Control of Transient Power during Unintentional Islanding of Microgrids

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Control of Transient Power during Unintentional Islanding of Microgrids

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Abstract—In inverter-based microgrids, the paralleled inverters need to work in grid-connected mode and stand-alone mode and to transfer seamlessly between the two modes. In grid-connected mode, the inverters control the amount of power injected into the grid. In stand-alone mode, however, the inverters control the island voltage while the output power is dictated by the load. This can be achieved using the droop control. Inverters can have different power set-points during grid-connected mode. But in stand-alone mode they all need their power set-points to be adjusted according to their power ratings. However, during sudden unintentional islanding (due to a grid loss), transient power can flow from inverters with high power set-points to inverters with low power set-points, which can raise the DC link voltage of the inverters causing them to shut down. This paper investigates the transient circulating power between paralleled inverters during unintentional islanding and proposes a controller to limit it. The controller monitors the DC link voltage and adjusts the power set-point in proportion to the rise in the voltage. A small signal model of an islanded microgrid is developed and used to design the controller. Simulation and experimental results are presented to validate the design.

Index Terms—microgrid, droop control, power sharing, unintentional islanding.

I. INTRODUCTION

The concept of the microgrid has emerged in response to the increased penetration of renewable energy systems. In a microgrid, distributed generation (DG) units, energy storage systems (ESS), and loads are all aggregated into one unit and can be connected to the grid via a Static Transfer Switch (STS), as illustrated in Fig. 1. Due to its controllability, microgrids will become the building blocks of future smart grid. Compared to a single DG, a microgrid has more capacity and control.
flexibility to provide system reliability and meet power quality requirements. A microgrid can operate in grid-connected mode and in stand-alone mode. In grid-connected mode, DG units can export power to the grid (when the price is advantageous for example) or import power and store it in ESS to be used later. In case of a power outage, the microgrid works autonomously and provides power to the local load. ESS can be used to balance any mismatch between available renewable energy power and load demand. In order to provide an uninterruptible power supply, it is necessary for the microgrid to be able to transfer seamlessly from grid-connected mode to island mode and vice-versa. In addition, a low speed communication setup, between the supervisory controller and all units, can be used for power management and monitoring beside decision-making outcomes transfer.

![Microgrid Structure](image)

Fig. 1. Microgrid Structure

There are a number of publications on the control of grid interactive PWM inverters [1]-[7]. Chandorkar et al. [8] proposed a grid interactive PWM inverter based on $P-\omega$ and $Q-V$ droop control where the inverter frequency and voltage amplitude are drooped linearly with the inverter output active and reactive power, respectively. Inverters can operate in parallel and load sharing is achieved without the need for communication signals between the inverters.

Using droop control it is possible for the inverter to transfer from the grid-connected to the stand-alone mode automatically. During, grid-connected mode, the frequency is stiff and maintained by the power grid. Hence, the power set-point of the droop controller can be used to control the power output of the inverter. This power set-point can be set by an energy management system implemented inside the supervisory controller. In stand-alone mode, however, the frequency can deviate from its nominal value depending on the amount of power drawn by the local load and the power set-point can be used to reduce this deviation. In the case of multiple inverters operating in stand-alone mode, each set-point has to be adjusted according to the power rating of the inverter, i.e. according to the drooping gain. Therefore, the power set-point has two different purposes depending on the mode of operation: 1) in grid-connected mode, power set-point is set to control the
output power. 2) In stand-alone mode, it is used to reduce the frequency deviation. Before changing from grid-connected to stand-alone mode the supervisory controller needs to bring all the power set-points of all paralleled inverters to their nominal values before disconnecting from the grid. However, if unintentional islanding occurs, it is not possible instantaneously adjust the power set-points to match the load demand and circulation power can flow from the inverters with higher power set-points to inverters with lower set-points. It is important to note that by using the anti-islanding strategy, the period from grid failure until the opening of the STS may vary depending on the mismatch between the power generated by the microgrid and the local distributed load. If the mismatch is large, the islanding detection will be quite quick. However, if the mismatch is small, it will take longer for the anti-islanding controller to detect the grid loss. In the worst case scenario of perfect mismatch between the power generated and the load, the anti-islanding controller should not take more than 2 seconds according to the IEEE Standard 1547 [12]. If one inverter imports power during this period, the DC-link voltage will rise and might exceed the maximum limit. This will cause the inverter to shut down to prevent any possible damage. Even though there has been a number of publications recently on seamless transfer of microgrids [9],[10], and [11], the effect of different power set-points on the transient power between inverters has not yet been discussed.

This paper investigates the issue of transient power between parallel inverters during unintentional islanding. This circulating power can raise the DC link voltage of the inverters causing the inverter to shut down if the voltage level exceeds its maximum limit. The paper also proposes a controller to limit this circulating power by adjusting the power set-point according to the rise in the DC link voltage. A small signal model of a microgrid consisting of two inverters in island mode is developed and used to design the controller. Simulation and experimental results are presented to validate the design. The main contributions of this paper are: 1) Analysis of a microgrid during unintentional islanding and the effect of this on the DC-link voltage, 2) Using small signal perturbation to develop a model of an island microgrid of two parallel inverters, 3) The design of a controller that limits the rise of the DC-link voltage during unintentional islanding.

The rest of the paper is organized as follows. Section II discusses the droop control operation and analysis of the unintentional islanding. Section III presents a small signal model of a microgrid consisting of two inverters. The proposed controller is presented in section IV. Simulation and Experimental results are presented in Section V.

II. DROOP CONTROL

In this paper, a microgrid consisting of two inverters as shown in Fig. 2 is considered. The circuit diagram of each inverter and its LCL filter and controller is illustrated in Fig. 3. The system parameters are listed in Table I. The frequency and voltage droop control of an inverter operating in microgrid is given by
\[
\omega = \omega^* - k_o (P - P^*) \tag{1}
\]
\[
V = V^* - k_v (Q - Q^*) \tag{2}
\]

where \(\omega^*, V^*, k_o,\) and \(k_v\) are the nominal frequency reference, nominal voltage reference, proportional frequency drooping coefficient, and proportional voltage drooping coefficient, respectively. The active and reactive power set-points \(P^*\) and \(Q^*\) are set to the reference power \(P_{ref}\) and \(Q_{ref}\) in grid connected mode. In stand-alone mode, however, they are set to nominal active and reactive power \(P_n\) and \(Q_n\) to improve frequency and voltage regulation [9]. The inverter controller receives a signal from the supervisory controller about the status of the STS, and the set-points \(P^*\) and \(Q^*\) are set accordingly as shown in Fig. 3. Without losing generality, it is assumed that the two inverters in Fig. 2 have the same power ratings and hence they have the same drooping gains \(k_o1 = k_o2 = k_o\). In grid-connected mode, the inverters are assumed to have different power set-points such as \(P_1^* \neq P_2^*\). Fig. 4 shows the droop control of the two inverters with different power set-points. During grid-connected mode, the frequency is fixed by the stiff grid \(\omega_{grid}\) which equals the nominal frequency \(\omega_o\) and the two inverters generate different power values \(P_{grid(1)}\) and \(P_{grid(2)}\). When the microgrid transfers to island mode (due to unintentional islanding) the island frequency \(\omega_{island}\) deviates from its nominal value \(\omega_o\) and inverters 1 and 2 generate \(P_{island(1)}\) and \(P_{island(2)}\), respectively. In this case, \(P_{island(2)}\) is negative and hence inverter 2 is importing power. In the event of unintentional islanding and from (1), the system will have steady state frequency such as

\[
\omega_{island} = \omega^*_o - k_o (P_1 - P_1^*)
= \omega^*_o - k_o (P_2 - P_2^*) \tag{3}
\]

Knowing that the two inverters have the same drooping gain \(k_o1 = k_o2 = k_o\), (3) becomes

\[
P_1 = P_1^* + P_2 - P_2^* \tag{4}
\]

The total power dissipated by the load should equal the output power generated by the two inverters i.e.,

\[
P_L = P_1 + P_2 \tag{5}
\]

Substituting (4) and (5) in (3) the steady state island frequency is given by

\[
\omega_{island} = \omega^*_o - \frac{k_o}{2} (P_L - P_1^* - P_2^*) \tag{6}
\]

Equation (6) shows that the deviation from the nominal frequency depends on the local load and the power set-points of the inverters. Substituting, (5) in (4), the steady state output power of inverter 1 in island mode is given by

\[
P_1 = \frac{1}{2} (P_L + P_1^* - P_2^*) \tag{7}
\]

Similarly, the steady state output power of inverter 2 is given by

\[
P_2 = \frac{1}{2} (P_L + P_2^* - P_1^*) \tag{8}
\]
Equations (7) and (8) show that the two inverters will only share the load equally if $P_1^* = P_2^*$. They also show that if the load power is less than the difference between the two set-points, i.e.,

$$P_L < |P_1^* - P_2^*|$$

then one of the inverters will import power. Consider for example the case where $P_1^* = 30$ kW and $P_2^* = 10$ kW, Fig. 5 shows how the inverters output power varies with respect to local load. If islanding happens when the load is less than 20 kW, i.e., $P_L < P_1^* - P_2^*$, the power output $P_2$ will be negative hence inverter 2 will import power. This power will cause the DC-link voltage (see Fig. 2) to rise and if the voltage exceeds the maximum allowed limit, the inverter will shut down. This phenomenon will reduce the reliability of the microgrid. In normal operation and after unintentional islanding is detected by the supervisory controller, a signal is sent to all inverters updating them with the status of the microgrid (grid-connected or stand-alone) so that the inverters local controller changes the set-points. However, this signal is sent via a low speed communication protocol (such as CAN-bus). There can be a delay in order of seconds between when the islanding happens and until the update signal is received, during this transitional period, the dynamic of the microgrid is important in determining the amount of energy imported by an inverter.

![Fig. 2. Two inverters in microgrid](image-url)
DC-Link Capacitor

PWM Generator

Vc
Ic

Power Calculation

Vo
Io

P\ast \sin(\omega t)

Kw

Q\ast

\omega_o

V_o

Fig. 3. Inverter circuit diagram

TABLE I

DC/AC CONVERTER PARAMETER VALUES

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<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>L_i</td>
<td>1350\mu H</td>
<td>Inverter-side filter inductor</td>
</tr>
<tr>
<td>C</td>
<td>240\mu F</td>
<td>Filter capacitor</td>
</tr>
<tr>
<td>L_2</td>
<td>300\mu H</td>
<td>Grid-side filter inductor</td>
</tr>
<tr>
<td>C_{dc}</td>
<td>2000\mu F</td>
<td>DC link capacitor</td>
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Fig. 4. Droop control of two inverters in microgrid

Fig. 5. Output power versus load power, $P_1^* = 30\text{ kW}$, $P_2^* = 10\text{ kW}$.

### III. SMALL SIGNAL MODEL

In this section, a small signal model is developed to help analyzing the system behavior during unintentional islanding. The model will be developed for two inverters in island mode. The inverter can be modeled by a two-terminal Thevenin equivalent circuit as shown in Fig. 6 where $G(s)$ and $Z_o(s)$ represent the closed loop and output impedance transfer functions, respectively [13]. The response time of $G(s)$ is quite fast with respect to that of the outer droop control and hence it will be assumed as unity [9].
The output impedance \( Z_o(s) \) is predominantly inductive around the fundamental frequency \([4],[5],[13]\) and hence \( Z_o(s) \) can be approximated as \( Z_o(s) \approx sL_o \). The inductance \( L_o \) can be determined by the slope of \( Z_o(s) \) around the fundamental frequency and in the experimental setup used in this paper it is 2500\( \mu \)H. Fig. 7 shows the equivalent circuit for the two inverters operating in island mode. For simplicity, it is assumed that both inverters have identical output impedance \( X = \omega L_o \) and they supply a local resistive load.

### A. Power flow equations and power measurement

The current that flows from each inverter can be described as follows:

\[
I_n = \frac{V_n \angle \theta_n - V_L \angle 90}{X \angle 90}, \quad n = 1, 2
\]  

(10)

Applying Kirchhoff current law at the load node gives

\[
\frac{V_L - V_1 \angle \theta_1}{X \angle 90} + \frac{V_L - V_2 \angle \theta_2}{R} = 0
\]

(11) Rearranging (11), the load voltage is given by

\[
V_L = \frac{R(V_1 \angle \theta_1 + V_2 \angle \theta_2)}{2R + X \angle 90}
\]

(12)

The apparent power of each inverter is given by
\[ P_n + jQ_n = V_nI_n^*, \quad n = 1, 2 \]  

(13)

Substituting (12) in (10) and the result into (13), the active and reactive power for the two inverters are given by

\[ P_1 = \frac{XV_1^2}{R} + 2V_1 \sin(\theta_1 - \theta_2) + \frac{XV_1}{R} \cos(\theta_1 - \theta_2) \]

\[ Q_1 = \frac{X^2}{R^2} + 2V_1 \left( -2V_1 \cos(\theta_1 - \theta_2) + \frac{XV_1}{R} \sin(\theta_1 - \theta_2) \right) \]

(14)

\[ P_2 = \frac{XV_2^2}{R} + 2V_2 \sin(\theta_2 - \theta_1) + \frac{XV_2}{R} \cos(\theta_2 - \theta_1) \]

\[ Q_2 = \frac{X^2}{R^2} + 2V_2 \left( -2V_2 \cos(\theta_2 - \theta_1) + \frac{XV_2}{R} \sin(\theta_2 - \theta_1) \right) \]

(15)

(16)

When practically implementing the droop control, average active and reactive powers need to be measured and thus the droop control equations described in (1) and (2) become

\[ \omega = \omega_o - k_o (P_{avg} - P^*) \]

(18)

\[ V = V_o - k_v (Q_{avg} - Q^*) \]

(19)

The average power can be calculated by a low pass filter using the following equations:

\[ P_{avg} = F(s)P(s) \]

(20)

\[ Q_{avg} = F(s)Q(s) \]

(21)

where \( P(s) \) and \( Q(s) \) are the instantaneous active power and reactive power, respectively and described in equation (14)-(17).

\( F(s) \) is the transfer function of the LPF and is given by

\[ F(s) = \frac{1}{\tau s + 1} \]

(22)

where \( \tau \) is the filter time constant.

**B. State Space Equations**

By perturbing the power flow equations (14)-(17) we obtain,
\[ \Delta P_1 = a_1 \Delta V_1 + b_1 \Delta V_2 + c_1 \Delta \theta_1 + d_1 \Delta \theta_2 \]  \hspace{1cm} (23)

\[ \Delta P_2 = a_2 \Delta V_1 + b_2 \Delta V_2 + c_2 \Delta \theta_1 + d_2 \Delta \theta_2 \]  \hspace{1cm} (24)

\[ \Delta Q_1 = a_3 \Delta V_1 + b_3 \Delta V_2 + c_3 \Delta \theta_1 + d_3 \Delta \theta_2 \]  \hspace{1cm} (25)

\[ \Delta Q_2 = a_4 \Delta V_1 + b_4 \Delta V_2 + c_4 \Delta \theta_1 + d_4 \Delta \theta_2 \]  \hspace{1cm} (26)

where \( \Delta \) means a small perturbation around the equilibrium points. The coefficients \( a, b, c, d \) (with the different subscripts) are obtained by calculating the corresponding partial derivatives and they are given in Appendix A.1.

By perturbing (18) and (19) we obtain,

\[ \Delta \omega_n = -k_n \Delta P_{avg} \]  \hspace{1cm} (27)

\[ \Delta V_n = -k_n \Delta Q_{avg} \]  \hspace{1cm} (28)

Substituting (22) in (20) and (21) the average power is related to the instantaneous power by

\[ \Delta P_{avg} = \frac{1}{r.s+1} \Delta P_n \]  \hspace{1cm} (29)

\[ \Delta Q_{avg} = \frac{1}{r.s+1} \Delta Q_n \]  \hspace{1cm} (30)

Substituting (23) in (29) and rearranging gives,

\[ s \Delta P_{avg} = \frac{1}{r} (a_1 \Delta V_1 + b_1 \Delta V_2 + c_1 \Delta \theta_1 + d_1 \Delta \theta_2) - \frac{1}{r} \Delta P_{avg} \hspace{1cm} n = 1, 2 \]  \hspace{1cm} (31)

Substituting (25) in (30) and rearranging gives

\[ s \Delta Q_{avg} = \frac{1}{r} (a_3 \Delta V_1 + b_3 \Delta V_2 + c_3 \Delta \theta_1 + d_3 \Delta \theta_2) - \frac{1}{r} \Delta Q_{avg} \hspace{1cm} n = 1, 2 \]  \hspace{1cm} (32)

The inverter power angle is related to the frequency by

\[ s \Delta \theta_n = \Delta \omega_n, \hspace{1cm} n = 1, 2 \]  \hspace{1cm} (33)

Equations (23)-(33) can be combined into a homogeneous state space equation such as,

\[ [sX_1] = [A_n][X_1] \]  \hspace{1cm} (34)

where \([X]\) contains the state variables and is given by

\[
\begin{bmatrix}
\Delta \delta_1 \\
\Delta \delta_2 \\
\Delta \omega_1 \\
\Delta \omega_2 \\
\Delta V_1 \\
\Delta V_2 \\
\Delta P_{avg1} \\
\Delta P_{avg2} \\
\Delta Q_{avg1} \\
\Delta Q_{avg2}
\end{bmatrix}
\]
The state variable matrix $[A]$ is given in Appendix A. Equation (34) represents a state space model of the two inverters in island mode.

IV. DC-LINK VOLTAGE CONTROLLER

A. DC link modeling

As explained earlier, the imported power may raise the DC-link voltage to an unacceptable limit. In this section, the state space model developed in III will be extended to include the DC-link voltage. Fig. 8(a) shows the DC-link capacitor when the inverter is importing power during the transient period. The energy $E$ absorbed by the capacitor is related to the capacitor voltage $V_{DClink}$ by,

$$E = \int P(t)dt = \frac{1}{2} C_{dc} V_{DClink}^2$$

where $P$ is the absorbed power and $C_{dc}$ is the DC-link capacitance. In order to have a linear relationship between $\Delta P$ and $\Delta V_{DClink}$, the square root relation needs to be linearized. Let $x = V_{DClink}^2$ and $y(x) = \sqrt{x}$, a small change in $y$ is given by:

$$\Delta y = \Delta x \frac{dy}{dx}$$

where $\Delta x$ is a small change in $x$ and $x_o$ is the equilibrium point. Given that the DC-link voltage needs to be around $x_o$, $\Delta y$ becomes

$$\Delta y = m \Delta x$$

where $m = \left. \frac{dy}{dx} \right|_{x=x_o} = \frac{1}{2\sqrt{x}} |_{x=x_o^2}$

Therefore, as shown in Fig. 8(b), the linear relationship between the DC-link voltage and the power is given by

$$\Delta V_{DClink} = \frac{2m}{C_{dc}s} \Delta P$$

Substituting (23) and (24) in (38) gives the state equation for $\Delta V_{DClink1}$ and $\Delta V_{DClink2}$. This can then be integrated with (34) to give (39) such as

$$[sX_2] = [A_2][X_2]$$
\[
[X_2] = \begin{bmatrix}
X_1 \\
\Delta V_{DClink1} \\
\Delta V_{DClink2}
\end{bmatrix}
\]

\[
[A_2] = \begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}
\]

\[A_2 \text{ is given in appendix A.2}\]

Equation (39) represents the state space equation for the complete model of the two inverters in island mode including the dynamics of the DC-link voltages.

![Fig. 8. DC-link capacitor (a) when DC/AC inverter is importing power, (b) small signal model.](image)

**B. Design of DC-link voltage controller to limit transient power**

In this section, a controller is proposed to limit the amount of imported power during the transitional period so the inverters carry on working without interruption until they receive the update signal from the supervisory controller as explained earlier. The controller reduces the power set-points if the DC-link voltage exceeds a certain limit. The proposed controller is illustrated in Fig. 9.

![Fig. 9. Proposed Controller based on DC-link voltage](image)
During normal operation when the power flows out of the inverter, the DC-link voltage is regulated by a DC/DC boost converter. The reference $V_{DC_{link}}^*$ is the nominal DC voltage such that it only becomes effective if the DC-link voltage exceeds a threshold which means the inverter is importing power. In this case, the controller will change the reference power set-point until the DC voltage difference is minimized.

The controller gain $k_{DC}$ must be selected carefully to guarantee good stability and good reduction in imported energy. In order to analyze stability, the small signal model described by (39) will be slightly modified to include the DC-link voltage controller. If the proposed controller is implemented for inverter 1, from Fig. 9 we can write,

$$P = P_{avg1} - P' + k_{DC} (V_{DC_{link1}} - V'_{DC_{link1}})$$  \( (40) \)

By perturbing (40) around the equilibrium points we get:

$$\Delta P = \Delta P_{avg1} + k_{DC} \Delta V_{DC_{link1}}$$  \( (41) \)

Substituting (41) in (27) gives,

$$\Delta \omega = -k_{\omega} (\Delta P_{avg1} + k_{DC} \Delta V_{DC_{link1}})$$  \( (42) \)

The state variable “s. $\Delta \omega$” becomes

$$s \Delta \omega = -k_{\omega} s \Delta P_{avg1} - k_{DC} s \Delta V_{DC_{link1}}$$  \( (43) \)

The state space equation of (39) can be modified to include this control loop. It can be done by modifying the 3rd row of the state matrix of $A_2$ according to (43). If the controller is implemented for inverter 2 then the 4th row of $A_2$ is also modified.

In order to analyze the effect of $k_{DC}$ on the system stability, the locus of the Eigen values of $A_2$ is plotted as shown in Fig. 10. The Eigen values of the system are plotted for $0 < k_{DC} < 10$. They are in the left half plane for the selected gain range.

The arrows depict the evolution of the Eigen values when the gain value increases, which show that the system becomes faster with higher overshoot by increasing the gain since the complex poles become the dominant poles whilst the effect of the real poles decreases, which exposing more oscillation and even instability if it is quite high.
Increasing the gain $k_{DC}$ decreases the absorbed energy and so the DC voltage is minimized. However, it will increase the oscillatory components resulting in higher overshoot. Thus, by choosing $k_{DC}$ to 1, a compromise between stability and absorbed energy is achieved.

V. SIMULATION AND EXPERIMENTAL RESULTS

The simulation results of the state space model developed earlier are compared with that of a detailed model developed using Matlab/SimPowerSystems and the results obtained from an experimental setup. The two inverters have been modeled as ideal voltage sources in Simulink as shown in Fig. 7. The simulation parameters are shown in Table II. A laboratory-scale microgrid was built. It consists of two DG units connected in parallel. Each DG is interfaced to the microgrid by a VSI with LCL filter. The experimental setup is shown in Fig. 11. In addition, setup parameters are listed in Table I. A circuit breaker is used to connect each unit to the PCC. Two Semikron SKAI IGBT blocks are used. The dSPACE 1103 control unit is used to implement and realize the proposed controller scheme in real time. The dSPACE interfacing board is equipped with eight analog to digital channels (ADC) to interface the measured signals. The software code is generated by the Real-Time-Workshop under Matlab/Simulink environment. The switching and sampling frequencies used are 10KHz and 20kHz, respectively. Limitations on some of the laboratory equipment prevent us from operating at full voltages so a step-down transformer is used at the grid side.

TABLE II

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<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$P_1^*$</td>
<td>Active Power set-point for inverter 1</td>
<td>20 W</td>
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<td>Value</td>
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<td>$P_2^*$</td>
<td>Active Power set-point for inverter 2</td>
<td>0 W</td>
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<td>$Q_1^*$</td>
<td>Reactive Power set-point for inverter 1</td>
<td>0 VAR</td>
</tr>
<tr>
<td>$Q_2^*$</td>
<td>Reactive Power set-point for inverter 2</td>
<td>0 VAR</td>
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<td>$P_L$</td>
<td>Load power</td>
<td>0</td>
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<tr>
<td>$X$</td>
<td>Inverter output inductance (small signal and detailed simulation model)</td>
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<td>$k_w$</td>
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</tr>
<tr>
<td>$m$</td>
<td></td>
<td>0.0125</td>
</tr>
</tbody>
</table>

A. Initial Conditions for the state space model

Two cases will be considered to validate the state space model: In case 1, two inverters are started in island mode with different power set-points. Even though this case is not practical, as the supervisory controller should set the power set-points equally before starting the inverters, it provides good test for validating the small signal model. Case 2: represents unintentional islanding when the two inverters have different set points. Each state variable of the small signal model described in (34) represents the deviation $\Delta x(t)$ from the equilibrium point $x_{eq}$. The time domain response $x(t)$ is calculated by adding the deviation to the equilibrium point such as

$$x(t) = x_{eq} + \Delta x(t) \quad (44)$$

The equilibrium points are calculated as follow: the average power equilibrium points are calculated using (7) and (8). The frequency equilibrium point is calculated using (6). The angle equilibrium point can be calculated using (45) which relates the active power transferred from each inverter to the load node

$$P_n = \frac{V_n V_L \sin(\theta_n)}{X}, \quad n = 1, 2 \quad (45)$$

All equilibrium points are listed in Table III. The initial deviations from the equilibrium points $\Delta x(0)$ are calculated using (46) such as

$$\Delta x(0) = x(0) - x_{eq} \quad (46)$$

where $x(0)$ represents the initial condition at the beginning of the simulations. In case 1, $x(0)$ are the initial conditions at rest before starting the inverters. In case 2, however, $x(0)$ represents the initial condition in grid-connected mode just before
the unintentional islanding. All the initial conditions and initial deviations are calculated for both cases and listed in Table III.

B. Validation of the small signal model

Results of case 1

Fig. 12 depicts the average active power for both inverters under case 1 conditions. The figure includes the results obtained from the small signal model, the detailed Simulink model and the experimental setup. As can be seen, the small signal model is in complete agreement with the detailed model and both agree with the experimental results. A similar conclusion can be obtained from Fig. 13 and Fig. 14, which show the response of the frequency and phase angle, respectively.

Results of case 2

In this case, the two inverters are initially operating in grid connected mode. At time $t = 2.1$ seconds, the grid is isolated so the two inverters operate in island mode. Fig. 15 shows the responses of the average active power of both inverters using the detailed and the small signal model and the experimental setup. Again, the dynamics of are all in full agreement. Fig. 15 also shows the frequency responses of both inverters. The behavior of the second inverter, which is importing 10W, develops high voltage across the DC-link capacitor resulting in a power trip as shown in Fig. 16, which depicts the experimental DC link voltage of inverter 2 before and after islanding. When the DC link exceeds the max limit, a trip signal is generated.

C. Results of the proposed DC-Controller

Fig. 17 shows the simulation and experimental results of the unintentional islanding case (case 2) with $P_L=0$. The first inverter was generating 20W while the second inverter was generating 0W in grid-connected mode. When the islanding occurs at $t=3$ sec the output powers become $P_1 = 10W$ and $P_2 = -10W$ which agree with (7) and (8). The DC-link voltage of inverter 2 starts to rise, and when it reaches 100V the DC-Link controller is activated. The active powers are then reduced to zero and the dc voltage is reduced to 50V. The charging time in simulation and practical results are slightly different since the dynamics of the DC source (applied on the DC-link capacitor) is beyond the scope of this paper besides considering ideal AC source as stated previously so the simulated DC value is a calculated value not measured one. The effectiveness of the proposed controller is clear, as it has prevented the DC-link voltage from reaching the trip limit by quickly adjusting the power demand and the inverters kept working waiting for an update signal to be received from the supervisory controller.
According to the Eigen values of the DC-link controller of Fig. 10, the predicted transient response of the dc voltage is
\[ c(t) = e^{-2.6t} \sin(2\pi 3.7t) \]. The magnified portion in Fig. 17 shows the transient oscillation of the dc voltage. The oscillation frequencies of the detailed model and the experimental setup are 3.57Hz and 3.125Hz, respectively. The small signal model has provided good prediction of the transient response. The exponential decaying term also agrees with the Eigen values of Fig. 10.

![Fig. 11. View of the laboratory setup](image)

**TABLE III**

**EQUILIBRIUM POINTS AND INITIAL DEVIATIONS FOR THE SMALL SIGNAL MODEL**

<table>
<thead>
<tr>
<th>State Variable</th>
<th>( X_{eq} )</th>
<th>Case 1 (starting in island mode)</th>
<th>Case 2 (unintentional islanding)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( X(0) )</td>
<td>( \Delta X(0) )</td>
<td>( X(0) )</td>
</tr>
<tr>
<td>( \Delta \theta_1 )</td>
<td>0.019 rad</td>
<td>0 rad</td>
<td>-0.019 rad</td>
</tr>
<tr>
<td>( \Delta \theta_2 )</td>
<td>-0.019 rad</td>
<td>0 rad</td>
<td>0.019 rad</td>
</tr>
<tr>
<td>( \Delta \omega_1 )</td>
<td>314.66 rad/s</td>
<td>315.16 rad/s</td>
<td>0.5 rad/s</td>
</tr>
<tr>
<td>( \Delta \omega_2 )</td>
<td>314.66 rad/s</td>
<td>314.16 rad/s</td>
<td>-0.5 rad/s</td>
</tr>
<tr>
<td>( \Delta V_1 )</td>
<td>23 Vrms</td>
<td>23 Vrms</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta V_2 )</td>
<td>23 Vrms</td>
<td>23 Vrms</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta P_{avg1} )</td>
<td>10W</td>
<td>0</td>
<td>-10 W</td>
</tr>
<tr>
<td>( \Delta P_{avg2} )</td>
<td>-10W</td>
<td>0</td>
<td>10 W</td>
</tr>
<tr>
<td>( \Delta Q_{avg1} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \Delta Q_{avg2} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 12. Average measured active power of inverters 1 and 2 in island mode (case 1)

Fig. 13. Frequency of inverters 1 and 2 in island mode (case 1)
Fig. 14. Phase of inverters 1 and 2 in island mode (case 1)

Fig. 15. Average active power (above) and frequency (below) of inverters 1 and 2 after grid loss - unintentional islanding (case 2)
Fig. 16. DC-Link voltage across the capacitor of inverter 2

Fig. 17. Average active power of both inverters and and DC link voltage of inverter 2 with proposed controller ($k_{DC}=1$) (a) Simulink detailed model, (b) experimental setup
VI. CONCLUSION

This paper has investigated the transient power between paralleled inverter during unintentional islanding and a controller to limit this circulating power has been proposed. The controller monitors the DC link voltage and in case the voltage rises above a specific limit, indicating power being imported, the controller adjusts the power set-point in proportion to the rise in the voltage. A small signal model of a microgrid consisting of two inverters in island mode has been developed and used to design the controller. Simulation and experimental confirmed the accuracy of the developed model and the validity of the design.

REFERENCES


**APENDIX A**

1. State space equations coefficients

\[
a_1 = \frac{\partial P}{\partial V_i} = 3 \left(\frac{X V_i + 2V_i \sin(\theta_{in} - \theta_{in}) + \frac{X}{R} V_i \cos(\theta_{in} - \theta_{in})}{M}\right)
\]

\[
b_1 = \frac{\partial Q_i}{\partial V_i} = 3 \left(\frac{2V_i \sin(\theta_{in} - \theta_{in}) + \frac{X}{R} V_i \cos(\theta_{in} - \theta_{in})}{M}\right)
\]

\[
c_1 = \frac{\partial P_i}{\partial \theta_i} = \frac{3}{M} \left(\frac{2V_i \sin(\theta_{in} - \theta_{in}) + \frac{X}{R} V_i \cos(\theta_{in} - \theta_{in})}{M}\right)
\]

\[
d_1 = \frac{\partial Q_i}{\partial \theta_i} = -c_1
\]

\[
a_2 = \frac{\partial P}{\partial V_i} = 3 \left(\frac{\frac{X}{R} V_i \cos(\theta_{in} - \theta_{in}) + \frac{X}{R} V_i \cos(\theta_{in} - \theta_{in})}{M}\right)
\]

\[
b_2 = \frac{\partial Q_i}{\partial V_i} = 3 \left(\frac{2V_i \sin(\theta_{in} - \theta_{in}) + \frac{X}{R} V_i \cos(\theta_{in} - \theta_{in})}{M}\right)
\]

\[
c_2 = \frac{\partial P_i}{\partial \theta_i} = \frac{3}{M} \left(\frac{2V_i \sin(\theta_{in} - \theta_{in}) + \frac{X}{R} V_i \cos(\theta_{in} - \theta_{in})}{M}\right)
\]

\[
d_2 = \frac{\partial Q_i}{\partial \theta_i} = -c_2
\]
where $M = 4X + \frac{X^T}{R'}$

2. Complete state space model

$$[A] = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{k_c c_1}{\tau} & \frac{k_c d_1}{\tau} & \frac{1}{\tau} & 0 & \frac{k_a a_1}{\tau} & \frac{k_b b_1}{\tau} & 0 & 0 & 0 & 0 \\
\frac{k_c c_2}{\tau} & \frac{k_c d_2}{\tau} & 0 & \frac{1}{\tau} & \frac{k_a a_2}{\tau} & \frac{k_b b_2}{\tau} & 0 & 0 & 0 & 0 \\
\frac{k_c c_3}{\tau} & \frac{k_c d_3}{\tau} & 0 & 0 & \frac{1}{\tau} & \frac{k_a a_3}{\tau} & \frac{k_b b_3}{\tau} & 0 & 0 & 0 \\
\frac{k_c c_4}{\tau} & \frac{k_c d_4}{\tau} & 0 & 0 & 0 & \frac{1}{\tau} & \frac{k_a a_4}{\tau} & \frac{k_b b_4}{\tau} & 0 & 0 \\
\frac{c_1}{\tau} & \frac{d_1}{\tau} & 0 & 0 & a_1 & \frac{b_1}{\tau} & -1 & 0 & 0 & 0 \\
\frac{c_2}{\tau} & \frac{d_2}{\tau} & 0 & 0 & a_2 & \frac{b_2}{\tau} & 0 & -1 & 0 & 0 \\
\frac{c_3}{\tau} & \frac{d_3}{\tau} & 0 & 0 & a_3 & \frac{b_3}{\tau} & 0 & 0 & -1 & 0 \\
\frac{c_4}{\tau} & \frac{d_4}{\tau} & 0 & 0 & a_4 & \frac{b_4}{\tau} & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$[A] = \begin{bmatrix}
\frac{2mc_1}{c_w} & \frac{2md_1}{c_w} & 0 & 0 & \frac{2mc_2}{c_w} & \frac{2md_2}{c_w} & 0 & 0 & 0 & 0 \\
\frac{2mc_3}{c_w} & \frac{2md_3}{c_w} & 0 & 0 & \frac{2mc_4}{c_w} & \frac{2md_4}{c_w} & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$