Uncertainty propagation in SEA using sensitivity analysis and Design of Experiments

Antonio Culla, Walter D'Ambrogio and Annalisa Fregolent

Abstract A limit of Statistical Energy Analysis (SEA) is that of providing only the mean values of the mechanical energy of a vibrating system. In the proposed paper, the variability of SEA solution under uncertain SEA parameters (coupling loss factors and internal loss factors) is investigated by comparing a sensitivity approach and a Design of Experiment (DoE) approach. Uncertainties of the SEA parameters depend on uncertainties in the physical properties of the considered mechanical system (Young modulus, material density, geometry, ...). Numerical results are derived using a benchmark structure made by three aluminum plates with a common junction.

1 Introduction

In Statistical Energy Analysis, the studied systems belong to a random population of similar systems [1]. Systems are considered similar if their physical parameters are slightly different. SEA considers a structure as the union of several subsystems. Each of them is a modal group, i.e. a set of similar modes. For instance, considering two plates welded together, six modal groups can be identified, one set of flexural modes and two sets of in plane modes for each plate.

Antonio Culla
Dipartimento di Meccanica e Aeronautica, Università di Roma 'La Sapienza', Via Eudossiana 18, 00184 Roma, Italy, e-mail: antonio.culla@uniroma1.it

Walter D'Ambrogio
Dipartimento di Ingegneria Meccanica, Energetica e Gestionale, Università dell'Aquila, Piazza E. Pontieri 2, 67040 Roio Poggio (AQ), Italy e-mail: walter.dambrogio@univaq.it

Annalisa Fregolent
Dipartimento di Meccanica e Aeronautica, Università di Roma 'La Sapienza', Via Eudossiana 18, 00184 Roma, Italy, e-mail: annalisa.fregolent@uniroma1.it

SEA estimates the mean value of the energy stored in the modal groups constituting the studied system. The mean value provided by SEA equations is in principle the average response of a set of similar systems. However, SEA equations are represented by a linear system of equations for each frequency, or better for each frequency band. The solution of each linear system gives the energy of each subsystem in a given frequency band. No average operation is explicitly performed, but all the statistics is not visible to the user. In general, this is not a problem because many simple relationships used by physicists and engineers are the result of more complicated mathematical procedures. Unfortunately, in this case this simple model holds only under many strong hypotheses, listed in Section 2.

The linear system results from some mathematical manipulations, that include also averages on frequency bands, on the classical equations of motion of multi degrees of freedom systems, and the observance of the strong hypotheses mentioned before. The coefficients of the linear system, named coupling loss factor (CLF) and internal loss factor (ILF), are the result of these average processes and account for the parameters of the native physical system. Therefore, SEA gives the energy of each modal group belonging to the studied system. This energy is the most representative sample of a statistical population of similar systems and on a frequency band. No information is given about the dispersion of the data around the result. In order to provide a true statistic solution it is necessary, at least, to know the variance of the result. A correct procedure should calculate the variance of the energy by starting from the equation of motion following a similar procedure like that performed for the mean calculation. Lyon [1, 2] estimates the variance by using a particular probability distribution of natural frequencies. Therefore this procedure neglects a direct dependence of the modal parameters on the randomness of the physical properties. However, he concludes that "There is a considerable area of interesting research work that needs to be done in analysing variance of interacting systems."

Radcliffe and Huang [3] study the problem by introducing a stochastic perturbation in SEA equations, by using a first order approximation of them in terms of this random perturbation and by calculating the variance of the new linear equations. They state that the lack of information of SEA solution may be filled by calculating the solution variance due to the random perturbation of SEA parameters (coupling loss factor, injected power, ...). Langley and Cotoni [4] follow the Lyon's approach and study the problem by considering the Gaussian orthogonal ensemble, detailed in Weaver [5]. Some other authors [6, 7] assume that uncertainties lie on the system parameters and they achieve results not exhibiting the same tendency of the Lyon's prediction. Bussow et al. [8] propose an analytical approach for the investigation of the problem of uncertain system parameters. The analysis is performed by partial derivatives of the energies. It shows the effect of uncertainties of a given parameter.

All these methods to approach the uncertainty problems use either *parametric* or *nonparametric* models [9]. In the first case the physical properties of the system are considered to be uncertain. The uncertainty of the response is calculated by considering the propagation of the physical uncertainties through the model. If the system is very complex and random, its natural frequencies statistics can be assumed, that

is the statistics of the response can be considered independent of the statistics of the physical quantities. This is the *nonparametric* case.

In this paper a *parametric* approach is adopted. It proceeds from the belief of the authors that SEA is a methodology able to analyse well both the actual *nonparametric* models (very complex and random systems) and the models (not so complex and random) which could be solved also by a FE approach (simple structure, not much uncertain, forced by random loads).

Moreover, aim of this paper is not only to evaluate the variability of SEA solution, but also to calculate the sensitivity of the energies stored into the subsystems by considering uncertainties on ILF's and CLF's. To be more precise, the nominal values of CLF's and their range of variability are those resulting from a previous analysis. The effect of uncertainties is modeled by using both a sensitivity approach and a DoE approach [10, 11]. DoE provides a regression model of energies: the coefficients of this model show the influence of the uncertain parameters on the energy stored in each subsystem. Finally, a comparison between sensitivity and DoE results is presented.

2 SEA equations

Under some particular hypotheses, it is possible to assume that the transmitted power between two subsystems is proportional to the difference of the energy stored in each subsystem. A list of these hypotheses is presented below:

- all the modes of a subsystem must be similar (i.e. they must have almost the same energy, damping, coupling with the other subsystems and they must be almost excited by the same input power),
- the coupling between the subsystems must be conservative,
- the eigenfrequencies must be uniformly probable in the frequency range,
- the force exciting the subsystems must be random and not-correlated,
- the interactions between the subsystems must be weak.

Thus, the SEA equations of N_{sub} coupled subsystems can be written as follows:

$$P_{i,inj} = \omega \eta_i E_i + \omega \sum_{j=1, j\neq i}^{N_{\text{sub}}} (\eta_{ij} E_i - \eta_{ji} E_j) \tag{1}$$

where i and j are indexes of the subsystems, η_i and η_{ij} are the internal loss factors (ILF) and the coupling loss factors (CLF), respectively, $P_{i,inj}$ is the power injected into the subsystem i, E is the energy in a given subsystem and ω is the central frequency of the considered band. Equations (1) represents the energy balance of the subsystems. The power dissipated in the subsystem i is:

$$P_{i,d} = \omega \eta_i E_i \tag{2}$$

The power transmitted from subsystem i to the subsystem j is:

$$P_{ij} = \omega \left(\eta_{ij} E_i - \eta_{ji} E_j \right) \tag{3}$$

The solution of the linear system (1) provides the energy stored in each subsystem. The set of equations (1) can be rewritten in a more convenient way as follows:

$$\mathbf{p} = \omega \mathbf{C} \mathbf{e} \tag{4}$$

where the coefficients of matrix $\mathbf{C}$ are combinations of ILF's and CLF's as shown in the following equations:

$$\begin{cases} C_{ij} = -\eta_{ji} \qquad i, j = 1 \ldots N_{\text{sub}}, \, i \neq j \\ C_{jj} = \eta_j + \sum_{i=1, i \neq j}^{N_{\text{sub}}} \eta_{ji} \end{cases} \tag{5}$$

If n_i and n_j are the modal densities of subsystems i and j, the following reciprocity relationship holds:

$$\eta_{ij} n_i = \eta_{ji} n_j \tag{6}$$

By enforcing reciprocity, under the assumption that only the η_{ji} with $j > i$ are known, it is:

$$\begin{cases} C_{ij} = \begin{cases} -\eta_{ji} & \text{if } j > i \\ -\eta_{ji} \dfrac{n_j}{n_i} & \text{if } j < i \end{cases} \\ C_{jj} = \eta_j - \sum_{i=1, i \neq j}^{N_{\text{sub}}} C_{ij} \end{cases} \tag{7}$$

3 Uncertainty propagation in SEA

SEA gives only the mean value of the energy of a set of similar systems. This is not a complete statistical information, because at least the dispersion of the data around the mean is lacking. The correct variance of the solution could be obtained by working directly on the equation of motion as it was done to provide the SEA equations. Here a study on the variability of SEA results is performed. Therefore, the goal of this research is not a way to achieve the variance matching the classical SEA solution, but to understand how much the energies (SEA solution) depend on uncertainties on CLF's and ILF's.

SEA equations are deterministic, and CLF's are deterministic functions of the physical parameters as well. The solution of this deterministic set of equations, the energies of the modal groups, depends on the ILF's, the CLF's and the injected powers. In order to study the variability of SEA solution, many techniques can be followed (Monte Carlo, sensitivity, Design of Experiments, etc.).

Let us consider a given mechanical system made of N_{sub} subsystems. CLF's depend on the material properties and the geometric parameters of the coupled subsystems. Therefore, a given η_{ij} depends, for instance, on the Young modulus of the systems i and j, Y_i and Y_j, and on the thickness of these subsystems, t_i and t_j. The

energy of each subsystem is calculated by solving equation (4) with the obvious implication that energies depend on the CLF's and the ILF's of the considered system.

By defining a range of variability of the physical parameters, a procedure can be developed in order to obtain the range of variability of the CLF's. At this point, both a sensitivity approach and a DoE procedure are developed to account for the dependence of the energy on the variability of SEA coefficients.

3.1 Approach using sensitivity

Sensitivity to loss factors is evaluated in correspondence to nominal values $\hat{\eta}$ of the CLF's and ILF's. To compare different sensitivity factors, it is assumed that changes $\Delta\eta_{kl}$ in the coupling loss factors are not arbitrarily chosen, but are those corresponding to variations of the physical parameters.

$$\Delta\mathbf{e}_{kl} = \left.\frac{\partial\mathbf{e}}{\partial\eta_{kl}}\right|_{\eta=\hat{\eta}} \Delta\eta_{kl} \tag{8}$$

and similarly for ILF's.

To find $\partial\mathbf{e}/\partial\eta_{kl}$, it is necessary to differentiate the solution of Eq. (4):

$$\mathbf{e} = \frac{1}{\omega}\mathbf{C}^{-1}\mathbf{p} \quad \Rightarrow \quad \frac{\partial\mathbf{e}}{\partial\eta_{kl}} = \frac{1}{\omega}\frac{\partial\mathbf{C}^{-1}}{\partial\eta_{kl}}\mathbf{p} \tag{9}$$

and similarly if internal loss factor η_k are considered instead of η_{kl}. Here it is assumed that the injected power is not affected by changes in CLF's and ILF's.

The derivative of $\mathbf{C}^{-1}$ can be easily obtained from the identity $\mathbf{C}\,\mathbf{C}^{-1} = \mathbf{I}$

$$\frac{\partial\mathbf{C}^{-1}}{\partial\eta_{kl}} = -\mathbf{C}^{-1}\frac{\partial\mathbf{C}}{\partial\eta_{kl}}\mathbf{C}^{-1} \tag{10}$$

where $\partial\mathbf{C}/\partial\eta_{kl}$ can be computed from Eq. (7):

$$\frac{\partial C_{ij}}{\partial\eta_{kl}} = \begin{cases} 1 & \text{if } i=j \text{ and } i=k \\ \dfrac{n_k}{n_l} & \text{if } i=l \text{ and } j=l \\ -1 & \text{if } i=l \text{ and } j=k \\ -\dfrac{n_k}{n_l} & \text{if } i=k \text{ and } j=l \\ 0 & \text{else} \end{cases} \tag{11}$$

and similarly for $\partial\mathbf{C}/\partial\eta_k$:

$$\frac{\partial C_{ij}}{\partial\eta_k} = \begin{cases} 1 & \text{if } i=j \text{ and } i=k \\ 0 & \text{else} \end{cases} \tag{12}$$

The variance of the energy stored in a given subsystem i can be approximately evaluated from the variance of CLF's and ILF's as follows:

$$\sigma_{E_i}^2 = \sum_{k=1}^{N_{\text{sub}}} \sum_{l=1}^{k-1} \left(\frac{\partial E_i}{\partial \eta_{kl}} \right)^2 \sigma_{\eta_{kl}}^2 + \sum_{k=1}^{N_{\text{sub}}} \left(\frac{\partial E_i}{\partial \eta_k} \right)^2 \sigma_{\eta_k}^2 \tag{13}$$

3.2 Approach using Design of Experiments

In Design of Experiments (DoE), the values of the variables that affect an output response are appropriately modified by a series of tests, to identify the reasons for changes in the response. This does not prevent from performing numerical tests whenever this may be convenient for a better understanding of the numerical problem under investigation.

Since many experiments involve the study of the effects of two or more variables or factors, it is necessary to investigate all possible combinations of the levels of the factors. This is performed by factorial designs which are very efficient for this task.

Specifically, if p factors at two levels are considered, a complete series of experiments requires 2^p observations and is called a two-level 2^p full factorial design. Usually, each series of experiments should be replicated several times using the same value of the factors to average out the effects of noise. Of course, this is unnecessary if experiments are numerical.

A feature of two-level factorial design is the assumption of linearity in the effect of each single factor and of multi-linearity in interactions among factors. To account for possible non linear effects, quadratic terms can be introduced, as in the following regression model for two factors:

$$f = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{12} x_1 x_2 + \alpha_{11} x_1^2 + \alpha_{22} x_2^2 + \varepsilon \tag{14}$$

where the α's are parameters whose values are to be determined, the variables x_1 and x_2 are defined on a coded scale from -1 to $+1$ (the low and high levels of the two factors) and ε is an error term.

Of course, a three level (low level -1, intermediate level 0, high level $+1$) factorial design, involving 3^p observations, is a possible option if quadratic terms are important. However, a more efficient alternative is the Central Composite Design (CCD) that starts from the 2^p design augmented with the *center point* i.e. a single observation with all factors at intermediate level, and *axial runs* where each factor is considered at two levels (the low level -1 and the high level $+1$) while the remaining factors are at the intermediate level, for a total of $2p$ observations.

Overall, a central composite design for p factors requires $n = 2^p + 2p + 1$ observations instead of 3^p observations required by the three level factorial design, with advantages for $p \geq 3$.

For p control factors, the experimental response can be expressed as a regression model representation of a 2^p full factorial experiment (involving 2^p terms),

augmented with p quadratic terms:

$$\begin{aligned} f = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_i + \sum_{i=1}^{p} \sum_{j=1}^{i-1} \alpha_{ji} x_j x_i + \ldots + \\ + \sum_{i=1}^{p} \sum_{j=1}^{i-1} \cdots \sum_{n=1}^{m-1} \alpha_{nm\cdots ji} x_n x_m \cdots x_j x_i + \sum_{i=1}^{p} \alpha_{ii} x_i^2 + \varepsilon \end{aligned} \tag{15}$$

The expression contains $2^p + p$ parameters α, each one providing an estimate of the effect of a single factor (linear or quadratic) or of a combination of them.

Note that Eq. (15) is linear in the parameters α, and it can be rewritten as:

$$f = \begin{bmatrix} 1 & x_1 & \cdots & x_p^2 \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{pp} \end{Bmatrix} + \varepsilon \tag{16}$$

having arranged the parameters in a vector α. A different equation can be written for each observation by varying the factors $(x_1, \ldots, x_p)$ as indicated by CCD.

By arranging the experimental responses in a vector $\mathbf{f}$, a linear relationship between $\mathbf{f}$ and α can be expressed in matrix notation as:

$$\mathbf{f} = \mathbf{X}\alpha + \varepsilon \tag{17}$$

where $\mathbf{X}$ is a $(2^p + 2p + 1) \times (2^p + p)$ matrix. The least square estimate of α is:

$$\hat{\alpha} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{f} \qquad \Rightarrow \qquad \hat{\mathbf{f}} = \mathbf{X}\hat{\alpha} \tag{18}$$

where $\hat{\mathbf{f}}$ is the fitted regression model.

The difference between the actual observations vector $\mathbf{f}$ and the corresponding fitted model $\hat{\mathbf{f}}$ is the vector of residuals $\mathbf{e} = \mathbf{f} - \hat{\mathbf{f}}$. The residuals account both for the modelling error ε and for the fitting error due to the least square estimation.

The total sum of squares SS_T is the sum of the squared deviations of each response f_i from its average value $\bar{f} = \sum_{i=1}^{n} f_i / n$:

$$\mathrm{SS}_T = \sum_{i=1}^{n} (f_i - \bar{f})^2 = \sum f_i^2 - n\bar{f}^2 = \mathbf{f}^T \mathbf{f} - \bar{\mathbf{f}}^T \bar{\mathbf{f}} \tag{19}$$

SS_T can be partitioned into a sum of squares due to the model SS_R and one due to residual SS_E:

$$\mathrm{SS}_T = \mathrm{SS}_R + \mathrm{SS}_E \tag{20}$$

It can be shown that the sum of squares of the residuals can be computed as:

$$\mathrm{SS}_E = \mathbf{f}^T \mathbf{f} - \hat{\alpha}^T \mathbf{X}^T \mathbf{f} \tag{21}$$

A low value of the ratio $\mathrm{SS}_E / \mathrm{SS}_T$ between the error sum of squares and the total sum of squares indicates that the chosen regression variables provide a good fit.

4 Results

The studied structure is a system of three Aluminum plates with the same thickness of 3 mm and different sizes: plate 1 (600 mm $\times$ 400 mm) along x axis, plate 2 (300 mm $\times$ 400 mm) and plate 3 (400 mm $\times$ 400 mm). These plates are welded along the 400 mm side (Figure 1). Power is input to plate 1 only.

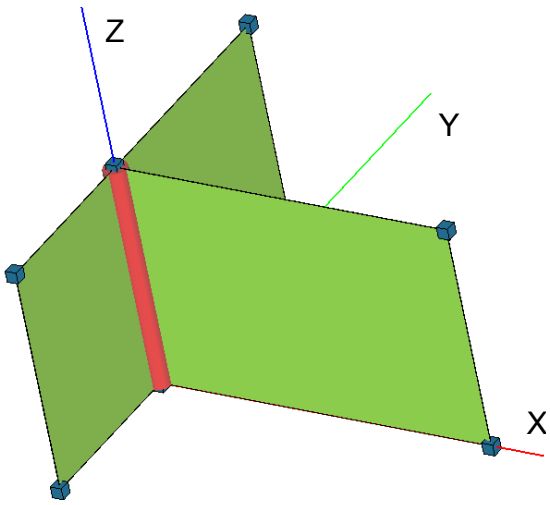

Fig. 1 Three plates system: power is input to plate 1 (along x axis).

The considered problem concerns the variability of the SEA solution, the energy of each subsystem, represented by the flexural modes of each plate, when the variability of CLF's and ILF's (6 parameters) is taken into account, while input power is assumed to be constant at all frequencies with a value P_1 of about 30 mW.

A range of variability of the Young modulus and the thickness is considered by varying these parameters of $\pm 10\%$ around the nominal values: 7×10^{10} Pa and 3 mm, respectively.

The correspondence between the variability of the physical parameters and the variability of the CLF's is preserved, because the nominal value of the CLF's, shown in Table 1, corresponds to the nominal value of the physical parameters. Furthermore, the variation of the CLF's, shown in Table 2, is the maximum difference from the nominal value obtained, by varying the physical parameters of $\pm 10\%$. It should be noted that the $\Delta\eta_{ij}$ are about 50% of the corresponding η_{ij} as a consequence of 10% variations of physical parameters.

The reciprocity relationship (6) is used to get the η_{ij} and the $\Delta\eta_{ij}$ with $i < j$. The modal densities are $n_1 = 12.46 \cdot 10^{-3}$, $n_2 = 6.233 \cdot 10^{-3}$ and $n_3 = 8.311 \cdot 10^{-3}$. The ILF's are varied of $\pm 10\%$ around 0.01.

Table 1 Nominal values $\hat{\eta}_{ij}$ of the coupling loss factors with $i > j$ at 14 third octave bands.

Frequency [Hz]	$\hat{\eta}_{21}$	$\hat{\eta}_{31}$	$\hat{\eta}_{32}$
	values to be multiplied by 10^{-3}		
100	15.440	11.580	11.288
125	13.808	10.356	10.015
160	12.203	9.152	8.739
200	10.913	8.185	7.688
250	9.759	7.319	6.731
315	8.691	6.518	5.959
400	7.710	5.783	6.042
500	6.894	5.170	6.252
630	6.138	4.603	5.533
800	5.443	4.083	4.762
1000	4.865	3.649	4.167
1250	4.347	3.260	3.668
1600	3.838	2.878	3.198
2000	3.427	2.570	2.831

Table 2 Variations $\Delta\eta_{ij}$, with $i > j$, of the coupling loss factors.

Frequency [Hz]	$\Delta\eta_{21}$	$\Delta\eta_{31}$	$\Delta\eta_{32}$
	values to be multiplied by 10^{-3}		
100	7.992	5.993	6.307
125	7.250	5.437	5.237
160	6.405	4.803	4.587
200	5.725	4.294	4.067
250	5.117	3.838	3.600
315	4.555	3.416	3.076
400	4.037	3.028	2.991
500	3.607	2.705	3.536
630	3.209	2.406	3.127
800	2.842	2.132	2.642
1000	2.537	1.903	2.285
1250	2.264	1.698	1.999
1600	1.995	1.496	1.739
2000	1.778	1.333	1.541

Sensitivities of energies in the three subsystems, with respect to coupling loss factors and internal loss factors, are evaluated according to the procedure outlined in section 3.1. In practise, each value represents the first order approximation of variation of the energy stored in a given subsystem (1,2, or 3) due to a change $\Delta\eta_{ij}$ of a given CLF or $\Delta\eta_i$ of a given ILF. Results are shown in Table 3 with reference to the 500 Hz third octave band, and to both CLF's and ILF's and in Figures 2, 3 and 4 for CLF's only but for all third octave bands.

A DoE procedure, with Central Composite Design requiring $2^6 + 2 \cdot 6 + 1 = 77$ experiments for each of 14 third octave bands from 100 Hz to 2000 Hz, is used. Table 4 shows the regression coefficients α at 500 Hz. Only the linear, quadratic and bilinear terms of the regression model are shown. The fit is very good because low values of SS_E/SS_T (not shown) are found. The results show that the α of the linear terms are always the most relevant. Furthermore, all energies decrease as the internal loss factors η_1, η_2 and η_3 increase, as expected. By comparing the sensitivities and the linear terms of the regression model, it can be noticed that they are quite similar: not only they indicate an energy variation in the same direction, but the values provided by the two models are very close to each other as well.

Figures 2, 3 and 4 show the regression coefficients α of the energies of the three subsystems. For the sake of comparison with sensitivities, only the linear terms of the regression model are drawn. In both cases, the trends are very similar. Also, it can be noticed that the magnitude of the sensitivities is always smaller than the magnitude of the corresponding regression coefficients. This is not unexpected because the regression model includes quadratic and multi-linear terms that can compensate for the noted difference.

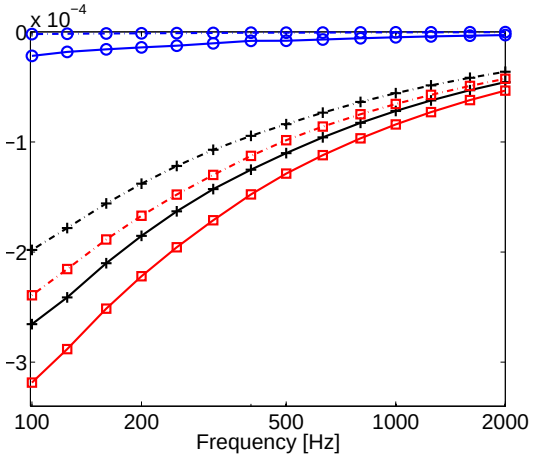

Fig. 2 Energy of subsystem 1: regression coefficients α (solid) vs sensitivities (dash–dotted) with respect to CLF's η_{21} (+), η_{31}(□) and η_{32} (O)

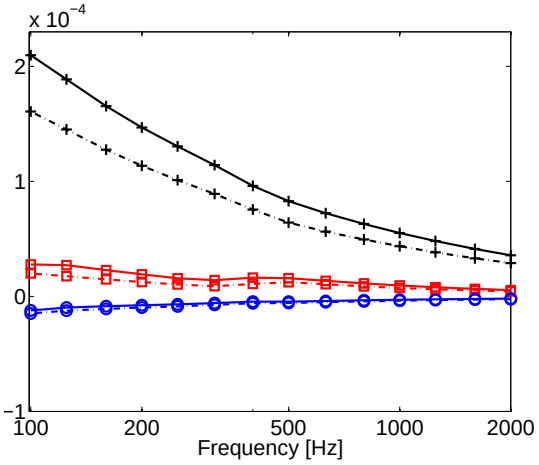

Fig. 3 Energy of subsystem 2: regression coefficients α (solid) vs sensitivities (dash–dotted) with respect to CLF's η_{21} (+), η_{31}(□) and η_{32} (O)

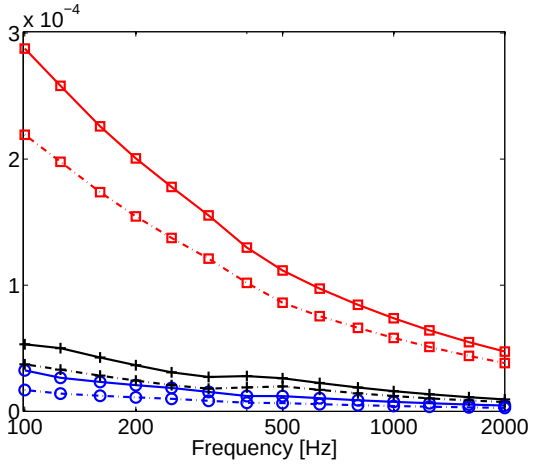

Fig. 4 Energy of subsystem 3: regression coefficients α (solid) vs sensitivities (dash–dotted) with respect to CLF's η_{21} (+), η_{31}(□) and η_{32} (O)

Table 3 Sensitivities[a] $\Delta\mathbf{e}$ at 500 Hz

index	loss factor	E_1	E_2	E_3
1	η_{21}	-0.0839	0.0641	0.0198
2	η_{31}	-0.0984	0.0123	0.0861
3	η_{32}	-0.0008	-0.0057	0.0065
4	η_1	-0.2412	-0.1039	-0.1366
5	η_2	-0.0894	-0.0557	-0.0622
6	η_3	-0.1161	-0.0614	-0.0954

[a] All values are multiplied by 10^3

Table 4 Regression coefficients[b] α at 500 Hz

index	loss factor	E_1	E_2	E_3
1	η_{21}	-0.1101	0.0828	0.0262
2	η_{31}	-0.1288	0.0158	0.1117
3	η_{32}	-0.0081	-0.0046	0.0121
4	η_1	-0.2504	-0.1046	-0.1372
5	η_2	-0.0882	-0.0572	-0.0614
6	η_3	-0.1137	-0.0603	-0.0969
7	$\eta_{21}\eta_{21}$	0.0316	-0.0233	-0.0081
8	$\eta_{21}\eta_{31}$	0.0408	-0.0181	-0.0224
9	$\eta_{21}\eta_{32}$	-0.0037	-0.0128	0.0165
10	$\eta_{21}\eta_1$	0.0113	-0.0019	0.0016
11	$\eta_{21}\eta_2$	-0.0071	0.0005	-0.0018
12	$\eta_{21}\eta_3$	-0.0021	0.0001	-0.0005
13	$\eta_{31}\eta_{31}$	0.0363	-0.0050	-0.0311
14	$\eta_{31}\eta_{32}$	0.0039	0.0149	-0.0187
15	$\eta_{31}\eta_1$	0.0132	0.0019	-0.0022
16	$\eta_{31}\eta_2$	-0.0013	-0.0002	-0.0001
17	$\eta_{31}\eta_3$	-0.0094	-0.0020	0.0002
18	$\eta_{32}\eta_{32}$	0.0023	0.0019	-0.0040
19	$\eta_{32}\eta_1$	0.0008	0.0004	-0.0004
20	$\eta_{32}\eta_2$	0.0004	0.0026	-0.0026
21	$\eta_{32}\eta_3$	-0.0010	-0.0029	0.0027
22	$\eta_1\eta_1$	0.0141	0.0045	0.0058
23	$\eta_1\eta_2$	0.0089	0.0048	0.0056
24	$\eta_1\eta_3$	0.0115	0.0055	0.0081
25	$\eta_2\eta_2$	0.0044	0.0008	0.0006
26	$\eta_2\eta_3$	0.0052	0.0031	0.0040
27	$\eta_3\eta_3$	0.0060	0.0015	0.0023

[b] All values are multiplied by 10^3

5 Conclusions

In this paper the effect of uncertainties of the loss factors is modeled by using both a sensitivity and a DoE approach. The present technique to model uncertainties is *parametric*, i.e. a model of the uncertainties of the system parameters is necessary to evaluate the variability of the solution.

The results obtained by DoE and sensitivity are compared and they show a good agreement. However, the DoE approach gives more information than sensitivity, because it allows to calculate the dependence of SEA solution both on the parameters (CLF's and ILF's) and on their combinations.

In addition to the obtained results on the uncertainty propagation, an important information can be obtained also for the design of a system with controlled energy levels. In fact, by assuming that the energy of one subsystem must not exceed a given level, the presented analysis allows to evaluate the sensitivity of such energy to the

uncertain parameters and, consequently, where to act in order to reduce vibration and noise.

Next activities will consider the study of more complicated systems to investigate the dependency of the SEA solution on the parameters of not directly connected subsystems and on the injected power.

Acknowledgements This research is supported by MIUR grants.

References

1. Lyon, R., De Jong, R.: Theory and Applications of Statistical Energy Analysis. The MIT Press, Cambridge (U.S.A.) (1995)
2. Lyon, R.: Statistical analysis of power injection and response in structures and rooms. Journal of the Acoustical Society of America **45**(3), 545–565 (1969)
3. Radcliffe, C.J., Huang, X.: Putting statistics into the statistical energy analysis of automotive vehicles. Journal of Vibration and Acoustics **119**(4), 629–634 (1997)
4. Langley, R., Cotoni, V.: Response variance prediction in the statistical energy analysis of built-up systems. Journal of the Acoustical Society of America **115**(2), 706–718 (2004)
5. Weaver, R.: Spectral statistics in elastodynamics. Journal of the Acoustical Society of America **85**, 1005–1013 (1989)
6. Culla, A., Carcaterra, A., Sestieri, A.: Energy flow uncertainties in vibrating systems: Definition of a statistical confidence factor. Mechanical Systems and Signal Processing **17**(3), 635–663 (2003)
7. de Langhe, R.: Statistical analysis of the power injection method. Journal of the Acoustical Society of America **100**(1), 294–304 (1996)
8. Bussow, R., Petersson, B.: Path sensitivity and uncertainty propagation in SEA. Journal of Sound and Vibration **300**(3-5), 479–489 (2007)
9. Langley, R., Cotoni, V.: Response variance prediction for uncertain vibro-acoustic systems using a hybrid deterministic-statistical method. Journal of the Acoustical Society of America **122**(6), 3445–3463 (2007)
10. Montgomery, D.: Design and Analysis of Experiments, 6th edn. Wiley, New York (2005)
11. D'Ambrogio, W., Fregolent, A.: Reducing variability of a set of structures assembled from uncertain substructures. In: Proceeding of 26th IMAC. Orlando (U.S.A.) (2008)