \hat{x}_1)^{r_{n+1}}, \end{aligned} \tag{2}$$

where c_i and r_i, $i = 1, 2, \cdots, n$ are constants that will be chosen in later steps. $\lambda_k \in \{0, 1\}$ is determined by whether the system output is successfully received over the network: if the output is received, then $\lambda_k = 1$; otherwise, $\lambda_k = 0$.

By defining the error terms as $\delta_i = x_i - \hat{x}_i$, the error dynamics can be written as

$$\begin{aligned} \dot{\delta}_1 &= \delta_2 - \lambda_k c_1 (y - \hat{x}_1)^{r_2} + f_1(x_1) - f_1(\hat{x}_1) \\ &\vdots \\ \dot{\delta}_{n-1} &= \delta_n - \lambda_k c_{n-1} (y - \hat{x}_1)^{r_n} + f_{n-1}(X_{n-1}) - f_{n-1}(\hat{X}_{n-1}) \\ \dot{\delta}_n &= -\lambda_k c_n (y - \hat{x}_1)^{r_{n+1}} + f_n(X_n) - f_n(\hat{X}_n), \end{aligned} \tag{3}$$

where X_i and $\hat{X}_i$ represents the states $x_1, \cdots, x_i$, for $i = 1, 2, \cdots, n$, and their estimations, respectively.

The error dynamics (3) can be decomposed into two parts

$$\begin{aligned}\dot{\delta} &= \Delta_1 + \Delta_2 \\ &= \begin{pmatrix} \delta_2 - \lambda_k c_1 (y - \hat{x}_1)^{r_2} \\ \vdots \\ \delta_n - \lambda_k c_{n-1} (y - \hat{x}_1)^{r_n} \\ -\lambda_k c_n (y - \hat{x}_1)^{r_{n+1}} \end{pmatrix} + \begin{pmatrix} f_1(x_1) - f_1(\hat{x}_1) \\ \vdots \\ f_{n-1}(X_{n-1}) - f_{n-1}(\hat{X}_{n-1}) \\ f_n(X_n) - f_n(\hat{X}_n) \end{pmatrix}. \end{aligned} \quad (4)$$

Next, we consider the scenario when the data package is received, i.e. $\lambda_k = 1$.

Theorem III.1. *When the communication is successful, there exists a positive definite Lyapunov function, so that the error dynamics (4) can be finite-timely stabilized.*

Proof. By defining $\tau = -\frac{q}{p}$, where $p > q$, p is a positive odd integer and q is a positive even integer, one can choose the homogeneous dilation weight $R = \{r_1, \cdots, r_i\}$ following $r_i = 1 + (i-1)\tau$, for $i = 1, 2, \cdots, n+1$. It is noted that the choice of τ is not unique, as long as $r_1, \cdots, r_{n+1}$ are all positive real numbers.

In a sampling period, $t \in [t_k, t_{k+1})$, the system output $y = x_1(t_k)$ can be considered as the initial value of the function $x_1(t)$. Thus, for any t in this period, based on the Mean Value Theorem, we can find a constant $\gamma > 0$ and $\eta \in \mathbb{R}$, such that

$$y = x_1(t_k) = \gamma x_1(t) + \eta. \quad (5)$$

Thus, there exist a series of positive constants β_i, such that

$$|y - \hat{x}_1|^{r_i} \geq \beta_i |\delta_1|^{r_i} \quad (6)$$

By Lemma III.2, the error dynamic term Δ_1 is globally finite-time stable. Further, there exists a Lyapunov function V with degree of k with respect to the dilation weight R, such that

$$\dot{V}(\delta)|_{\Delta_1} \leq \frac{\partial V(\delta)}{\partial \delta} \begin{pmatrix} \delta_2 - c_1 \beta_1 \delta_1^{r_2} \\ \vdots \\ \delta_n - c_{n-1} \beta_{n-1} \delta_1^{r_n} \\ -c_n \beta_n \delta_1^{r_{n+1}} \end{pmatrix} := -\omega(\delta), \quad (7)$$

where $\omega(\delta)$ is a homogeneous positive definite function with degree of $k + \tau$ with respect to R.

On the other hand,

$$\dot{V}(\delta)|_{\Delta_2} \leq \sum_{i=1}^{n} \left| \frac{\partial V}{\partial \delta_i} \right| |f_i(X_i) - f_i(\hat{X}_i)| \leq c\omega(\delta), \quad (8)$$

where c is positive constant. Note that the proof of Inequality (8) shares a similar scheme as in the previous work [11], and hence, is omitted in this letter.

Now, we can always choose a proper series of constants $c_1, \cdots, c_n$ in (7), to guarantee that the following inequality holds for a positive constant $\tilde{c}$

$$\dot{V}|_{(4)} = -\tilde{c}\omega(\delta). \quad (9)$$

Since $\omega(\delta)$ is homogeneous with degree of $k + \tau$, and the Lyapunov function V is homogeneous with degree of k both with respect to R, by Lemma III.1 and III.2, there exists a positive constant ξ, such that

$$\dot{V}|_{(4)} = -\xi V(\delta)^{\frac{k+\tau}{k}}. \quad (10)$$

Note that in (10), since $\tau < 0$, it is easy to see that the power $0 < \frac{k+\tau}{k} < 1$. This implies the error dynamics (4) is finite-time stable when $\lambda_k = 1$, i.e. the output data is received over the network. □

Remark III.1. *The finite-time stability result (10) requires the errors δ_i to be small. To guarantee the small errors, the following saturation function can be adopted:*

$$sat(\frac{\hat{x}_i}{M}) = \begin{cases} 1, & \text{for } \frac{\hat{x}_i}{M} \geq 1 \\ \frac{\hat{x}_i}{M}, & \text{for } -1 < \frac{\hat{x}_i}{M} < 1 \\ -1, & \text{for } \frac{\hat{x}_i}{M} \leq -1. \end{cases} \quad (11)$$

By using the saturation function (11), one can adjust the constant M, where $|x_i| \leq M$, so that the error

$$\delta_i = x_i - M sat(\frac{\hat{x}_i}{M}) \quad (12)$$

can be bounded within a small range.

Remark III.2. *In the case of communication interruption, i.e., $\lambda_k = 0$, the estimator will lose its correction term, and may lead to inaccurate results. However, the saturation function will still be able to bound the error within a limited range. Thanks to the nice feature of finite-time convergence, the estimator shall be able to correct the inaccuracy within a limited amount of time as soon as the communication resumes.*

IV. Numerical Results

In this section, we evaluate the performance of our proposed finite-time convergent estimator by considering the following Duffing equation, a typical chaotic synchronization system that can be applied to achieve secure communications in MEC [8][9][12].

$$\begin{aligned} &\dot{x}_1 = x_2, \quad \dot{x}_2 = 1.8x_1 - 0.1x_2 - x_1^3 + 1.1\cos(t), \\ &y = x_1. \end{aligned} \quad (13)$$

Two practical unreliable communication scenarios are considered: the short-term signal interruption and the long-term signal interruption. The first scenario may be resulted by tall building channel fading, while the second may occur when vehicles going through traffic tunnels. We compare the proposed finite-time convergent estimator with three representative schemes: the conventional linear estimator, conventional Kalman filter (KF) and the extended Kalman filter (EKF).

Example IV.1. *In this example, we simulated the state estimation problem over an unreliable communication network with short-term connection interruptions. With a sampling period $T = 10$ ms and in a total of 10-second simulation, we assumed the communication broke during the time intervals of $t \in [1, 3)$ and $t \in [4, 6)$ seconds. As depicted in Fig. 2, the conventional KF had the poorest performance mainly because when the system states exited the compact neighborhood around the nominal operating point, the linearization became inaccurate. The EKF had a significant improvement in handling nonlinear systems compared with the conventional KF. However, our proposed estimator obtained the fastest convergent speed. In particular, in the period of $t \in [3, 4)$ seconds, the broken communication has been temporarily resumed for 1 second.*

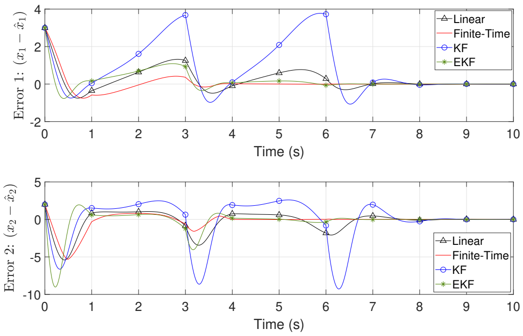

Fig. 2. Comparison of estimation errors: two short-term data losses

Our estimator successfully eliminated the estimation error within this window, while the conventional linear estimator and the EKF, which converged asymptotically, only slightly reduced the estimation error in this period and apparently needed longer time to converge. This simulation indicated that a short-term data loss can be quickly recovered using our estimator.

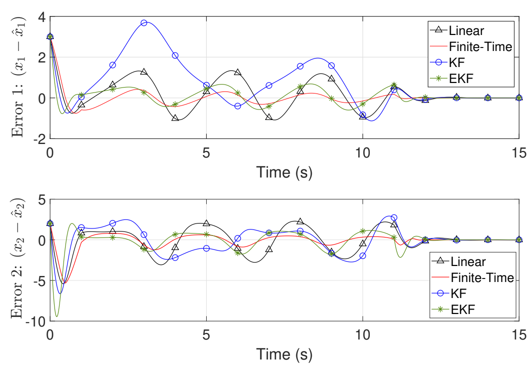

Fig. 3. Comparison of estimation errors: a long-term data loss

Example IV.2. *Besides the short-term connection interruption, we also considered the long-term connection interruption scenario in this example. Using the same system as in Example IV.1, we investigated the estimator performance under a longer period of consecutive data package loss. In particular, we assumed the communication interruption occurred in the period of* $t \in [1, 11)$ *seconds. As depicted in Fig. 3, our finite-time convergent estimator performed better than other schemes thanks to its fast convergent speed that bounded the estimation error in a smaller range. When communication resumed after* $t = 11$ *second, our estimator yielded the fastest convergent rate, which is similar to the results in Example IV.1.*

Remark IV.1. *In addition to the above results, we can also learn from Fig. 2 and Fig. 3 that a longer consecutive data loss made a greater harm to the stability of nonlinear systems. The numerical results as well as the findings inspire some further research topics, e.g., how to deploy a limited number of communication relays/sensors inside traffic tunnels in order to reduce cost while maintaining system performance. This topic will be conducted in the future.*

V. Conclusion

By bringing cloud computing capabilities to local radio access networks, MEC is able to reduce latency, ensure highly efficient network operation and service delivery, and enhance user experience. Therefore, scenarios like connected vehicles, augmented reality, cloud gaming, and real-time drone detection will generate numerous edge devices in the near future. The success of MEC greatly relies on the quality of wireless communications, where the existence of channel fading and signal interruption may result in unreliable communications. We investigated the unreliable communication problem in MEC and proposed a finite-time convergent estimator to accelerate the convergence rate of the estimation error in this letter. We found that the proposed estimator converges quickly and is efficient in restoring the interrupted data flow. We also found that it took more time to restore the long-term data loss in comparison with the short-term data loss. The numerical results as well as the findings in this letter shed light on strategies of relay deployment, which is also a significant topic for researchers to study in the future.

References

[1] Y. Mao, C. You, J. Zhang, K. Huang, and K. B. Letaief, "A survey on mobile edge computing: The communication perspective," *IEEE Communications Surveys & Tutorials*, vol. 19, no. 4, pp. 2322–2358, 2017.

[2] Z. Ning, P. Dong, X. Kong, and F. Xia, "A cooperative partial computation offloading scheme for mobile edge computing enabled internet of things," *IEEE Internet of Things Journal*, vol. 6, no. 3, pp. 4804–4814, 2018.

[3] S. Kucera, K. Fahmi, and H. Claussen, "Latency as a service: Enabling reliable data delivery over multiple unreliable wireless links," *2019 IEEE 90th Vehicular Technology Conference (VTC2019-Fall)*, pp. 1–5, 2019.

[4] M. S. Elbamby, C. Perfecto, C.-F. Liu, J. Park, S. Samarakoon, X. Chen, and M. Bennis, "Wireless edge computing with latency and reliability guarantees," *Proceedings of the IEEE*, vol. 107, no. 8, pp. 1717–1737, 2019.

[5] X. Liang and J. Xu, "Control for networked control systems with remote and local controllers over unreliable communication channel," *Automatica*, vol. 98, pp. 86–94, 2018.

[6] Y. Ouyang, S. M. Asghari, and A. Nayyar, "Optimal infinite horizon decentralized networked controllers with unreliable communication," *IEEE Transactions on Automatic Control*, 2020.

[7] Z. Li, B. Hu, M. Li, and G. Luo, "String stability analysis for vehicle platooning under unreliable communication links with event-triggered strategy," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 3, pp. 2152–2164, 2019.

[8] C. Peng, J. Chen, M. S. Obaidat, P. Vijayakumar, and D. He, "Efficient and provably secure multi-receiver signcryption scheme for multicast communication in edge computing," *IEEE Internet of Things Journal*, 2019.

[9] C. Zhou, Q. Liu, and R. Zeng, "Novel defense schemes for artificial intelligence deployed in edge computing environment," *Wireless Communications and Mobile Computing*, vol. 2020, 2020.

[10] C. Qian, W. Lin, and W. Zha, "Generalized homogeneous systems with applications to nonlinear control: A survey," *Mathematical Control & Related Fields*, vol. 5, no. 3, pp. 585–611, 2015.

[11] W. Tian, H. Du, and C. Qian, "A semi-global finite-time convergent observer for a class of nonlinear systems with bounded trajectories," *Nonlinear Analysis: Real World Applications*, vol. 13, no. 4, pp. 1827–1836, 2012.

[12] A. A. Zaher, "Duffing oscillators for secure communication," *Computers & Electrical Engineering*, vol. 71, pp. 77–92, 2018.