Model of Biological Pattern Recognition with Spatially Chaotic Dynamics

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Abstract—This article describes computer simulation of the dynamics of a distributed model of the olfactory system that is aimed at understanding the role of chaos in biological pattern recognition. The model is governed by coupled nonlinear differential equations with many variables and parameters, which allow multiple high-dimensional chaotic states. An appropriate set of the parameters is identified by computer experiments with the guidance of biological measurements, through which this model of the olfactory system maintains a low dimensional global chaotic attractor with multiple "wings." The central part of the attractor is its basal chaotic activity, which simulates the electroencephalographic (EEG) activity of the olfactory system under zero signal input (expiration). It provides the system with a ready state so that it is unnecessary for the system to "wake up" from or return to a "dormant" equilibrium state every time that an input is given (by inhalation). Each of the wings may be either a near-limit cycle (a narrow band chaos) or a broad band chaos. The reproducible spatial pattern of each near-limit cycle is determined by a template made in the system. A novel input with no template activates the system to either a nonreproducible near-limit cycle wing or a broad band chaotic wing. Pattern recognition in the system may be considered as the transition from one wing to another, as demonstrated by the computer simulation. The time series of the manifestations of the attractor are EEG-like waveforms with fractal dimensions that reflect which wing the system is placed in by input or lack of input. The computer simulation also shows that the adaptive behavior of the system is scaling invariant, and it is independent of the initial conditions at the transition from one wing to another. These properties enable the system to classify an uninterrupted sequence of stimuli.

Keywords—Chaos, Neural networks, Olfaction modeling, Pattern classification, Phase coherence, Scaling invariance, Serial recognition.

1. INTRODUCTION

The study of neural networks has opened a new direction in the investigation of novel computer design principles through understanding complex dynamics in biological brains. The marriage of nonlinear dynamics and neurobiology may be very helpful to the field of artificial intelligence by capturing the speed and power of brains during pattern recognition.

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1.1 Biological Phenomena

Physiological experiments suggest that the main components of neural activity in olfactory systems are chaotic (Skarda & Freeman, 1987). It has been found that distributions of intervals between pulses in trains are typically Poisson; histograms of electroencephalographic (EEG) amplitudes are close to Gaussian curves; and spectra of brain waves are broad with low and variable peaks consistent with $1/f$ noise (Freeman, 1975). Fractal dimensions are estimated to be low (Freeman, 1987, 1988). An equilibrium occurs only under deep anesthesia, in coma, or in areas of cortex that have been isolated from the rest of the brain. There is no evidence that pure limit cycle activity appears in olfactory systems, although at times the activities of neurons may come close to a limit cycle. Our physiological experiments tell us: (a) The EEG manifests a particular kind of chaos that is spatially coherent across the cortex when sampled with an array of electrodes; (b) the adaptive
neurodynamics that we access through the EEG for different animals, even different species (e.g., rabbit and rat), are indistinguishable; (c) a reproducible spatial pattern of amplitude of a chaotic neural time series lasting about 100 milliseconds is generated whenever a stimulus of a previously learned class is presented to the animal; (d) the time required for a transition from one spatial pattern to another in the olfactory system may be as brief as 6 milliseconds.

1.2 Dynamic Overview

After the study of differential dynamics, especially structural stability (Palis & Melo, 1982), one of the main achievements in dynamics is the work on chaos in deterministic nonlinear dynamic systems (Gleick, 1987; Schuster, 1984; Thompson & Stewart, 1986) although it had already been discovered by the mathematician H. Poincare (1892) that certain Hamiltonian systems could display aperiodic motions. In the past, the studies were mainly of low dimensional (less than five) systems, for example, Lorenz equations describing the disorder of the atmosphere (Lorenz, 1963); ecological population models showing the oscillations of wildlife (May, 1976); and a Josephson-junction circuit displaying irregular activities (Yao, 1987). The reason for this is the difficulty in the theoretical analysis of higher dimensional systems. The algorithms of Grassberger and Procaccia (1983) and of Guckenheimer (1984) can be used to estimate the correlation dimension, entropy, and largest Lyapunov exponent from a single time series. These methods enable us to study cooperativity in high-dimensional systems through their outputs such as the fluctuations of heart rates and the time series of human EEG signals (Pool, 1989).

Based on our experience with an isolated Kill model (Freeman, 1987) of the olfactory system, we discuss a particular chaotic system: a distributed olfactory model with the number of independent state variables up to 658. The distributed olfactory model developed below is a natural extension of the lumped KIII model (Freeman, 1987) and a distributed KII model (Freeman, 1979c; Freeman, Yao & Burke, 1988). The lumped KIII model showed how to generate an EEG-like chaos; the distributed KII model showed how to classify patterns with Hopf bifurcation. Here we propose that our simplified Hebbian input correlation learning rule, which we developed by studying the adaptive properties of the KII model, is suitable for application to the chaotic KIII model for pattern recognition.

This article is organized as follows: We discuss how to model an olfactory system in Section 2. The topological diagram of the olfactory system and its detailed mathematical implementation are presented. In Section 3, the functions of various parameters in the distributed KIII model are discussed. More than 30 parameters are identified by computer simulation and biological experiments. Spatial properties of the model such as scaling invariance, independence of initial conditions, and spatial coherence are analyzed in Section 4. Section 5 addresses the adaptive dynamics of the model. The pattern classification capability of the model is explored.

2. MODELING OF OLFACITION

The olfactory system is the simplest and most accessible sensory system in vertebrates. From the view of dynamics, like other sensory systems the olfactory system may be considered as an interactive dynamic system with multiple stable states corresponding to templates formed in the system by associative learning. To grasp our working model, a knowledge of olfactory anatomy, an understanding of related neural functions, and appropriate abstractions are necessary.

2.1 Anatomic Picture

The topology of connections defining the neural sets in the olfactory system is shown in Figure 1 (Freeman, 1972). Each receptor (R) has one axon that extends to the bulb via the primary olfactory nerve (PON) and ends in one of the glomeruli. The ending is a densely branched tuft. The approximately 40 million receptors for each bulb have no interconnections. Periglomerular neurons (P) receive excitatory axodendritic synapses from PON axons and are densely interconnected with each other by axodendritic excitatory synapses in and between glomeruli. The mitral cells (M) receive synapses from PON axons and periglomerular neurons. They are densely interconnected by excitatory axodendritic and axosomatic synapses. Mitral cell axons carry the output signal to the rest of the olfactory tissues. The granule cells (G) mutually interact, and form a set of densely interconnected inhibitory neurons. The
mitral cells both deliver and receive synaptic input to and from granule cells through reciprocal synapses. The mitral cells excite the granule cells, and the granule cells inhibit the mitral cells. These two cell types are thus connected in negative feedback loops and form neural oscillators. Output by the lateral olfactory tract (LOT) is to the superficial pyramidal cells of the anterior olfactory nucleus (AON) and the prepyriform cortex (PC), each with excitatory neurons (E and A) and inhibitory neurons (I and B). Output of the PC is by deep pyramidal cells (C) into the external capsule (EC) and centrifugally to the AON and OB in the medial olfactory tract (MOT). The AON also feeds back to the granule cells and the periglomerular neurons. Interactions among superficial pyramidal neurons in the AON and PC layers are similar to those in the OB layer.

2.2 Topological Diagram

From the olfactory anatomy above, the olfactory system may be represented by a block connection diagram (Yao & Freeman, 1989). The diagram with N units in the bulb is shown in Figure 2. In the remainder of the article, this diagram is referred to as an N-channel model or an N-channel system. The top arrows come from the N receptors, which stand for the system's inputs. Excitation is (+); inhibition is (−). The means and ranges of latencies (L1 to L4) are calculated from measurements of the conduction velocities and distances between structures. Each circle denotes a “unit” that represents a local population of neurons. Physiological experiments confirm that the linear dynamics of a neuronal population is described adequately by a second-order nonlinear
differential equation (Freeman, 1975; Freeman, Yao & Burke, 1988). In the neuronal population the output of the equation is then operated on by a sigmoid curve. Hereafter, the notations of “cell” or “unit” refer to the model of the neuronal population, that is, one of the circles in the diagram. Input to periglomerular and mitral cells is from receptors by the PON through the glomeruli subject to attenuation and logarithmic range compression (Freeman, 1975). Each path is assigned a gain, the scalar value of the corresponding parameter. For example, $K_{pr}$ and $K_{mr}$ are the gains from R to P and from R to M, which represent the degree of the amplifications. $K_{mg}$, $K_{ir}$, and $K_{bb}$ are from G to M, from E to I, and from B to B, respectively; $K_{mM[i,j]}$ is from the jth unit to the ith unit associated with M cells, $K_{gG}$ is the connection strength from G to G and between two units. Double arrows denote that the signals are added to all of the neurons within those levels, and that the signals are obtained from all of the neurons within those levels. The block diagram includes the major operations of the olfactory neurons: linear conversion of afferent axonal impulses to dendritic currents; linear spatiotemporal integration; static nonlinear conversion of summed dendritic current to a pulse density; and linear delay, temporal dispersion, translation, and spatial divergence. The excitatory couplings between the mitral cells provide modifiable synapses in learning and perception, while the invariant inhibitory couplings between the granule cells provide stability, self-organization, and spatial contrast (Freeman, 1987).

2.3 Mathematical Model

It is suggested by Figure 2 that a massively parallel architecture with multiple layers can be utilized to describe the olfactory dynamics. Mathematically, the dynamics may be governed by a set of nonlinear coupled-differential equations. Notice that each circle in Figure 2 is described by a second-order differential equation followed by a static sigmoid nonlinearity, and the connections among those circles are specified by scalar gains. Therefore, a mathematical model of the olfactory system may be written as follows:

$$
\frac{d^2 X_p[i]}{dt^2} + \frac{A}{t} \frac{dX_p[i]}{dt} + BX_p[i] = \frac{1}{N} \sum_{j=1}^{L} K_{mp}Q(X_p[j]) + K_{pr}Input[i] + \frac{K_x}{T_{x1} - T_{x2}} \sum_{k=1}^{L} Q(X_p[k]); \quad (P)
$$
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\[ \frac{dX_i}{dt} + A \frac{dX_j}{dt} + BX_{m} = K_{w}Q(X_m)[i] \]

where \( i = 1, 2, \ldots, N; A = a + b; B = ab; K_1, K_2, K_3, \) and \( K_4 \) are feedback gains; \( t - T_{ij}, T_{ij} - T_{ji} \) are the delay starting time and the terms of the delay; \( Input[i] \) is the input signal to the \( i \)th channel; and \( Q(\cdot) \) is a sigmoid function, that is, the nonlinearity of each neuron pool (Freeman, 1979).

\[ Q(v) = Q_m[1 - e^{-v/\mu}], \text{if } v > -\mu; \text{otherwise}, Q(v) = -1, \mu = -\ln[1 - Q_m\ln(1 + 1/Q_m)]. \]

\[ \text{Here } a \text{ and } b \text{ are open loop rate constants.} \]

The activity of each variable in the above equations, or the output of each circle in Figure 2, at time \( t \) depends on the values of its associated variables and on the outputs of its linked cycles at time \( t - 1 \), as well as its related delay terms. That is, the left side of each equation is in time \( t \) and the right side of each equation is in time \( t - 1 \), except \( Input[i] \), which is in time \( t \). It simulates the transition delay of information from one unit to another. This implies that the solution in time \( t \) for each \( KO \) or \( KII \) subset is independent of the solutions of the rest in time \( t \). In other words, the model is parallel. As an example, let \( N = 8 \), that is, an eight-channel model. Instead of solving 98 simultaneous first-order equations, what should be done is to solve 9 second-order and 10 eighth-order differential equations by a parallel algorithm. When \( N \) becomes larger, the parallel algorithm is even faster than solving the model serially. The price for the increase of the speed is to memorize the outputs of all units in time \( t - 1 \).

3. PARAMETER SETTING

In conformance to the olfactory system in an animal, the value of each “synaptic” and “axonal” gain parameter in the model is a variable on a much longer time scale. At synapses, the conversion from pulse frequency to dendritic current amplitude is linear and is expressed by a gain coefficient, \( K \). The value of \( K \) is subject to sudden increase or decrease with learning and persists at a new level indefinitely. At an ensemble of axonal trigger zones when the integrated dendritic current is expressed as axonal pulse frequency, the input-output relation of the local neural ensemble in the olfactory system is a sigmoid curve, that is, \( Q(v) \) in the model. The slope of the curve is subject to sudden changes with arousal and motivation, but typically persists unchanged for minutes at a time. On the other hand, the shape of the sigmoid curve evolves with its related neuronal activity. It is thought that the maximum of the gain increases exponentially with excitation of the neurons above their resting level, that is, \( K_{max} = K(1 - e^{-T/T_0}), \) where \( T \) is excitation duration and \( \alpha \) is a constant. In this study, we model the adult animal in an aroused state. Thus, every gain in the model is set to be an appropriate fixed value, except the adaptive gains \( K_{w} \) with “learning.” The values are determined by computer experiments using neurophysiological data as constraints.
3.1 Feedback Gain and Delay

Both local and global feedback gains and delays in the model are necessary to generate EEG-like chaos owing to physiological constraints described by Freeman (1987). In the model, local feedback gives rise to periodic oscillation. There are three kinds of feedback: excitatory, inhibitory, and negative (Freeman, 1967). If the neurons of a loop are all excitatory, the action is called mutual excitation, which spreads the activity of the neurons widely and destabilizes through regenerative feedback. If the neurons of a loop are all inhibitory, the action is called mutual inhibition, which promotes spatial contrast and stabilization. The loops between excitatory and inhibitory neurons provide negative feedback. It is the main cause of oscillation in the gamma range (30–100 Hz). Chaos arises from the interaction between areas of cortex that are separated by long or slowly conducting pathways that introduce distributed communication delays, which...
are expressed by $L_1, L_2, L_3,$ and $L_4$ in the connection diagram in Figure 2. Without the global feedback pathways, the olfactory system generates regular oscillatory activity or goes to equilibrium like the KII model (Freeman, 1979b; Gray, Freeman, & Skinner, 1986).

The feedback gain and delay terms are determined by means of computer simulations and physiological experiments. In addition, in conformance with the properties of neural feedback, all the feedback pathways are temporally dispersed and spatially divergent. In the distributed KIII model, the dispersion and divergence are implemented by storing and averaging previous activities of superficial pyramidal cells E and A in the AON, PC, as well as deep pyramidal cell (C) and providing these as inputs to their associated units. In particular,

$$L_i = \frac{K_i}{T_i} - \frac{1}{T_i} \sum_{s=1}^{N} Q(X_e(k)),$$

$$L_i = \frac{K_i}{T_i} - \frac{1}{T_i} \sum_{s=1}^{N} Q(X_i(k)),$$

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These distributed feedback delays act as low pass filters (Freeman, 1987).

### 3.2 Adaptive Linkage

During learning in the biological olfactory system, it is supposed that there is selective strengthening of those synapses that interconnect excitatory neurons within the olfactory bulb and within the prepyriform cortex. The change with learning does not take place at other synapses, particularly at the input to the bulb and cortex. The input synapses of the bulb, AON and PC, are “use-dependently” modifiable in respect to the volume of input and behavioral arousal, but not in respect to habituation and associative learning. Under this hypothesis, only $K_{mm[i,j]}, i, j = 1, 2, \ldots, N$ are adaptive parameters because in this study only lumped AON and PC layers are considered. The molecular basis for those biological changes in excitatory synapses is not known. Here we apply the following input correlation learning rule, which was developed in the study of the distributed KII model (Freeman, Yao & Burke, 1988), to this modeling approach.

Let $V^1, V^2, \ldots, V^m$ be the $m$ patterns that need to be stored. Here each pattern $V^i$ is a binary $N$-vector. First, let all of the synaptic strengths $K[i,j]$ be set to $K_{low}$. Then for $V^i$, if $V^i \cdot V^j \neq 0$, set $K[i,j] = K_{high}$, otherwise keeping $K[i,j]$ unchanged, where $i, j = 1, 2, \ldots, N, i \neq j, s = 1, 2, \ldots, m,$ and $K[i,j] = 0, i = 1, 2, \ldots, N$.

The correlation rule is used to selectively strengthen pairwise interconnections in respect to inputs classified as “to be learned.” It bypasses exploration of the behavior of learning and allows direct access to generating and classifying patterns in models of the vertebrate olfactory system. With this rule the distributed KIII model behaves as content addressable memory. Here $K_{mm[i,i]} = 0$, which denotes that no self-excitation exists in brain, because in cortex the likelihood that the axonal of a neuron will synapse onto its own dendrites is vanishingly small, and if it did the neuron would be refractory and hence unable to respond to its own impulse (Freeman, 1975). For convenience, the other $K_{mm[i,j]}$ are binary, either low or high. Also, whereas the time scale of learning in a living animal is much longer than that of recognizing and requires multiple trials, the teaching process in an artificial system is compressed into one trial by this rule.

![Diagram](image_url)
3.3 Input Parameters

The top vertical arrows in the topological diagram (Figure 2) are from the receptors. Olfactory receptors can not form cooperative collectives since they are not synaptically connected, but each individual receptor still has the thresholding property of neurons, and in the aggregate of a sufficient number of such receptors the resulting threshold function is sigmoid, which may be written as \( IN = IN_{\text{max}} \left(1 - e^{-(v - V_0)/IN_{\text{max}}}\right) \) if \( v > V_0 \); otherwise, \( IN = IN_{\text{min}} \), where \( V_0 = \ln(1 - IN_{\text{max}}, \ln(1 - IN_{\text{min}}/IN_{\text{max}})) \) (see Figure 3a). Then in terms of this input–output characteristic curve, the stimulus inputs to the periglomerular cells (P) and mitral cells (M) can be simulated by the following curve:

\[
\text{Input}[i] = \begin{cases} 
(IN_{\text{max}} - IN_{\text{min}}) \times \sin(\pi t/2T_1) + IN_{\text{min}} & \text{if } 0 \leq t \leq T_1, \\
IN_{\text{max}} & \text{if } T_1 < t < T_1 - T, \\
(IN_{\text{max}} - IN_{\text{min}}) \times \sin(\pi(T_L - t)/2T_1) + IN_{\text{min}} & \text{if } T_1 - T_L \leq t \leq T_1,
\end{cases}
\]

where \( T_L \) and \( T_1 (\leq T_L/2) \) are the time duration of the stimulus, and the transition time of the receptors from their background bias \( IN_{\text{min}} \) to their maximum...
output $\text{IN}_{\text{max}}$ (see Fig. 3b). In particular, $\text{Input}[i]$ is a step function if $T_i = 0$. The activity of the system is called basal activity if $\text{Input}[i] = \text{IN}_{\text{max}}, i = 1, 2, \ldots, N$, that is, for all of the channels there is no signal input. Since the input to the system depends on the input-output characteristic curve of the receptors, appropriate values of $\text{IN}_{\text{max}}$ and $\text{IN}_{\text{min}}$ are important. In particular, the function of $\text{IN}_{\text{min}}$ is to maintain the basul chaotic activity whenever stimulus inputs are terminated, and to permit easy departure from the basal activity when a stimulus input is presented. The importance of $\text{IN}_{\text{max}}$ value is to generate and maintain certain high DC offset activity. In this respect, the present distributed KIII set differs from the lumped KIII set (Freeman, 1987), in which no input bias was used or required to maintain the basal chaotic activity.

### 3.4 Internal Parameters

Chaos can arise by the delayed feedback between coupled oscillators having frequencies that are incommensurate. Physiopathological measurements of the characteristic frequencies with the evoked potential technique show that the OB has the highest frequency, the PC the lowest, and the AON a frequency in between. The differences are large enough to prevent entrainment in a limit cycle but lie within the range of the first harmonic of the lowest frequency. They are set by assigning values to $(K_{mm}, K_{mg}, K_{cm}, K_{cg})$ in OB, $(K_{mr}, K_{rg}, K_{sr}, K_{sr})$ in AON, and $(K_{ar}, K_{ag}, K_{ag}, K_{bg})$ in PC.

Other parameters in the model include the open loop rate constants $a$, $b$ and the slope $Q_m$ of the sigmoid curve of the cells (i.e., the circles in Figure 2); interactive gains $K_{ii}$, $K_{ij}$, and feed forward parameters: $K_{m}$, $K_{r}$, $K_{mr}$, $K_{mr}$, $K_{cm}$, $K_{cm}$. The settings of these parameters are not crucial.

The working set of the parameters used in the following sections are listed below. Structural parameters: $Q_m = 5.0, a = 0.2/\text{ms}, b = 0.72/\text{ms}$; global parameters: $K_{mm} = 0.5$, $K_{mr} = 4.0$, $K_{ag} = 1.5$, $K_{ag} = 0.2$; feed forward parameters: $K_{m} = 1.0$, $K_{r} = 1.0$, $K_{mr} = 0.5$, $K_{mr} = 1.0$, $K_{mr} = 1.0$, $K_{cm} = 1.0$, $K_{cm} = 1.0$; feedback parameters: $K_{r} = 1.6$, $T_{e1} = 11$, $T_{e2} = 20$; $K_{e1} = 0.25$, $T_{c1} = 15$, $T_{c1} = 26$; $K_{e2} = 2.0$, $T_{c1} = 12$, $T_{c1} = 25$; $K_{e2} = 1.5$, $T_{e2} = 24$, $T_{e2} = 39$. Internal parameters: $(K_{mm}, K_{mg}, K_{cm}, K_{cg}) = (0.25, 1.5, 1.5, 1.8)$, $(K_{mr}, K_{mr}, K_{mr}, K_{mr}) = (1.5, 1.5, 1.5, 1.5, 1.8)$, $(K_{mr}, K_{mr}, K_{mr}, K_{mr}) = (0.25, 1.4, 1.4, 1.8)$; input parameters: $\text{IN}_{\text{min}} = 2 \text{ or } 0$, $\text{IN}_{\text{max}} = 8$, $T_i = 50 \text{ ms} \text{ or } 0 \text{ ms}$. The integral method used is Runge–Kutta method, the time step is 0.5 ms.

### 4. SPATIAL PROPERTIES

Properties not found in components of a lower level can emerge at a higher level through the self-organization of these components at the higher level. Such properties constitute the collective behavior of
the system. The impulse response of a single KO subset is the sum of two damped exponential curves, and a single KII subset is the sum of two cosines (Freeman, 1975). Under physiological constraints, the dynamics of an array of such KII subsets is not chaotic, but is periodic or quasiperiodic (Freeman, 1979c; Freeman, Yao, & Burke, 1988). However, the distributed KIII model composed of those KII subsets is strongly chaotic, as we show in what follows.

4.1 Scaling Invariance

Scaling invariance is crucially important for practical applications and for the plausibility of the model. Different species have different numbers of neurons. For a given animal, the number of its neurons decreases with age. The scaling invariant property that the principles of pattern recognition in different animals are similar.

Consider the model with 4 KII subsets (or channels) in OB layer. Use the values of the parameters defined in last section, where $IN_{\min} = 0$, $IN_{\max} = 6$, and $T_i = 0$. No template has been established in the model, that is, $K_{nm}[i,j] = K_{nm,\text{ini}}$ for $i \neq j$, $i, j = 1, 2, \ldots, N$ in the equations in Section 2. The stimulus input is given to all of the four channels between 1,300 ms and 2,200 ms. The first trace in Figure 4 is the output waveform from one of the four G-cells during the period 1,000 ms to 2,500 ms. Within nonstimulus intervals $[1,000, 1,300]$ and $[2,200, 2,500]$, the system is in its basal activity. It can be seen that the activity is with rather low "frequency" and "amplitude." The system jumps to a high "amplitude" and high "frequency" chaotic activity when the stimulus input is given.2 In Figure 4, other three traces correspond to 8-channel, 16-channel, and 32-channel cases, respectively. By inspection, we can see the similarity among these four traces. The correlation

2 The corresponding spectrum analysis will be presented in a numerical analysis report. But we may visually identify that there are more high frequency components when the input is presented in Fig. 4.
dimensions of their high-level chaotic activities range between 2.26 and 2.66. The system's states only depend on the existence of the stimulus whatever the number of its channels. The simplest way to visualize chaotic activities in high-dimensional systems is to plot pairs of traces from different components against each other. Figure 5 illustrates results of simulation for the eight-channel case under the input, that is, the phase portraits for the activities from different pairs of the components of the system when the input is given to all of the channels. The top left plot is M-cell against G-cell, the top right is E-cell against I-cell, the bottom left is A-cell against B-cell, and the bottom right is P-cell against C-cell. Each plot consists of 2,500 points.

The scaling invariance property also holds for the model with "learned" templates. Consider an eight-channel case. Suppose Patt A = (1 0 0 1 0 0 1 0) and Patt B = (0 1 0 0 1 0 0 0). Make templates Patt A and Patt B, that is, build the matrix \( K_{mn}[i, j] \) in terms of input correlation learning rule. Let \( \Gamma_{mn} = 2, \Gamma_{mn} = 8, T_1 = 0 \), and let the rest of the parameters be the same as the settings in Section 3. Figure 6 shows the simulation results. Figure 6a is the output from the array of G-cells under the input Patt A, and Figure 6b is the output under the input Patt B. The inputs start at 400 ms and end in 800 ms. The time duration of each trace is between 200 ms and 1,000 ms. When the inputs are presented, the outputs are more regular, and the "on" channels have a higher DC offset. Now consider a 16-channel case: There are two templates made in the system—Patt C = (0 1 0 0 0 1 0 1 0 0 0 1 0 1 0 0) and Patt D = (0 0 0 0 1 0 0 0 1 0 1 0 0 0 1 0 0). Giving Patt C as an input during the period 200 ms to 500 ms and Patt D as an input during 500 ms to 800 ms, we obtain an array waveform from the 16 G-cells (see Figure 7). Here \( T_1 = 50 \) ms; the adaptive matrix \( K_{mn}[i, j] \) is determined by Patt C and Patt D. In Figure 7 (see Figure 6 as a comparison), the "on" channels have high DC offsets and the "off" channels have low DC offsets. In both Figures 6 and 7, from the basal state to the activated state 50 ms to 100 ms are needed for the phase transition. That is, the 16-channel system behaves qualitatively similar to the 8-channel system. Based on the above scaling invariant property, we will mainly concentrate on the 8-channel system with Patt A and Patt B as two templates.

The algorithm used to measure the correlation dimensions is Grassberger and Procaccia's algorithm. In this study, the precise value of the dimension is not very crucial. In fact, different epoch length and sampling interval, as well as embedding space, will effect the measurement of the dimension (Rapp, Albano, & Mees, 1988). The detailed simulation results about the correlation dimension of the system will be presented in the numerical analysis report.

4.2 Independence of Initial Conditions
An artificial system can be returned repeatedly to a particular initial condition. However, this is nonsense for a living animal. Thus, it is plausible to assume that perception and cognition in the animal are independent of the initial conditions as long as the initial conditions are within certain limits. Does the dynamics of our model have this property, too? To see this, let us give the system a sequence of inputs. Figure 8 illustrates the simulation results. Figure 8a is the output waveform from the first granule cell in the eight-channel system. No template is made in the system like the case of the second trace in Figure 4. Although these traces are quite different from each other, the values of their correlation dimensions are close. Theoretically, the values should be exactly the same. Here the settings of the parameters are the same as those in Fig. 6.
periods from 950 ms to 1,050 ms and from 1,450 ms to 1,550 ms. The system goes back and forward between its basal activity and its high-amplitude chaotic activity depending on the presence of the stimulus. Figure 8b is the output waveforms from the eight-channel system with Patt A and Patt B as two templates like the case in Figure 6 except \( T_{l} = 50 \) ms. By presenting the input Patt A sequentially at 200 ms, 500 ms, and 800 ms, the array of time series from the G-cells is obtained (see Figure 8b), which shows the same transition property in Figure 8a. Notice that here we consider the information to be in the amplitude and not in the frequency content. The input lasts 300 ms, that is, \( T_{l} = 300 \) ms. As we can see both in Figures 8a and in 8b, the transition from a basal state to an activated state, and vice versa, is not affected by the initial conditions although the initial conditions do reflect the transition time. Let us come back to Figure 7, which shows that the 16-channel system leaves Patt C state for Patt D state when input Patt C is terminated and input Patt D is presented. The simulations in Figures 8 and 7 imply that the qualitative behavior of the model olfactory system is independent of its initial conditions. The system is driven by its input from one of its states to another. We will come back to this topic later (see Figure 12).

4.3 Spatial Coherence

The olfactory bulb reveals a remarkable spatial coherence of the activity at all times in contrast to the irregularity of the time series at each channel (Freeman & Viana di Prisco, 1986). This biological behavior appears in our model as shown in Figure 6. Although these time series are irregular and differ in amplitude across the array, at any instant the frequency of rate of change of the simulated EEG traces tends to be the same when an input is presented. This tendency of spatial coherence also exists when input is not presented, especially among those template channels (channel 1, 2, 4, 5, 7, and 8). It is this phase-coherent property that expresses the dramatically reduced dimension of the attractor. For example, the number of variables in the eight-channel system is 98. The correlation dimension of the basal activity in the system is 2.33 ± 0.004 when stimuli are not presented in Figure 6a. Under a stimulus the correlation dimension of the activity of the system

![Some phase portraits of the basal activities.](image)

**FIGURE 10.** Some phase portraits of the basal activities. Each picture in the figure consists of 2,500 points. (a) M-cell against G-1 cell; (b) E-cell against I-cell in AON; (c) A-cell against B-cell in PC; (d) P-cell against C-cell; (e) M-1 cell against M-2 cell; (f) M-1 cell against M-3 cell; (g) G-1 cell against G-2 cell; (h) G-1 cell against G-3 cell. See the text for the detailed explanation.
within the interval [400, 800] in Figure 6a is reduced still further to 1.13 ± 0.003.

5. PATTERN CLASSIFICATION

Pattern classification in biological sensory systems is a dynamic operation. In this section, we explore the capability of pattern classification of the distributed KIII model developed in the previous section. Consider the eight-channel case with Patt A and Patt B as two templates made by the input correlation rule. Problem: Given any eight-dimensional vector \( X \): Does \( X \) belong to A, or B, or neither A nor B?

5.1 Basal Activity

As mentioned above, in the system \( \text{Input}[j] = IN_{min} \) (for all \( j \)) simulates no significant stimulus input. Under the same parameter settings as in Figure 6, Figure 9 shows the basal activities from various parts of the model. The first trace is from P-cell. Others are from M-1, G-1, E, I, A, B, C-cells. These time series are aperiodic, and look like noise. Since every term in the system (see the model) is deterministic, the aperiodic motions are chaotic. By plotting different pairs of traces in Figure 9 against each other, we get different views of a chaotic attractor (see Figure 10). Figure 10a is M-1 against G-1. Figure 10b is E-cell against I-cell, and so on. Although the time series of M-1, M-2 are chaotic, the result of plotting them against each other is a straight line in Fig. 10e because the first two channels of the system are made templates (see Patt A and Patt B) as well as the spatial coherence is manifested among the channels. The same reasons give rise to a straight line for the plot of G-1 against G-2 in Figure 10g. In fact, we find the trace 1 and trace 2 in Figures 6a or 6b are identical before the input is presented. Visually, in Figure 9 the traces are quite different with each other, for example, the third trace is more “irregular” than the fourth trace. But the correlation dimension of the basal state ranges between 2.3 and 2.4 no matter which trace is adopted to measure the dimension.

5.2 Pattern Formation

Giving Patt A and Patt B as inputs, respectively, to the eight-channel model olfactory system, we obtain
FIGURE 12. The system comes back to its basal state whenever the input is terminated. The inputs drive the system back and forward among the wings and the basal state. The top left plot and the bottom left plot are look like each other and are similar to Fig. 10a. The top right plot is similar to Fig. 11a. See the text for details.

FIGURE 13. Time series of other main components in the model corresponding to Fig. 6a. The system leaves its basal activity and return to the basal activity when Patt A is presented or terminated, respectively. The second trace and the third trace are the outputs from M-1 and M-3. The former is from an “on” channel; the latter is from an “off” channel. They are different. The reasons for the difference are that the input activates the M-1 directly and there are high connect strengths $K_{11}$, $K_{21}$ associated with M-1, while there are no input and no high strength directly associated with M-3. The correlation dimensions of these time series between 400 ms and 800 ms, that is, the near-limit cycle activities, range from 1.12 to 1.13.

The output waveforms of the system shown in Figure 6. The outputs are from the array of G-cells. The amplitude of the shared waveform differs between channels, so that one of two spatial patterns of waveform amplitude, which corresponds to Patt A or Patt B, is formed. It can be seen that the DC offset of the “on” channels is larger than that of the “off” channels. When the system is out of its basal activity, it behaves more like a limit cycle (see Figure 11). Figures 11a, b, c, and d are M-cell against G-cell, E-cell against I-cell, A-cell against B-cell, and P-cell against C-cell, respectively. Figures 11e, f, g, and h are M-cell, G-cell, E-cell, and A-cell against themselves lagged 50 ms, respectively. In fact, the pictures in Figure 11 are still chaotic, but they are quite different from those in Figure 10. We call them near-limit cycles. The input given to the system is Patt B. Each plot in Figure 11 consists of 2,500 points. It can be concluded: (a) The geometry of the phase portrait changes in different states of brain activity; (b) the system shifts to a near-limit cycle activity from its basal activity when input is presented, and returns to the basal activity following termination of the stimulus (see Figure 12) wherever its initial condition is. In Figure 12, Patt B is presented during 1,500 ms to 2,500 ms, while Patt A is presented during 3,500 ms to 4,500 ms. Each plot in the figure is composed of 2,000 points. The x axis is M-1 and the y axis is G-1. The top left plot is from 500 ms to 1,500 ms, the top right plot is from 1,500 ms to 2,500 ms, the bottom left is from 2,500 ms to 3,500 ms, and the bottom right is from 3,500 ms to 4,500 ms. The top left and the bottom left plots look similar to the plot in Figure 11a, and the top right plot looks like the plot in Figure 11a. It is expected that they are not exactly the same due to chaos. The similarity is evidence that the attractor (including its wings) is stable with respect to the initial condition.

Figure 13 shows the activities of other main components of the system under the input Patt A during [400, 800], which corresponds to Figure 6a. The ac-
5.3 Information Retrieval

Our hypothesis is that: the olfactory model maintains a global chaotic attractor with multiple “wings” in the eight-channel system. The core of the attractor is manifested by the basal activity, and the wings are expressed by near-limit cycles, one corresponding to Patt A and another to Patt B. By changing the initial condition of the model and shifting the starting point of the inputs, we have found that the shape of the global attractor remains the “same” (Figures 7, 8, and 12).

A question that might interest neural network researchers is whether the system behaves like an associative memory? That is, can the system retrieve Patt A or Patt B from partial information? To see this, we present the system with the following inputs $A^* = (00010010)$ and $B^* = (00001000)$, and observe its outputs of the G-cell array. Figure 14 shows the simulation results. As measured by the DC offsets, the patterns of A and B are retrieved even though in the Figure 14b case spatial pattern B is induced by only one “on” channel. Figure 14 and Figure 6 are not identical, but they are qualitatively

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![Figure 14. Pattern retrieval corresponding to Fig. 6 (a) Patt A is retrieved by the input (0 0 0 1 0 0 1 0); (b) Patt B is retrieved by the input (0 0 0 0 1 0 0 0). By retrieval it means that the Fig. 6a and Fig. 14a, and Fig. 6b and Fig. 14b have the same high DC offset channels and the same low DC offset channels, respectively.](image-url)
the same, that is, they have the same high DC offset channels and the same low DC offset channels.

The property of reproducible spatial pattern is only for an input belonging to a "trained" set, that is, a template made in the system. It is not true for a "nontrained" pattern. Consider the same system discussed in Figure 6 under the following inputs (0 0 1 0 0 1 0 0) and (0 0 1 0 0 0 0). Their simulated outputs are illustrated in Figures 15a and 15b, respectively. It is clear that they do not look like because only one on channel is in Figure 15b.

5.4 Successive Recognition

The results presented in this section provide an understanding of the model olfactory system with eight channels for categorization of inputs in accordance with learned nerve cell assemblies to define the categories. From the above, we know the property of convergence of the model is independent of its initial conditions. Thus, it is possible to study the state transitions between wings of a global attractor corresponding to successive stimuli of different classes. Based on the independence of its initial conditions, the system can be used to accomplish certain successive pattern classification tasks. Consider the system discussed in Fig. 6. Let the input start with Patt A, then Patt B, and end with Patt A. Let the time duration of each input pattern be 300 ms. The simulation result is shown in Figure 16a. The spatial amplitude patterns of the system are in turn Patt A, Patt B, and Patt A. The same property is also found in a 16-channel case (see Figure 7) since the property of scaling invariance of the model. In this case, Patt C and Patt D are made in the system as two templates, that is, the matrix $\mathbf{K}_{ij}$ is determined by Patt C and Patt D. The input starts with Patt E and ends with Patt F.
The above simulation results provide support for the following properties of the distributed olfactory model developed in this report. First, there is a global chaotic attractor in the system. Its stability is shown by the reproducibility of its geometric form in state space. The attractor may be described as composed of a central part and multiple wings. The transition back and forth between the wings or between the central part and one wing stands for phase transition in the sense of physics and for pattern recognition in the sense of neural networks. Second is its scaling invariant property; which implies the plausibility of the model. A third is the independence of initial conditions, which implies that each wing has its own basin. This may make it possible to do successive pattern recognition as well as pattern completion. A fourth is that the olfactory system reveals a spatial coherence across the array, which is expressed in the reduced dimension of model states, that is, the dimensions of the model activities are much lower than the number of the related elements. The dimensions are fractal and may vary with the complexity of ongoing behavior. Fifth, for inputs that go to template elements there is a reproducible spatial pattern of output, but this is not so for nontemplate or "novel" inputs. In these five respects the model simulates the corresponding properties of the olfactory system. In two important respects the model fails to do so. First, in the model the response component that best serves to describe the convergence to a reproducible spatial pattern is the baseline shift, whereas in bulbar output it is the amplitude of the burst oscillation and not the baseline shift. Second, whereas in the bulb the transition time required to go from an interburst state to a burst state is only a few milliseconds, in the model the transition often required up to 50 ms.

As a model of a real olfactory system, this study is not completed, for example, in the animal, instead of being lumped units, the AON and PC are distributed and adaptive. We still cannot answer "how does the biological chaotic system do better than an equilibrium or an oscillatory system in pattern recognition?", although we can intuitively suspect the advantages of certain chaotic systems, for example, chaos seems to provide a "restless" state that allows a biological sensory system to access any direction and to provide a "variability" that allows the system to respond quickly to a variety of stimuli. In other words, chaos serves the brain to keep it primed to accept capricious and unpredictable new input. Further exploration of the properties of the model and its real application in the area of pattern recognition is under way.

REFERENCES


