

## DETERMINING OPTIMAL NON-UNIFORM DAMPING COEFFICIENTS FOR ADJACENT BUILDINGS VIA A NELDER-MEAD APPROACH

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### ABSTRACT

Passive coupling of adjacent buildings using dampers is known to be an effective method to reduce seismic induced pounding damage. Many damping devices, such as visco-elastic, viscous fluid, and MR dampers, etc., have been introduced and shown to be effective. Many studies have investigated efficiency and applicability of a specific device. However, few papers studied *optimization of mechanical characteristics of connectors* to achieve the best performance of the system. In most papers, damping coefficients of dampers are assumed to be uniform throughout the building. This paper aims to assume a general case of non-uniform damping coefficient for the connectors and find the optimal values for damping coefficients. Since the derivatives of objective function to damping coefficients are not known, to optimize damping coefficients the Nelder-Mead method is employed. Each building is modeled as a multi-degree of freedom dynamic system consisting of lumped-masses, adjacent linear springs and dampers. Adjacent buildings are connected to each other at all floors using linear viscous dampers. Dynamic analysis of the system is done in frequency domain using a pseudo-excitation based on Kanai-Tajimi spectrum. It is shown that the assumption of uniform dampers is not reliable and removing this assumption can generate significant improvement in coupling effectiveness.

**Keywords:** Coupled structures, Earthquake damage control, viscous dampers, Derivative-free optimization, Nelder-Mead.

### INTRODUCTION

In most metropolitan areas, limited land availability and high demand for residency and office buildings, leads high-rise buildings constructed in close proximity. During an earthquake the effect of pounding of two such adjacent buildings is highly destructive. The pounding effect and frequency becomes even more pronounced in dense urban centers. For example, severe damage has been observed in Mexico City earthquake (1985), Loma Prieta earthquake (1989), the Kobe earthquake (1994), the Bhuj earthquake (2001), and the recent New Zealand earthquake (2011).

Many studies have introduced and investigated different dampers (visco-elastic, viscous fluid, MR dampers, etc.) in order to prevent pounding induced damage, while increasing seismic resistance of the structure [1,2]. Experimental studies also confirmed efficiency of damper connectors to mitigate vibrations of structures [3,4]. Disregarding the type of damping device, it is common to assume that all adjacent floors of adjacent buildings are connected using identical dampers [1,5,6].

Some preliminary studies investigated non-uniform damping coefficients of connectors for adjacent buildings [7]. However, they did not present an optimization method to find optimal damping coefficients. In [7], to find non-uniform damping coefficients, the authors assume a simple linear function along the building. This assumption makes an intuitive sense, as the force generated in viscous dampers is a linear function of relative velocity. However, (to the best of our knowledge) no research has examined the use of mathematical optimization on the optimal damping coefficients of viscous dampers.

In this study, we, for the first time, employ a derivative-free method (specifically the Nelder-Mead (NM) method) to

find the optimal damping coefficients of damper connectors. The NM method is a direct search method based on evaluations of the objective value at a finite number of points. In order to examine efficiency of the presented method, various numerical examples are presented. It is shown that optimal damping coefficients is non-uniform and assumption of uniformity of dampers imposes an additional constraint to the problem that prevents the finding of the optimal damper coefficients.

## DYNAMIC MODEL

The dynamic model used in this study is in line with past studies [1]. The ground motion and dynamic response of the buildings are assumed to be unidirectional. In order to prevent the effect of torsional vibrations of the buildings, both buildings are assumed to be symmetric and their centers of mass are located in the same plane. Figure 1 shows that each building is modeled as a multi-degree of freedom system consisting of lumped masses, representing mass for each floor, linear spring, representing stiffness of columns, and, linear viscous damper. In order to connect adjacent building, both buildings are assumed to have the same floor elevations. However, the height of each building does not need to match. Damper connectors are modeled as viscous dampers with various damping coefficient for each floor.

Let  $x_{i1}(t)$  and  $x_{i2}(t)$  be the displacement in time domain of the  $i^{th}$  floor of buildings 1 and 2, respectively. Consequently,  $\dot{x}_{i1}(t)$  and  $\dot{x}_{i2}(t)$  represent the velocity, and  $\ddot{x}_{i1}(t)$  and  $\ddot{x}_{i2}(t)$  represent the acceleration of the  $i^{th}$  floor of buildings 1 and 2, respectively. To find a general and reliable objective function, the model is analyzed in pseudo-excitation frequency domain. Therefore, displacement, velocity and acceleration as functions of time are transformed into functions of frequency  $\omega$ :  $x_{i1}(\omega)$ ,  $x_{i2}(\omega)$ ,  $\dot{x}_{i1}(\omega)$ ,  $\dot{x}_{i2}(\omega)$ ,  $\ddot{x}_{i1}(\omega)$ , and  $\ddot{x}_{i2}(\omega)$ . As before, the first index corresponds to the floor number and the second index corresponds to the building number. The governing equation of the dynamic system can be written as:

$$\mathbf{M}\ddot{\mathbf{X}} + (\mathbf{C} + \mathbf{Cd})\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}(t), \quad (1)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are mass, damping and stiffness matrices of the uncoupled buildings, respectively;  $\mathbf{Cd}$  is the damping matrix of the connectors; and  $\mathbf{F}(t)$  is the equivalent base excitation force which is applied to each floor. The governing equation of the system is solved in frequency domain where the excitation is determined based on Kanai-Tajimi spectrum [1].

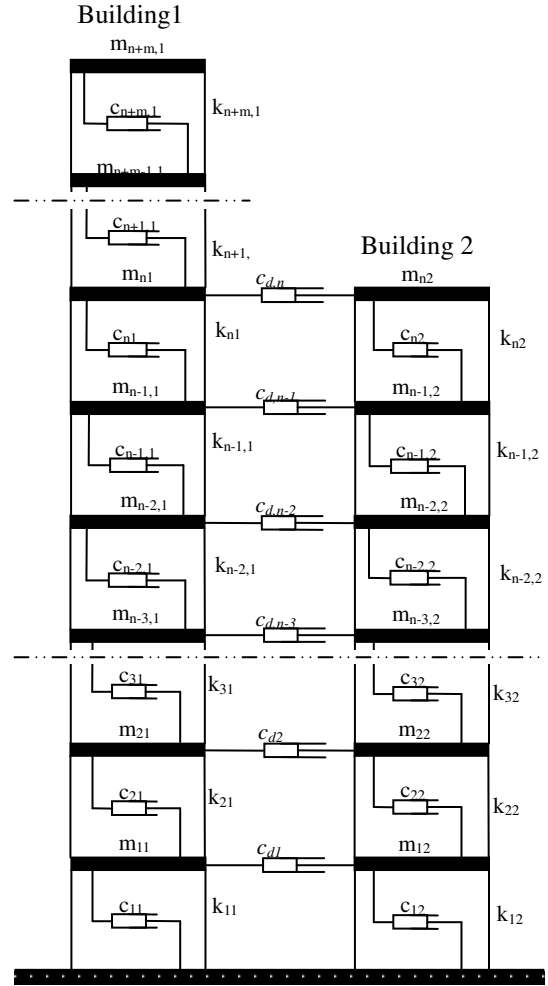


Figure 1 - Schematic model of buildings

Since calculated parameters in frequency domain are complex values, the squared magnitude, known as the auto-spectral density, is used for optimization. For example, auto-spectral density of displacement for the first floor of building 2 is written as:

$$S_{x_{12}}(\omega) = x_{12}(\omega) \bullet \text{conj}(x_{12}(\omega)) \quad (2)$$

Subsequently, applying integration over frequency domain will result in statistical values that represent standard deviations of displacement, velocity and acceleration, for each floor. For instance, for the first floor of Building 1, the standard deviation of displacement response is

$$\sigma_{x_{11}} = \left[ \int_{-\infty}^{+\infty} S_{x_{11}}(\omega) d\omega \right]^{1/2} \quad (3)$$

In this study, the objective function is defined as the maximum value of squared inter-floor drift:

$$\max \left\{ \begin{array}{l} \sigma_{di1}^2 : i = 1..n+m \\ \sigma_{di2}^2 : i = 1..n \end{array} \right\}, \quad (4)$$

where  $\sigma_{dib}$  is the standard deviation of inter-floor drifts for  $i^{\text{th}}$  floor of building  $b$ , and defined as:

$$\sigma_{dib} = \left[ \int_{-\infty}^{+\infty} S_{dib}(\omega) d\omega \right]^{1/2} \quad (5)$$

with

$$S_{dib} = \left\{ x_{ib}(\omega) - x_{(i-1)b}(\omega) \right\} \bullet \text{conj} \left( x_{ib}(\omega) - x_{(i-1)b}(\omega) \right) \quad (6)$$

The final output of the system can be defined as real value that is a function of damper coefficients, i.e. objective function:  $\left\{ \begin{array}{l} \text{DamperCoeff.} \rightarrow \max(\text{drift}^2) \\ \mathfrak{R}^n \rightarrow \mathfrak{R} \end{array} \right\}$ ,

where  $n$  is the height of the shorter building.

## OPTIMIZATION APPROACH

As explained in the previous section the objective value representing performance of the system is not an analytic function of the design variables. Instead, a simulation is performed for each set of damping coefficient in order to determine the objective value. As such, optimization methods that are based on the derivatives of the objective function cannot be employed in this problem. To solve an optimization problem of several design variables, where the derivatives are unknown, a derivative-free optimization (DFO) method can be used. One classical DFO method, the Nelder-Mead (NM) method, was introduced in 1965 [8]. The NM method has been used to analyze many engineering optimization problems [9] where the derivatives are unknown. This paper employs the NM method to solve the optimization problem of non-uniform damping coefficients for damper connectors between two adjacent buildings. The NM method is a direct search method based on finite evaluations of the model at certain vertices of a simplex. A simple and concise outline of the NM method can be found in [10].

## RESULTS

In order to illustrate the applicability and efficiency of the above methods and formulations, various examples are studied. Table 1 shows three different sets of mechanical properties of buildings used for numerical tests. It should also be noted that, for all numerical examples, ground acceleration parameters are considered same as [1]. For different configurations and different sets of mechanical properties of the buildings, numerical tests are performed and results are tabulated in Tables 2 to 4. Due to the fact that damping coefficients should be strictly positive, the logarithmic scales of damping coefficients are used as design

variables rather than actual values. It is clear that non-uniform damping coefficient leads to a smaller objective value for all cases. In some cases, improvement due to non-uniform damping coefficient is over 30%. Therefore, it is clear that assuming uniform damping coefficients imposes an additional constraint to the problem that prevents maximal effectiveness of the dampers.

Table 1 – Mechanical properties of buildings

	Building (a)		
	$m_a$	$k_a$	$c_a$
Set I	1.29E+06	4.00E+09	1.00E+05
Set II	2.60E+06	1.20E+10	2.40E+06
Set III	4.80E+06	1.60E+10	1.20E+06

	Building (b)		
	$m_b$	$k_b$	$c_b$
Set I	1.29E+06	2.00E+09	1.00E+05
Set II	1.60E+06	1.20E+10	2.40E+06
Set III	4.00E+06	2.30E+10	1.20E+06

Table 2 – optimal damping coefficients for set I

$f_a$	$f_b$	Objective Value		Improvement
		Non-uniform	uniform	%
10	10	8.59E-07	1.03E-06	20.0
10	20	1.97E-06	2.41E-06	22.3
20	10	1.59E-06	1.66E-06	4.4
10	40	5.49E-06	6.86E-06	24.9
40	10	5.88E-06	6.99E-06	19.0
20	20	1.51E-06	1.74E-06	15.4
20	40	3.66E-06	4.09E-06	11.7
40	20	2.76E-06	2.92E-06	5.7

Table 3 – optimal damping coefficients for set II

$f_a$	$f_b$	Objective Value		Improvement
		Non-uniform	uniform	%
10	10	4.03E-07	4.57E-07	13.54
10	20	2.39E-07	2.48E-07	3.82
20	10	3.13E-07	3.83E-07	22.23
10	40	2.61E-06	2.89E-06	10.55
40	10	1.21E-06	1.33E-06	9.57
20	20	6.09E-07	7.03E-07	15.39
20	40	4.09E-07	4.34E-07	6.23
40	20	5.95E-07	7.30E-07	22.72

Table 4 – optimal damping coefficients for set III

$f_a$	$f_b$	Objective Value		Improvement
		Non-uniform	uniform	%
10	10	4.60E-07	5.38E-07	17.10
10	20	4.38E-07	4.59E-07	4.97
20	10	4.95E-07	6.77E-07	36.68
10	40	8.62E-06	9.58E-06	11.19
40	10	2.35E-06	2.75E-06	17.41
20	20	7.00E-07	8.50E-07	21.44
20	40	7.65E-07	8.13E-07	6.28
40	20	9.59E-07	1.18E-06	22.77

To find the difference between uniform and non-uniform damping coefficient, optimal values of damping for three examples are plotted in Figure 2. It can be seen that the variation of damping coefficient along the building is not linear. Therefore, assuming that the non-uniform damping coefficient could be considered as linear function along the building based on the relative velocity of adjacent floors is not valid.

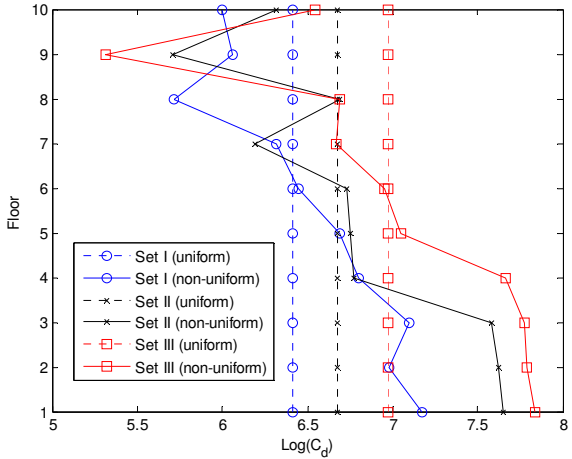


Figure 2 – Variation of optimal damping coefficient

Figure 3 shows the variation of inter-story drift for one test problem along the two buildings examined (in this case  $f_a=f_b=10$ ) and the mechanical properties are tabulated as set I in Table 1. Notice that the inter-story drift of building 1 is relatively unchanged, while the maximal inter-story drift of building 2 is notably diminished.

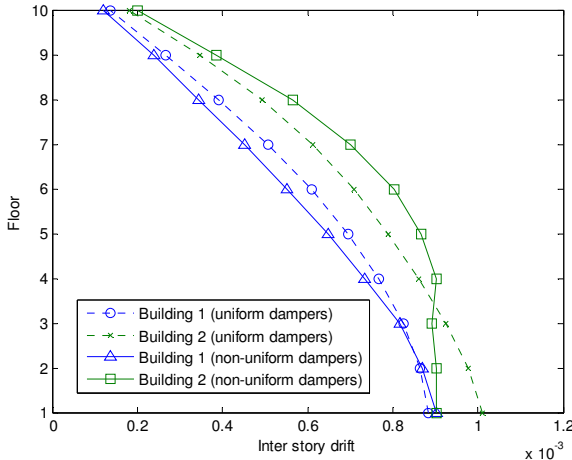


Figure 3 – Variation of inter-story drift

**CONCLUSIONS**

Two adjacent buildings are assumed to be connected with viscous dampers. Each building is modeled as a linear discrete system consisting a lumped mass, linear spring and linear damper for each floor. This paper aims to find the

optimal damping coefficients of each damper connector. The derivative-free method (specifically the Nelder-Mead (NM) method) is employed to find the optimal damping coefficients of damper connectors. Numerical examples have been presented to examine efficiency and reliability of the presented method. Results reveal that non-uniform damping coefficients lead to a more reliable system. In other words, assuming uniform damping coefficients imposes an additional constraint to the problem that prevents determining the best damping coefficients to minimize inter-floor drift. As a side result, it is also found that the variation of optimal the damping coefficient along the building is non-linear; consequently, it is not possible to estimate non-uniform damping coefficient as a linear function.

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