Dynamic Modeling and Vibration Control of a Flexible Satellite

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In this paper, the modeling and vibration control problem of a satellite with two flexible solar panels are addressed. Symmetric flexible solar panels attached to the center body are used to represent the dynamics of a flexible satellite system. The left and the right panels are modeled as two Euler-Bernoulli beams, and the main body of the satellite is modeled as a lumped mass in the center of two panels. The single-point control input is applied at the center body to suppress the vibrations of both panels. Based on the construction of a physically motivated Lyapunov function, exponential stability is proved with the proposed control. Both the control design and the stability analysis are based on the original infinite-dimensional dynamic equations. Numerical examples illustrate the effectiveness of the proposed control scheme.

Manuscript received December 11, 2013; revised September 4, 2014; released for publication October 11, 2014.

DOI. No. 10.1109/TAES.2014.130804.

This work was supported by the National Natural Science Foundation of China under Grant 61203057, 61403063, the National Basic Research Program of China (973 Program) under Grant 2014CB744206, the Fundamental Research Funds for the China Central Universities of UESTC under Grant ZYGX2013Z003 and Y03001023601034, and the Foundation of Shanghai Key Laboratory of Deep Space Exploration Technology under Grant DS201504-001.

Refereeing of this contribution was handled by J. Li.

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I. INTRODUCTION

Satellites have gained considerable interest in the past decades as a result of applications in communication, remote sensing, etc. Under a complex environment, space mission constraints have pushed demands such as lighter weight structure, limitation of mass, low energy consumption, and reduced launch cost. A number of satellites with a rigid hub and long flexible solar panels are used in space missions. However, due to the flexible property of solar panels, the deflection of the flexible panels has a significant influence on the dynamics and control performance of the satellites. Therefore, vibration suppression is an important research topic related to flexible spacecraft. Recently, a number of approaches have been developed for the vibration control of flexible satellites, including positive position control [1], sliding mode control [2, 3], linear quadratic regulator control [4], and adaptive control [5]. In [5], the vibration stabilization problem is addressed for a flexible spacecraft described by a cantilever flexible beam by using adaptive output feedback sliding-mode control techniques. In [6], a novel control strategy combining both the command input shaping and the sliding mode output feedback control techniques is proposed to suppress the vibrations of a flexible spacecraft. However, the papers mentioned above consider only one flexible panel, and the control design is based on the convention of changing the original partial differential equations (PDEs) into ordinary differential equation (ODEs) by using spatial discretization.

For a highly flexible satellite, the flexibility effect should be directly accommodated into the control design. The control of flexible structures described by hybrid PDEs-ODEs need to provide the control effort to suppress vibrations [7–15]. Thus, many conventional control methods for ODE systems cannot be directly used for flexible structures. Mathematically, flexible structures are infinite dimensional systems [16–20]. One of the control strategies is the discretization of original PDE model into a system of finite dimensional ODEs by neglecting the higher frequency modes. However, due to the high dimensionality of the original model, the model reduction will lead to spillover instability [21, 22], which should be avoided in the control design. In recent years, there are significant research efforts for flexible structures where the control design is based on the original distributed parameter systems [23–26]. In [23], a flexible marine riser is modeled as an Euler-Bernoulli beam, and boundary control at the top of the riser is proposed for suppressing the vibrations of the riser. In [24], adaptive boundary control is proposed to regulate the moving beam with varying traveling speed. In [25], a complete framework of dynamical analysis and control design is developed for various marine mechanical systems such as flexible riser system, installation system, and mooring system. In [26], adaptive boundary control is designed at the top and bottom boundaries of the riser to position the subsea payload to the desired set point and suppress the vibration

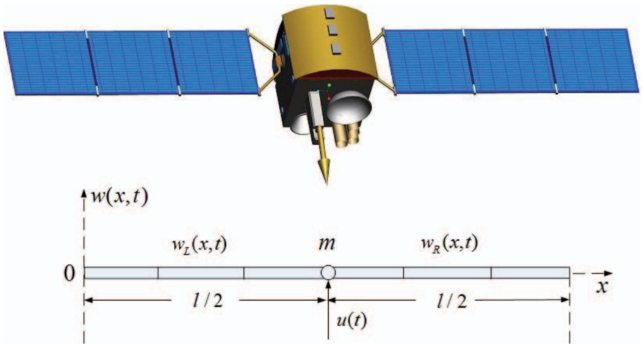

Fig. 1. Diagram depicting satellite with flexible panels.

of the riser. However, the model of flexible satellite with two solar panels is different from the models in [23–26], and the previous control methods cannot be applied to the problem in this paper. A single Euler-Bernoulli beam is considered in [23–26], but the flexible satellite consists of two Euler-Bernoulli beams, making the control problem in this paper more difficult to handle compared to the previous works.

In this paper, we aim to deal with the active vibration suppression problem for flexible structures. The configuration of the flexible satellite with two solar panels is shown in Fig. 1. The effect of rotation angle for the flexible satellite is not considered. Two Euler-Bernoulli beams connected to a center body are used to model the dynamics of flexible satellite. The left and the right panels are modeled as two beams, and the center body of the satellite is modeled as a lumped mass in the center of two panels. The flexible satellite systems can be actually regarded as a free-free beam with a point load in the center. The structure dynamics of flexible satellite belongs to the distributed parameter systems described by hybrid PDEs-ODEs, shown in Section II. The control and control-related issues are presented through theoretical analysis and simulations. A single-point control input is proposed on the basis of the original distributed parameter system to control the deformation of both flexible panels. With the proposed control, the closed-loop system is exponentially stable via the Lyapunov's direct method. The control performance of the system is guaranteed by suitably tuning the control parameters.

The outline of the paper is as follows. In Section II, the dynamic model of flexible satellite and some lemmas are given for the subsequent development. Based on the Lyapunov stability theory, boundary control schemes are proposed to control the deformation of panels in Section III, where it is shown that the exponential stability of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate the performance of the proposed control in Section IV. The conclusion of this paper is presented in Section V.

II. PROBLEM FORMULATION

The satellite dynamic model is composed of a center body with two identical flexible panels. Hamilton's principle [27] is used to derive the equations of motion for the satellite, starting from the expression of the kinetic and potential energy of the system. Let $w_L(x, t)$ and $w_R(x, t)$ denote the transverse displacements of the left and right panels from their initial equilibrium position at position x and time t, respectively, $w(l/2, t)$ denotes the transverse displacements of the lumped mass, ρ is the density of the beam material, A is the cross-sectional area of the beam, E is Young's modulus, I is the area moment of inertia of the beam, γ_1 is the coefficient of viscous damping, m is the mass of the center body, and $u(t)$ is a single-point controller at the center body. The actuator is located at the center body to regulate the vibrations of two flexible panels.

The kinetic energy of the beam $E_k(t)$ can be represented as

$$\begin{aligned} E_k(t) = &\frac{1}{2}\rho A \int_0^{l/2} \left(\frac{\partial w_L(x,t)}{\partial t}\right)^2 dx \\ &+\frac{1}{2} m \left(\frac{\partial w(x,t)}{\partial t}\right)^2 \Bigg|_{x=l/2} \\ &+\frac{1}{2}\rho A \int_{l/2}^{l} \left(\frac{\partial w_R(x,t)}{\partial t}\right)^2 dx, \end{aligned} \tag{1}$$

where x and t represent the independent spatial and time variables, respectively. The potential energy $E_p(t)$ due to the bending can be obtained from

$$\begin{aligned} E_p(t) = &\frac{1}{2} EI \int_0^{l/2} \left(\frac{\partial^2 w_L(x,t)}{\partial x^2}\right)^2 dx \\ &+\frac{1}{2} EI \int_{l/2}^{l} \left(\frac{\partial^2 w_R(x,t)}{\partial x^2}\right)^2 dx. \end{aligned} \tag{2}$$

The virtual work done by damping on the system is represented by

$$\begin{aligned} \delta W_d = &-\int_0^{l/2} \gamma_1 \frac{\partial w_L(x,t)}{\partial t} \delta w_L(x,t) dx \\ &-\int_{l/2}^{l} \gamma_1 \frac{\partial w_R(x,t)}{\partial t} \delta w_R(x,t) dx. \end{aligned} \tag{3}$$

The virtual work done by the axial control force $u(t)$ that produces a transverse force for vibration suppression can be written as

$$\delta W_u(t) = u(t)\delta w(l/2, t). \tag{4}$$

Then, we have the total virtual work done on the system as

$$\delta W(t) = \delta W_d(t) + \delta W_u(t). \tag{5}$$

The variations of (1) and (2) are obtained as

$$\delta E_k(t) = \rho A \int_0^{l/2} \frac{\partial w_L(x,t)}{\partial t} \delta \frac{\partial w_L(x,t)}{\partial t} dx + m \frac{\partial w(x,t)}{\partial t} \delta \left. \frac{\partial w(x,t)}{\partial t} \right|_{x=l/2} + \rho A \int_{l/2}^{l} \frac{\partial w_R(x,t)}{\partial t} \delta \frac{\partial w_R(x,t)}{\partial t} dx, \quad (6)$$

$$\delta E_p(t) = EI \int_0^{l/2} \frac{\partial^2 w_L(x,t)}{\partial x^2} \delta \frac{\partial^2 w_L(x,t)}{\partial^2 x} dx + EI \int_{l/2}^{l} \frac{\partial^2 w_R(x,t)}{\partial x^2} \delta \frac{\partial^2 w_R(x,t)}{\partial x^2} dx, \quad (7)$$

and we further obtain

$$\int_{t_1}^{t_2} \delta E_k(t) dt = -\rho A \int_{t_1}^{t_2} \int_0^{l/2} \frac{\partial^2 w_L(x,t)}{\partial t^2} \delta w_L(x,t) dx dt - m \int_{t_1}^{t_2} \left. \frac{\partial^2 w(x,t)}{\partial t^2} \delta w(x,t) \right|_{x=l/2} dt - \rho A \int_{t_1}^{t_2} \int_{l/2}^{l} \frac{\partial^2 w_R(x,t)}{\partial t^2} \delta w_R(x,t) dx dt, \quad (8)$$

$$\begin{aligned} \int_{t_1}^{t_2} \delta E_p(t) dt = {} & EI \int_{t_1}^{t_2} \int_0^{l/2} \frac{\partial^4 w_L(x,t)}{\partial x^4} \delta w_L(x,t) dx dt + EI \int_{t_1}^{t_2} \int_{l/2}^{l} \frac{\partial^4 w_R(x,t)}{\partial x^4} \delta w_R(x,t) dx dt \\ & + EI \int_{t_1}^{t_2} \left[\frac{\partial^2 w_L(x,t)}{\partial x^2} \delta \frac{\partial w_L(x,t)}{\partial x} - \frac{\partial^3 w_L(x,t)}{\partial x^3} \delta w_L(x,t) \right] \Bigg|_0^{l/2} dt \\ & + EI \int_{t_1}^{t_2} \left[\frac{\partial^2 w_R(x,t)}{\partial x^2} \delta \frac{\partial w_R(x,t)}{\partial x} - \frac{\partial^3 w_R(x,t)}{\partial x^3} \delta w_R(x,t) \right] \Bigg|_{l/2}^{l} dt. \end{aligned} \quad (9)$$

Applying Hamilton's principle $\int_{t_1}^{t_2} \delta[E_k(t) - E_p(t) + W(t)] dt = 0$ [25, 27, 28], we obtain the following structure dynamics of the spacecraft with the governing equations as

$$\rho A \ddot{w}_L(x,t) + EI w_L''''(x,t) + \gamma_1 \dot{w}_L(x,t) = 0 \quad (10)$$

$\forall x \in [0, l/2]$, $t \in [0, \infty)$, and

$$\rho A \ddot{w}_R(x,t) + EI w_R''''(x,t) + \gamma_1 \dot{w}_R(x,t) = 0 \quad (11)$$

$\forall x \in [l/2, l]$, $t \in [0, \infty)$, and boundary conditions as

$$\mathrm{w}'_L(l/2,t) = \mathrm{w}'_R(l/2,t) = 0, \quad (12)$$

$$w_L''(0,t) = w_R''(l,t) = 0, \quad (13)$$

$$w_L'''(0,t) = w_R'''(l,t) = 0, \quad (14)$$

$$w_L(l/2,t) = w_R(l/2,t) = w(l/2,t), \quad (15)$$

$$m\ddot{w}(l/2,t) = EI w_L'''(l/2,t) - EI w_R'''(l/2,t) + u(t) \quad (16)$$

$\forall t \in [0, \infty)$.

REMARK 1 For clarity, notations $(\cdot)' = \partial(\cdot)/\partial x$ and $(\dot{\cdot}) = \partial(\cdot)/\partial t$ are used throughout this paper.

REMARK 2 Boundary condition (16) is a motion equation of the center body in the satellite system. $\ddot{w}(l/2,t)$ denotes acceleration of the center body, $EI w_L'''(l/2,t)$ describes the shear force from the left panel, $EI w_R'''(l/2,t)$ describes the shear force from the right panel, and $u(t)$ is the control force from the actuator.

LEMMA 1 Poincaré inequalities: For any $\phi(x,t)$ continuously differentiable on $[L_1, L_2]$, we have

$$\int_{L_1}^{L_2} [\phi(x,t)]^2 dx \le 2(L_2 - L_1)\phi^2(L_2,t) + 4(L_2 - L_1)^2 \int_{L_1}^{L_2} [\phi'(x,t)]^2 dx, \quad (17)$$

$$\int_{L_1}^{L_2} [\phi(x,t)]^2 dx \le 2(L_2 - L_1)\phi^2(L_1,t) + 4(L_2 - L_1)^2 \int_{L_1}^{L_2} [\phi'(x,t)]^2 dx. \quad (18)$$

PROOF Using integration by parts, we have

$$\begin{aligned} 2\int_{L_1}^{L_2} (x - L_1)\phi(x,t)\phi'(x,t) dx &= (x - L_1)\phi^2(x,t)|_{L_1}^{L_2} - \int_{L_1}^{L_2} [\phi(x,t)]^2 dx \\ &= (L_2 - L_1)\phi^2(L_2,t) - \int_{L_1}^{L_2} [\phi(x,t)]^2 dx. \end{aligned} \quad (19)$$

We further have

$$\begin{aligned} \int_{L_1}^{L_2} [\phi(x,t)]^2 dx &= (L_2 - L_1)\phi^2(L_2,t) - 2\int_{L_1}^{L_2} (x - L_1)\phi(x,t)\phi'(x,t) dx \\ &\le (L_2 - L_1)\phi^2(L_2,t) + \frac{1}{2}\int_{L_1}^{L_2} [\phi(x,t)]^2 dx + 2\int_{L_1}^{L_2} (x - L_1)^2 [\phi'(x,t)]^2 dx \\ &\le (L_2 - L_1)\phi^2(L_2,t) + \frac{1}{2}\int_{L_1}^{L_2} [\phi(x,t)]^2 dx + 2(L_2 - L_1)^2 \int_{L_1}^{L_2} [\phi'(x,t)]^2 dx. \end{aligned} \quad (20)$$

Then we obtain

$$\int_{L_1}^{L_2} [\phi(x,t)]^2 dx \le 2(L_2 - L_1)\phi^2(L_2, t) + 4(L_2 - L_1)^2 \int_{L_1}^{L_2} [\phi'(x,t)]^2 dx. \quad (21)$$

Inequality (18) is obtained in a similar fashion.

REMARK 3 From Poincaré inequalities (17) and (18), we further have

$$\begin{aligned}\int_{L_1}^{L_2} [\phi(x,t)]^2 dx \le\ & 2(L_2 - L_1)\phi^2(L_2, t) \\ & + 8(L_2 - L_1)^3 \phi'^2(L_2, t) \\ & + 16(L_2 - L_1)^4 \int_{L_1}^{L_2} [\phi''(x,t)]^2 dx,\end{aligned} \quad (22)$$

$$\begin{aligned}\int_{L_1}^{L_2} [\phi(x,t)]^2 dx \le\ & 2(L_2 - L_1)\phi^2(L_1, t) \\ & + 8(L_2 - L_1)^3 \phi'^2(L_1, t) \\ & + 16(L_2 - L_1)^4 \int_{L_1}^{L_2} [\phi''(x,t)]^2 dx.\end{aligned} \quad (23)$$

III. CONTROL DESIGN

The control objective is to propose an active control law to regulate the deformation of the two flexible panels. A single-point control force $u(t)$ is applied on the center body of the satellite. Consider the Lyapunov candidate function as

$$V(t) = V_1(t) + V_2(t) + \Delta(t), \quad (24)$$

where $V_1(t)$, $V_2(t)$, and $\Delta(t)$ are defined as

$$\begin{aligned}V_1(t) =\ & \frac{\beta}{2}\rho A \int_0^{l/2} [\dot{w}_L(x,t)]^2 dx + \frac{\beta}{2} EI \int_0^{l/2} [w_L''(x,t)]^2 dx \\ & + \frac{\alpha}{2}\gamma_1 \int_0^{l/2} [w_L(x,t)]^2 dx + \frac{\beta}{2}\rho A \int_{l/2}^{l} [\dot{w}_R(x,t)]^2 dx \\ & + \frac{\beta}{2} EI \int_{l/2}^{l} [w_R''(x,t)]^2 dx + \frac{\alpha}{2}\gamma_1 \int_{l/2}^{l} [w_R(x,t)]^2 dx,\end{aligned} \quad (25)$$

$$V_2(t) = \frac{\beta}{2} m S^2(t) + \frac{\beta k_p}{2} [w(l/2, t)]^2, \quad (26)$$

$$\Delta(t) = \alpha\rho A \int_0^{l/2} \dot{w}_L(x,t) w_L(x,t) dx + \alpha\rho A \int_{l/2}^{l} \dot{w}_R(x,t) w_R(x,t) dx, \quad (27)$$

where α and β are positive weighting constants, k_p is the control gain, and

$$S(t) = \frac{\alpha}{\beta} w(l/2, t) + \dot{w}(l/2, t). \quad (28)$$

$V_1(t)$ is bounded as

$$V_1(t) \ge \theta_1 \left[\int_0^{l/2} \left([\dot{w}_L(x,t)]^2 + [w_L(x,t)]^2 \right) dx + \int_{l/2}^{l} \left([\dot{w}_R(x,t)]^2 + [w_R(x,t)]^2 \right) dx \right], \quad (29)$$

where $\theta_1 = \min\left(\frac{\beta\rho A}{2}, \frac{\alpha\gamma_1}{2}\right) > 0$.
From the definition of $\Delta(t)$, we know $\Delta(t)$ is bounded as

$$\begin{aligned}|\Delta(t)| \le\ & \alpha\rho A \left[\int_0^{l/2} \left([\dot{w}_L(x,t)]^2 + [w_L(x,t)]^2 \right) dx + \int_{l/2}^{l} \left([\dot{w}_R(x,t)]^2 + [w_R(x,t)]^2 \right) dx \right] \\ \le\ & \theta_2 V_1(t),\end{aligned} \quad (30)$$

where $\theta_2 = \frac{\alpha\rho A}{\theta_1}$. Considering $\theta_1 > \alpha\rho A$, we have

$$0 \le \theta_4 V_1(t) \le V_1(t) + \Delta(t) \le \theta_3 V_1(t), \quad (31)$$

where $\theta_3 = 1 + \theta_2 > 1$ and $0 < \theta_4 = 1 - \theta_2 < 1$. Then considering the Lyapunov candidate function (24), we have

$$0 \le \lambda_2 [V_1(t) + V_2(t)] \le V(t) \le \lambda_1 [V_1(t) + V_2(t)], \quad (32)$$

where $\lambda_1 = \max(\theta_3, 1) = \theta_3$ and $\lambda_2 = \min(\theta_4, 1) = \theta_4$.

Differentiating $V(t)$ leads to

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{\Delta}(t), \quad (33)$$

where $\dot{V}_1(t)$ is given as

$$\begin{aligned}\dot{V}_1(t) =\ & \beta\rho A \int_0^{l/2} \dot{w}_L(x,t)\ddot{w}_L(x,t) dx \\ & + \beta EI \int_0^{l/2} w_L''(x,t)\dot{w}_L''(x,t) dx \\ & + \alpha\gamma_1 \int_0^{l/2} w_L(x,t)\dot{w}_L(x,t) dx \\ & + \beta\rho A \int_{l/2}^{l} \dot{w}_R(x,t)\ddot{w}_R(x,t) dx \\ & + \beta EI \int_{l/2}^{l} w_R''(x,t)\dot{w}_R''(x,t) dx \\ & + \alpha\gamma_1 \int_{l/2}^{l} w_R(x,t)\dot{w}_R(x,t) dx.\end{aligned} \quad (34)$$

Substituting the governing equations (10) and (11), we obtain

$$\dot{V}_1(t) = A_1(t) + A_2(t) + A_3(t) + A_4(t), \quad (35)$$

where

$$A_1(t) = -\beta EI \int_0^{l/2} \dot{w}_L(x,t) w_L''''(x,t)dx + \beta EI \int_0^{l/2} w_L''(x,t)\dot{w}_L''(x,t)dx, \quad (36)$$

$$A_2(t) = -\beta EI \int_{l/2}^{l} \dot{w}_R(x,t) w_R''''(x,t)dx + \beta EI \int_{l/2}^{l} w_R''(x,t)\dot{w}_R''(x,t)dx, \quad (37)$$

$$A_3(t) = -\beta\gamma_1 \int_0^{l/2} [\dot{w}_L(x,t)]^2 dx - \beta\gamma_1 \int_{l/2}^{l} [\dot{w}_R(x,t)]^2 dx, \quad (38)$$

$$A_4(t) = \alpha\gamma_1 \int_0^{l/2} w_L(x,t)\dot{w}_L(x,t)\,dx + \alpha\gamma_1 \int_{l/2}^{l} w_R(x,t)\dot{w}_R(x,t)\,dx. \quad (39)$$

Using integration by parts and boundary conditions (12) and (13), we have

$$\begin{aligned} A_1(t) &= -\beta EI\dot{w}_L(l/2,t)w_L'''(l/2,t) \\ &\quad + \beta EI\dot{w}_L(0,t)w_L'''(0,t) \\ &\quad + \beta EI\dot{w}_L'(l/2,t)w_L''(l/2,t) \\ &\quad - \beta EI\dot{w}_L'(0,t)w_L''(0,t) \\ &= -\beta EI\dot{w}_L(l/2,t)w_L'''(l/2,t), \end{aligned} \quad (40)$$

$$\begin{aligned} A_2(t) &= -\beta EI\dot{w}_R(l,t)w_R''(l,t) \\ &\quad + \beta EI\dot{w}_R(l/2,t)w_R'''(l/2,t) \\ &\quad + \beta EI\dot{w}_R'(l,t)w_R''(l,t) \\ &\quad - \beta EI\dot{w}_R'(l/2,t)w_R''(l/2,t) \\ &= \beta EI\dot{w}_R(l/2,t)w_R'''(l/2,t). \end{aligned} \quad (41)$$

Combining $A_1(t)$–$A_4(t)$ and applying boundary conditions (12), (14), and (15), we obtain the derivative of $V_1(t)$ as

$$\begin{aligned} \dot{V}_1(t) &\le -\gamma_1\beta \int_0^{l/2} [\dot{w}_L(x,t)]^2 dx \\ &\quad - \gamma_1\beta \int_{l/2}^{l} [\dot{w}_R(x,t)]^2 dx \\ &\quad + \alpha\gamma_1 \int_0^{l/2} w_L(x,t)\dot{w}_L(x,t)dx \\ &\quad + \alpha\gamma_1 \int_{l/2}^{l} w_R(x,t)\dot{w}_R(x,t)dx \\ &\quad - \beta\dot{w}(l/2,t)\left[EIw_L'''(l/2,t) - EIw_R'''(l/2,t)\right]. \end{aligned} \quad (42)$$

The derivative of $V_2(t)$ is given as

$$\dot{V}_2(t) = \beta m S(t)\dot{S}(t) + \beta k_p w(l/2,t)\dot{w}(l/2,t). \quad (43)$$

Using boundary condition (16), we have

$$\begin{aligned} \dot{V}_2(t) &= \beta S(t)[u(t) + EIw_L'''(l/2,t) - EIw_R'''(l/2,t) \\ &\quad + \frac{\alpha}{\beta} m\dot{w}(l/2,t)] + \beta k_p w(l/2,t)\dot{w}(l/2,t). \end{aligned} \quad (44)$$

Design the proposed control law as

$$u(t) = -kS(t) - \frac{\alpha}{\beta} m\dot{w}(l/2,t) - k_p w(l/2,t), \quad (45)$$

where $k > 0$ and $k_p > 0$ are the control gains. Using the above control, we have

$$\begin{aligned} \dot{V}_2(t) &= [\alpha w(l/2,t) + \beta\dot{w}(l/2,t)][EIw_L'''(l/2,t) \\ &\quad - EIw_R'''(l/2,t)] - k\beta S^2(t) - \alpha k_p [w(l/2,t)]^2. \end{aligned} \quad (46)$$

The derivative of $\Delta(t)$ is given as

$$\begin{aligned} \dot{\Delta}(t) &= \alpha\rho A \int_0^{l/2} \ddot{w}_L(x,t)w_L(x,t)dx \\ &\quad + \alpha\rho A \int_0^{l/2} [\dot{w}_L(x,t)]^2 dx \\ &\quad + \alpha\rho A \int_{l/2}^{l} \ddot{w}_R(x,t)w_R(x,t)dx \\ &\quad + \alpha\rho A \int_{l/2}^{l} [\dot{w}_R(x,t)]^2 dx. \end{aligned} \quad (47)$$

Substituting the governing equations (10) and (11), we obtain

$$\dot{\Delta}(t) = B_1(t) + B_2(t) + B_3(t), \quad (48)$$

where

$$B_1(t) = -\alpha EI \int_0^{l/2} w_L(x,t)w_L''''(x,t)dx - \alpha EI \int_{l/2}^{l} w_R(x,t)w_R''''(x,t)\,dx, \quad (49)$$

$$B_2(t) = \alpha\rho A \int_0^{l/2} [\dot{w}_L(x,t)]^2 dx + \alpha\rho A \int_{l/2}^{l} [\dot{w}_R(x,t)]^2 dx, \quad (50)$$

$$B_3(t) = -\alpha\gamma_1 \int_0^{l/2} w_L(x,t)\dot{w}_L(x,t)\,dx - \alpha\gamma_1 \int_{l/2}^{l} w_R(x,t)\dot{w}_R(x,t)\,dx. \quad (51)$$

Using integration by parts and boundary conditions (12), (14), and (15), we obtain

$$\begin{aligned} B_1(t) &= -\alpha w(l/2,t)\left[EIw_L'''(l/2,t) - EIw_R'''(l/2,t)\right] \\ &\quad - \alpha EI \int_0^{l/2} [w_L''(x,t)]^2 dx \\ &\quad - \alpha EI \int_{l/2}^{l} [w_R''(x,t)]^2 dx. \end{aligned} \quad (52)$$

Combining $B_1(t)$–$B_3(t)$, we obtain

$$\begin{aligned}\dot{\Delta}(t) \le & -\alpha w(l/2,t)\left[EIw_L'''(l/2,t) - EIw_R'''(l/2,t)\right] \\ & -\alpha EI\int_0^{l/2}[w_L''(x,t)]^2dx \\ & -\alpha EI\int_{l/2}^{l}[w_R''(x,t)]^2dx \\ & +\alpha\rho A\int_0^{l/2}[\dot{w}_L(x,t)]^2dx \\ & +\alpha\rho A\int_{l/2}^{l}[\dot{w}_R(x,t)]^2dx \\ & -\alpha\gamma_1\int_0^{l/2}w_L(x,t)\dot{w}_L(x,t)dx \\ & -\alpha\gamma_1\int_{l/2}^{l}w_R(x,t)\dot{w}_R(x,t)dx. \end{aligned} \tag{53}$$

Therefore, we have the derivative of the Lyapunov candidate function as

$$\begin{aligned}\dot{V}(t) \le & -(\gamma_1\beta - \alpha\rho A)\int_0^{l/2}[\dot{w}_L(x,t)]^2dx \\ & -(\gamma_1\beta - \alpha\rho A)\int_{l/2}^{l}[\dot{w}_R(x,t)]^2dx \\ & -\alpha EI\int_0^{l/2}[w_L''(x,t)]^2dx - \alpha EI\int_{l/2}^{l}[w_R''(x,t)]^2dx \\ & -k\beta S^2(t) - \alpha k_p[w(l/2,t)]^2. \end{aligned} \tag{54}$$

From inequalities (22) and (23) and applying boundary conditions (12) and (15), we have

$$\begin{aligned}\int_0^{l/2}w_L^2(x,t)dx & \le l[w_L(l/2,t)]^2 + l^3[w_L'(l/2,t)]^2 \\ & \quad + l^4\int_0^{l/2}[w_L''(x,t)]^2dx \\ & \le l[w(l/2,t)]^2 + l^4\int_0^{l/2}[w_L''(x,t)]^2dx, \end{aligned} \tag{55}$$

$$\begin{aligned}\int_{l/2}^{l}w_R^2(x,t)dx & \le l[w_R(l/2,t)]^2 + l^3[w_R'(l/2,t)]^2 \\ & \quad + l^4\int_{l/2}^{l}[w_R''(x,t)]^2dx, \\ & \le l[w(l/2,t)]^2 + l^4\int_{l/2}^{l}[w_R''(x,t)]^2dx. \end{aligned} \tag{56}$$

Then we can obtain the following inequalities

$$\begin{aligned}-\eta_1 l[w(l/2,t)]^2 \le & -\eta_1\int_0^{l/2}w_L^2(x,t)dx \\ & +\eta_1 l^4\int_0^{l/2}[w_L''(x,t)]^2dx, \end{aligned} \tag{57}$$

$$\begin{aligned}-\eta_2 l[w(l/2,t)]^2 \le & -\eta_2\int_{l/2}^{l}w_R^2(x,t)dx \\ & +\eta_2 l^4\int_{l/2}^{l}[w_R''(x,t)]^2dx, \end{aligned} \tag{58}$$

where η_1 and η_2 are positive constants. Then we obtain

$$\begin{aligned}\dot{V}(t) \le & -(\gamma_1\beta - \alpha\rho A)\int_0^{l/2}[\dot{w}_L(x,t)]^2dx \\ & -\left(\alpha EI - \eta_1 l^4\right)\int_0^{l/2}[w_L''(x,t)]^2dx \\ & -\eta_1\int_0^{l/2}w_L^2(x,t)\,dx - (\gamma_1\beta - \alpha\rho A) \\ & \times\int_{l/2}^{l}[\dot{w}_R(x,t)]^2dx \\ & -\left(\alpha EI - \eta_2 l^4\right)\int_{l/2}^{l}[w_R''(x,t)]^2dx \\ & -\eta_2\int_{l/2}^{l}w_R^2(x,t)dx - k\beta S^2(t) \\ & -\left(\alpha k_p - \eta_1 l - \eta_2 l\right)[w(l/2,t)]^2. \end{aligned} \tag{59}$$

We further have

$$\dot{V}(t) \le -\lambda_3[V_1(t) + V_2(t)], \tag{60}$$

where

$$\begin{aligned}\lambda_3 = \min & \left(\frac{2\gamma_1\beta - 2\alpha\rho A}{\beta\rho A}, \frac{2\alpha EI - 2\eta_1 l^4}{\beta EI}, \frac{2\alpha EI - 2\eta_2 l^4}{\beta EI},\right. \\ & \left.\frac{2\eta_1}{\alpha\gamma_1}, \frac{2\eta_2}{\alpha\gamma_1}, \frac{2k}{m}, \frac{2\alpha k_p - 2\eta_1 l - 2\eta_2 l}{\beta k_p}\right) > 0, \end{aligned} \tag{61}$$

Combining (32) and (60), we have

$$\dot{V}(t) \le -\lambda V(t), \tag{62}$$

where $\lambda = \lambda_3/\lambda_1$. From the above statement, the control design for the flexible panels subjected to external loads can be summarized in the following theorem.

THEOREM 1 For the dynamical system described by governing equations (10) and (11) and boundary conditions (12)–(15), under the proposed boundary control (45), if the initial conditions are bounded, then the closed-loop system is exponentially stable.

REMARK 4 In the real system, the distributed disturbance along the flexible satellite system is relatively small. Thus, we neglect the influence of the distributed disturbance in this paper. If there is a distributed disturbance in the flexible satellite system, we can only ensure uniform ultimate boundedness for the closed-loop system with the proposed boundary control.

REMARK 5 How to construct the control law from the Lyapunov function is the main issue of this paper. First, we can obtain the energy term $V_1(t)$ from the analysis in Section II. Because the Lyapunov candidate function $V(t)$ needs to be positive definite and $\dot{V}(t)$ satisfies $\dot{V}(t) \le -\lambda V(t)$, we design an auxiliary term $V_2(t)$ and a

TABLE I
Parameters of the Flexible Satellite System

Parameter	Description	Value
$l/2$	Length of panel	10 m
m	Mass of the center body	100 kg
ρ	Density of the material	2.700×10^3 kg/m^3
A	Cross-sectional area of the panel	0.12 m^2
E	Young's modulus	6.894×10^{10} N/m^2
I	Area moment of inertia of the panel	1.734×10^{-7} m^4
γ_1	Viscous damping	0.005 kg/(ms)

crossing term $\Delta(t)$. We can then design the control law $u(t)$ for vibration suppression and substitute it in $\dot{V}_2(t)$. In turn, substitute the governing equations and boundary conditions in $\dot{V}(t)$ and see what term should be added in the Lyapunov function $V(t)$ and control law $u(t)$ in order to satisfy $\dot{V}(t) \leq -\lambda V(t)$. After continuous revision and calculation of the Lyapunov function and control law, we can obtain the appropriate $V(t)$ and $u(t)$ to achieve the control objective.

IV. SIMULATION

Several numerical methods such as finite difference method, assumed mode method, finite element method, and Galerkin method can be used to discretize the PDE system for simulations. In this paper, we select the finite difference method to simulate the system performance with the proposed boundary control. It is hard to find the mode functions for the system described by (11)–(17) for the assumed modes method. For the Galerkin method, it is not easy to calculate the eigenvalues. Many research studies of PDE systems have performed simulations by using the finite difference method, for example, [29–35]. In this paper, by choosing the proper temporal and spatial step size to approximate the solution of the PDE model, the performance of the proposed control is well demonstrated via the finite difference method.

In order to verify the effectiveness of the proposed control schemes, simulations have been performed and presented in this section by using the finite difference method. It should be noted that the finite difference method is not used for the control law design, and spillover instability will not arise. Parameters of the system are listed in Table I.

The corresponding initial conditions of the flexible satellite are given as $w_L(x, 0) = -0.3x$, $w_R(x, 0) = 0.3x$, $\dot{w}_L(x, 0) = \dot{w}_R(x, 0) = 0$. Deformation of the flexible satellite without control (i.e., $u(t) = 0$) are given in Fig. 2. For the results presented here, we can observe that there are large vibrations along the two panels. Moreover, we can obtain $|w(x, t)|_{\max} = 3.4721$ m.

Fig. 3 shows the actual closed-loop profiles of evolution of $w(x, t)$ for the flexible satellite by using the proposed control derived in (45). The values of the control gains are given as $k = 1250$, $k_p = 37.5$, and weighting constants α and β are chosen as 1 and 100, respectively. It can be seen that the designed control scheme is able to

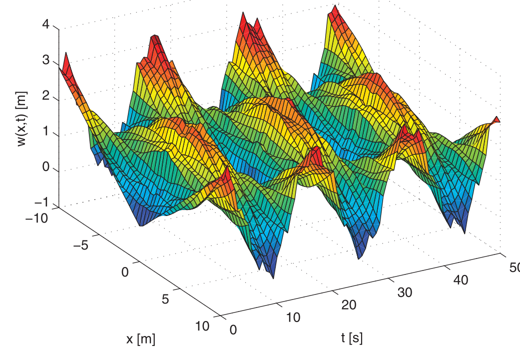

Fig. 2. Deformation of flexible satellite without control.

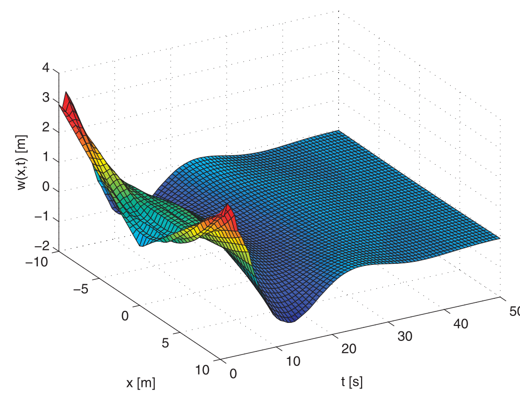

Fig. 3. Deformation of flexible satellite with proposed control.

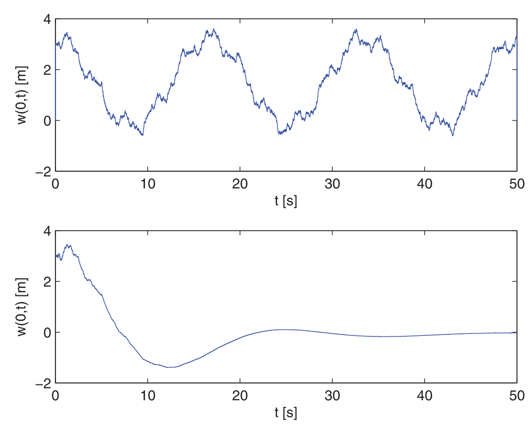

Fig. 4. Boundary displacement $w(0, t)$ of flexible satellite: without control and with control.

regulate the vibration greatly within 30 seconds and $w(x, t)$ numerically converge to the zero after 40 seconds, which means good convergence of the transverse vibration $w(x, t)$ can be achieved with the proposed control.

For comparison, the tip deformation of the flexible panels $w(0, t)$ and $w(l, t)$ are shown in Figs. 4 and 5, respectively. In addition, the transverse displacement of the center body $w(l/2, t)$ is shown in Fig. 6. It can be observed that the transverse displacements $w(0, t)$, $w(l/2, t)$, and $w(l, t)$ converge to zero, illustrating that control

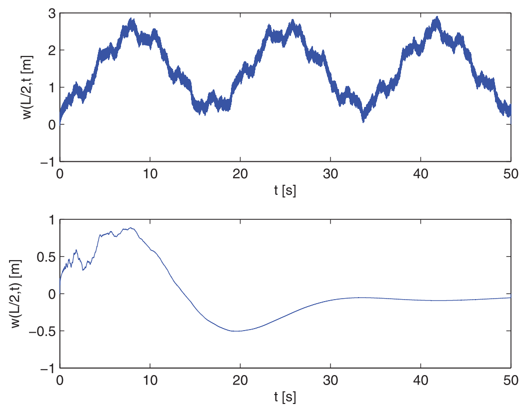

Fig. 5. Boundary displacement $w(l, t)$ of flexible satellite: without control and with control.

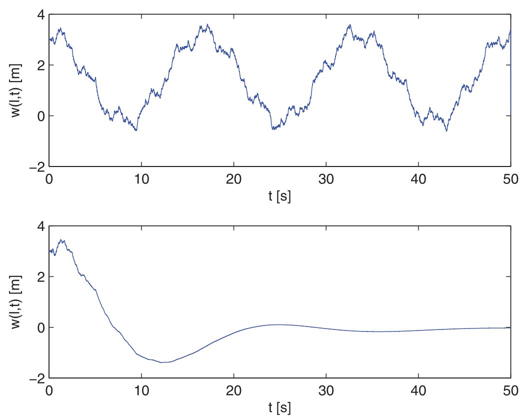

Fig. 6. Displacement of center body $w(l/2, t)$ of flexible satellite: without control and with control.

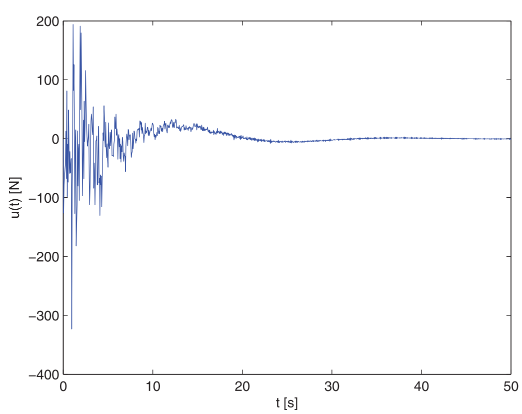

Fig. 7. Control input.

performance is ensured. Fig. 7 depicts the time histories of the control signal $u(t)$.

V. CONCLUSION

In this paper, the control problem of a satellite with flexible solar panels has been addressed by using a single-point control input. The panels with flexibility have been modeled as a distributed parameter system described by hybrid PDEs-ODEs. The control input has been proposed on the original PDE dynamics to suppress the vibrations of two panels. Then exponential stability has been proved by introducing a proper Lyapunov function. The effectiveness of the proposed control has been verified by simulations.

In this paper, we have addressed the vibration problems of a flexible satellite system in one-dimensional space. Future work includes control design for a flexible satellite system in three-dimensional space. In addition, we plan to extend the proposed control method for the flexible satellite system with input saturations.

ACKNOWLEDGMENT

The authors would like to sincerely thank the Editor-in-Chief, the Associate Editor, and the anonymous reviewers for their constructive comments that helped improve the quality and presentation of this paper.

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