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OPTIMIZATION OF MEDICAL EQUIPMENT REPLACEMENT USING DETERMINISTIC DYNAMIC PROGRAMMING

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ABSTRACT

In this paper, we use a multistage deterministic dynamic programming (DDP) approach to optimize medical equipment replacement for several revenue and depreciation scenarios. Each scenario is an optimal path which shows whether to keep an existing piece of medical equipment (defender) or replace it with a more economical alternative (challenger). Such an optimal path is a keep-replace sequence of the highest returns (or lowest costs) obtained with backward recursion in time. For each scenario, we estimated the optimal-sequence benefit as the difference between the highest returns (or the lowest costs) and returns (costs) of keeping medical equipment until the end of its expected life. We investigated this benefit for the scenarios of no revenue for the defender and the challenger, the scenario of equal revenues for both, and the scenario of higher revenue for the challenger. Our experiments show that the percentages of optimal-sequence benefits relative to the current acquisition cost for the three scenarios are 124%, 164%, and 204%, respectively. Moreover, the number of replacement actions increases with increasing challenger revenue and decreases with increasing depreciation rate. Last, the effect of the inflation rate on the optimal-sequence benefit was investigated.

KEYWORDS: Medical equipment, deterministic dynamic programming, optimal replacement, clinical engineering, healthcare technology management.

1. INTRODUCTION

The equipment replacement problem has been under study in the field of engineering economics since the 1940s [1]. Landmark contributions to this problem include those of [2, 3]. Christer and Goodbody in 1980 asked "At what time should a

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currently operating equipment or fleet of equipment be replaced?" Several approaches have been proposed since then to answer this question for different settings and applications [4]. In healthcare facilities, a piece of medical equipment is usually replaced because of performance degradation and/or technological obsolescence [5, 6]. Traditional replacement policies are based upon simple calculations of equipment depreciation and economic life. Needless to say, such policies are suboptimal since they do not take into consideration any optimality criteria. To overcome this gap, different approaches have been proposed in the literature for medical equipment replacement. For instance [7] developed a quantitative approach which takes into consideration technical factors, safety factors, and financial factors. The replacement decision is made based on the resultant relative replacement number for each device.

The use of risk assessment tools to prioritize medical equipment (whether for replacement or maintenance) was first proposed by [8] who devised a simple mathematical model to prioritize equipment based on certain attributes such as equipment service and support, equipment function, cost benefits, and clinical efficacy. A list of decision criteria was prepared to get ranked for an effective replacement analysis of critical medical equipment [9]. To determine replacement priority of old medical equipment, an evaluation tool based on multi-criteria decision analysis was developed, regarding technical and economic portion [10]. A scoring system that produced real-time equipment replacement prioritization results was designed by [11]. An analytical-hierarchy-process group-decision-making model was proposed to prioritize medical equipment replacement using a priority index [12]. Other decision support methods seeking quantitative solutions to the equipment replacement problem were tested. Examples include fault tree analysis [9, 13] quality function deployment, genetic algorithms [14], artificial neural networks [15], and fuzzy logic [16]. The search for an optimal solution to the equipment replacement problem was equally prolific. The total expected cost was formulated of retaining an existing medical equipment for a further K^{th} year versus replacing it with a new one for a period L [17]. The objective function was the total expected discounted cost per unit of usage over (K+L) years. The minimization was carried out with respect to K and L. The authors showed the applicability of their model in the case of medical ventilators. The results showed that, the relationship between the optimal replacement time of the current machine, and the penalty cost seems to follow a fixed pattern, regardless of the current machine age. Deterministic and stochastic models were proposed to determine the optimal replacement time for linear accelerators in a radiotherapy department [18]. A cost analysis was performed to quantify the losses incurred by equipment downtime. On average, the stochastic model yielded longer replacement times than the deterministic one [19].

Much earlier, Bellman was the first to formulate the equipment replacement problem as a dynamic programming (DP) problem [20]. He formulated a discounted DP version of the economic life of an asset model and determined analytically the optimal age to replace the asset. Basically, a DP model is a recurrence equation that connects different stages of the problem in order to assure that the optimal solution for each stage is the optimal solution for the whole problem [21]. DP has been used in several fields to solve replacement problems such as the automobile replacement where deterministic dynamic programming (DDP) was used to develop a general solution methodology that can be used to make optimal keep/replacement decisions for both brand-new and used vehicles both with and without annual budget considerations [22]. In a specific study in Benin City, Edo State, Nigeria which covered the period 2008 to 2013 and for the Toyota brand of buses only, the replacement problem was approached using backward recursive dynamic programming analysis [23]. The conventional keep-replace dynamic programming model was extended to allow the options to overhaul the asset [24]. Also, the framework of dynamic programming was reviewed for hydropower scheduling, and highlight the differences between deterministic and stochastic approaches [25]. A dynamical model was proposed to optimize replacement scheduling for the drainage pump station which is one of main facilities of flood control infrastructure [26].

The medical equipment replacement problem studied in this paper is solved using DDP under the assumption that the following model parameters are constant or predetermined and can be estimated using historical data: the annual operation and maintenance cost, annual revenue of equipment, first cost of a new equipment and salvage value. If there is a significant uncertainty in some model parameters such as the maintenance cost, then the stochastic dynamic programming approach (SDP) may be more appropriate [27]. The SDP approach shall be investigated in a future publication. In this paper, we make four main contributions. First, we use a multistage deterministic dynamic programming (DDP) approach to decide optimally whether to keep or replace an existing piece of medical equipment based on economic viability. To the best of our knowledge, this methodology wasn't used before in the medical equipment field. Second, we simulate three interesting possibilities for medical equipment replacement based on the relative revenues of an existing piece of medical equipment (defender) and the alternative equipment (challenger). Third, a simulation of three different depreciation models is made. Finally, we study the combined effects of different inflation rates and depreciation models on the three possibilities.

The rest of this paper is organized as follows: Section 2 reviews the DDP method, and discusses the relevant operation and maintenance costs, revenue models, future acquisition costs, as well as inflation and depreciation models. Section 3 shows the results of three numerical simulation scenarios. Discussion and Conclusions are given in Sections 4 and 5, respectively.

2. METHODS

In this section, we review Bellman's dynamic programming approach for equipment replacement. Figure 1 shows the basic structure of a Bellman's grid where each node represents the equipment's age at a specific point in time. Each arc represents the decision to either keep or replace the equipment. A solid arc connecting nodes of ages n and n+1 indicates a "Keep" decision, while a dashed arc indicates a "Replace" decision. If the equipment is replaced at the end of a period containing N stages, the number of all possible paths is equal to 2^N -1. Because we have two decisions at each state the maximum number of states equal to O (1+2+3+…+N-1 +1). Thus, O (N^2) represents the time complexity of Bellman's dynamic programming algorithm. Using the backward recursion, the solution procedure begins from the end and moves backward stage by stage till the optimal path starting at the beginning stage is found. We used a table to store all results ever calculated by recursive procedure. Once the recursive procedure requests a set of inputs which were already used, the results are just obtained from the table. In this case the time of algorithm will be reduced. We describe the DDP scheme following an approach similar to [22].

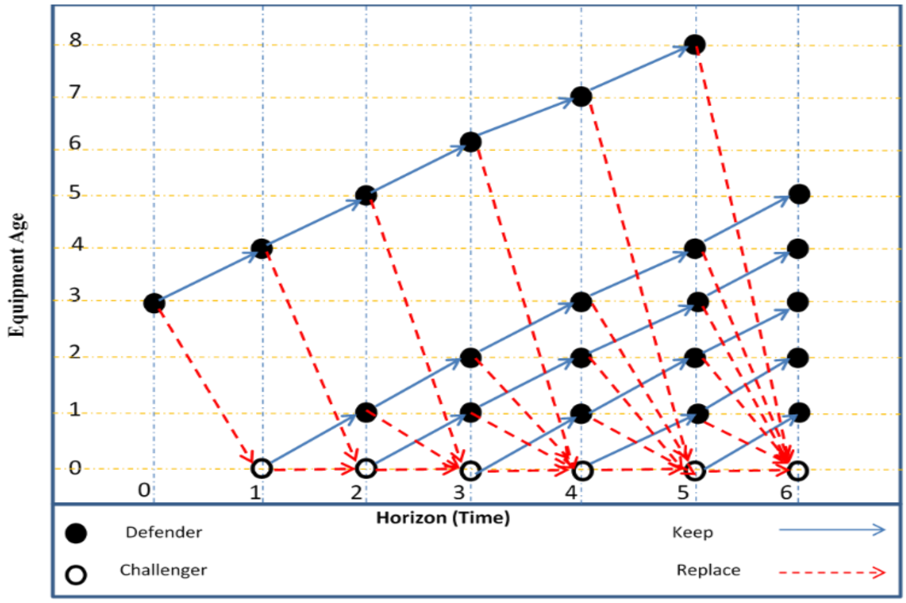


Fig. 1. Bellman's grid for equipment replacement.

2.1. DDP Formulation of the Replacement Problem

In order to formulate the replacement problem using the DDP model, we start with defining all stages and states, mathematical notations, the optimal-value function, and the DDP recursive equations.

2.1.1 Stages and states

The yearly interval refers to the stage variable while the age of the equipment at the beginning of each year refers to the state variable. The following is a list of all mathematical notations.

Y = age of equipment at the starting stage.

N = horizon or number of stages, the period of (Keep/ Replace) decisions.

i = current year in which a piece of equipment is waiting for the Keep/Replace decision at the starting stage. i = Y +1, Y + 2, ..., Y + N.

'd', *'c'*: A subscript '*d*' denotes existing (old) equipment (defender), while a subscript '*c*' denotes alternative equipment (challenger).

TC_i = Total operation and maintenance cost of equipment (including downtime) during the decision year at the end of which the equipment turns i years old,

TR_i = Total Revenue of equipment during the decision year at the end of which the equipment turns i years old,

FC_{cj} = First Cost or unadjusted basis of a new equipment (challenger) during year j, j = Y, Y+1, …, Y+N-1.

RC_{dj} = Replacement Cost of the old equipment (defender) during year j.

S_{di} = Salvage value of equipment during the decision year at the end of which the equipment turns i years old,

We assume that all equipment will be salvaged at the end of equipment expected life, and that future equipment value decreases by way of depreciation.

2.1.2 Optimal-value function

The optimal-value function is defined as a function that returns the maximum total estimated return from any point in the design period (state/stage) forward to the end of the equipment lifespan (horizon). Hence, the optimal-value function is defined as follows:

$M_j(i)$ = "maximum total estimated return of equipment that is i-years-old in year j onward until the end of lifespan N".

2.2.3 Model equation

At the beginning of year j, the decision maker has two available actions for an i-year-old equipment: either to Keep or Replace the equipment. If the decision is to replace, the equipment will be used throughout that year and replaced at the start of the next stage. The costs of the two actions are as follows:

Keep action: For this action, the i-year-old equipment is kept, so the cost of stage j is $(TR_{di} - TC_{di})$. The subsequent stage $j+1$ and state $i+1$ as a result of this action have an optimal-value function $M_{j+1}(i+1)$. Therefore, the recursive optimal value for the keep action is:

$$(TR_{di} - TC_{di}) + M_{j+1}(i+1) \quad (1)$$

Replace action: For this action, the i-year-old equipment is replaced, so the cost of stage j is equal to the cash flow value which consists of FC_{cj} (the first cost or unadjusted basis of a challenger during the year j), RC_{dj} (the replacement cost of the defender during year *j*), TC_{di} (the annual O and M cost of the *i*-year-old defender when ordering a challenger during that decision year), S_{di} (the revenue from the salvage value of the defender at the end of the decision year when the defender is *i* years old at the beginning of the decision year), and TR_{di} (the annual revenue of the defender during the decision year at the end of which the defender turns *i* years old). The next stage and state as a result of this action are $j+1$ and 0, respectively, and hence their optimal-value function is $M_{j+1}(0)$. Therefore, the recursive optimal value for the replace action is:

$$\left[(S_{di} + TR_{di}) - (FC_{cj} + RC_{dj} + TC_{di}) + M_{j+1}(0)\right] \quad (2)$$

Notice that the challenger becomes the defender after every replacement decision. Hence, the optimal-value function is expressed as:

$$\begin{aligned} M_j(i) = \ max \{ & \left[(TR_{di} - TC_{di}) + M_{j+1}(i+1)\right], \\ & \left[(S_{di} + TR_{di}) - (FC_{cj} + RC_{dj} + TC_{di}) + M_{j+1}(0)\right] \} \end{aligned} \quad (3)$$

Indeed, the optimal policy for the remaining stages is independent of the policy of the previous ones. The solution to $M_j(i)$ subproblem does not affect the solution to $M_{j+1}(i+1)$ subproblem of the same problem. Therefore, subproblems are solved independently as Cormen did for the shortest path problem [28]. The subproblems are solved one after one, thus each time finding the optimal solutions for that subproblem provides the whole optimal solution of the main problem. A recursive relation exists which finds the optimal solution for $M_j(i)$, given that $M_{j+1}(i+1)$ has already been solved.

2.2. O and M Costs, Revenue, and Future Acquisition Costs

Experts express the service fees as a percentage of the purchase value of equipment depending on the equipment type and age [29, 30]. In this paper, we divide the equipment lifespan into four periods: the warranty period plus, three equal subdivisions of the remaining lifespan. We suggest the service fees for these periods to be proportional to the equipment age:

- Warranty period: Set the service fees to 2% of the purchase value,
- First non-warranty period: Set the service fees to 3-7% of the purchase value,
- Second non-warranty period: Set the service fees to 5-10% of the purchase value, and
- Last non-warranty period: Set the service fees to 8-15% of the purchase value.

The expected annual operation costs and annual revenues are calculated as fixed amounts or as percentages of the first-year operational costs and revenue, respectively. Note that the revenue could decrease annually because of the increase in downtime or the deterioration in equipment performance over time. For estimating the future acquisition costs, we calculate the rate of increase in purchasing price f_p as [31, 32]:

$$f_p = \left(\frac{P_{t_n}}{P_{t_0}}\right)^{1/n} - 1 \tag{4}$$

Where:

P_{t_0} = Equipment purchase price at installation time, t_0.
P_{t_n} = Equipment purchase price at current time, t_n (Now).

2.3. Inflation Models

Inflation is an increase in the amount of money necessary to obtain the same amount of goods or services before the inflated price was present [31, 32]. Inflation decreases the purchasing power of money. Inflation can be expressed as the equipment future value in terms of the present value [32]:

$$F = P\,(1+f)^n \tag{5}$$

Where *F* is the future value in monetary units, *P* is the present value in monetary units, and f is the inflation rate over a time period *n*.

To account for the effect of varying yearly inflation rates over a period of several years, we can compute a single rate that represents an average inflation rate. Since each individual year's inflation rate is based on the previous year's rate, the years have a compounding effect. The average inflation rate is the rate of increase in average prices of goods and services over a specified time period, usually a year. This rate is given by:

$$f = \left(\frac{F}{P}\right)^{1/n} - 1 \tag{6}$$

2.4. Depreciation Models

The salvage value *S* is the estimated market value at the end of the equipment useful life. It can be expressed as an estimated monetary amount or as a percentage of the first cost. In this paper, we investigate straight line (SL), declining balance (DB) and double declining balance (DDB) depreciation models are shown in Fig. 2 [33].

For a straight-line (SL) depreciation model, the annual depreciation, D_t, is defined as:

$$D_t = (B - S)d_t = \frac{B - S}{n} \tag{7}$$

Where: *t* = age in years (*t* = 1, 2, …., *n*), *B* = first cost or unadjusted basis, *S* = estimated salvage value, *n* = recovery period, and d_t = depreciation rate = 1/*n*.

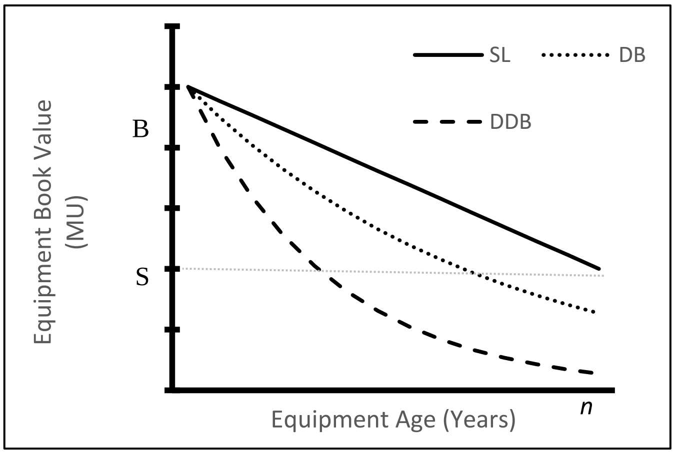


Fig. 2. Three equipment depreciation models: straight line (SL), declining balance (DB), and double declining balance (DDB).

Since the equipment is depreciated by the same amount each year, the book value after *t* years of service, denoted by BV_t, will be equal to the first cost *B* minus the annual depreciation times, *t*:

$$BV_t = B - t \times D_t \tag{8}$$

The declining-balance (DB) depreciation is also known as the book depreciation, the fixed-percentage depreciation, or uniform-percentage depreciation. Under this model, the annual depreciation is determined by multiplying the book value at the beginning of a year by a fixed (uniform) percentage *d*, expressed in decimal form. If *d* = 0.1, then 10% of the book value is removed each year. Therefore, the depreciation amount decreases each year (See Fig. 2). The maximum annual depreciation rate for the DB model is twice the straight-line depreciation rate, that is,

$$d_{max} = 2/n \tag{9}$$

In this case, the method is called the DDB depreciation.

Another commonly used percentage for the DB method is 150% of the SL rate, where *d* = 1.5/*n*.

The depreciation for year *t* is the fixed rate *d* times the book value at the end of the previous year.

$$D_t = d \times BV_{t-1} \tag{10}$$

The actual depreciation rate for each year *t*, relative to the basis *B*, is:

$$d_t = d(1-d)^{t-1} \tag{11}$$

If BV at *t*-1 is not known, the depreciation in year *t* can be calculated using *B* and *d*:

$$D_t = d \times B(1-d)^{t-1} \tag{12}$$

The book value in year *t* is determined in one of two ways: by using the rate *d* and basis *B* or by subtracting the current depreciation charge from the previous book value. The equations are:

$$BV_t = B(1-d)^t = BV_{t-1} - D_t \tag{13}$$

Where the range of *d* is $0 < d \le 2/n$.

3. RESULTS

3.1. Numerical Simulation Settings

In the following numerical simulations, we assume a 5-year-old equipment [34] with a lifespan of 12 years, a first cost of 10,000 monetary units (MU), a current acquisition cost of 11,500 MU, a salvage value of 800 MU, a SL depreciation model, a zero-inflation rate, and maintenance cost percentages of 1%, 3%, 5% and 7% for the warranty, 1st, 2nd, and last non-warranty periods, respectively. Our goal is to find the Keep-Replace sequence of highest return (lowest cost). We investigate three different scenarios based on the relative revenues of the defender and the challenger options. These three scenarios represent common possibilities for medical equipment replacement.

3.2. Scenario I: No-Revenue for Defender and Challenger

In this scenario, there is no uninstallation cost and no revenue for both defender and challenger. We look for the lowest cost in this scenario. Table1 shows the optimal sequence (R – K – K – R – K – K – K) with the optimal-sequence benefit value of 14,244 MU which is equal to 124% of the current acquisition cost. For further illustration, Fig. 3 shows the decision tree for this scenario and the optimal (lowest cost) path (bold line) from the 127 available paths [27].

Table 1. Results of the three scenarios including both optimal and conventional costs, sequences, and benefits.

Scenario	Year	**y**	**y+1**	**y+2**	**y+3**	**y+4**	**y+5**	**y+6**	**Total Benefit**
	Equipment Age (Year)	6	7	8	9	10	11	12	
	Equipment Market Value	5400	4633	3867	3100	2333	1567	800	
	Purchase Price	11826	12161	12506	12860	13225	13600	13985	
	Equipment O&M Cost	2476	2690	2928	3196	3495	3831	4206	
Scenario I	Optimal Sequence (OS)	R	K	K	R	K	K	K	14244
	Optimal Annual Costs (OC)	-8576	-1836	-1980	-2807	-1908	-2055	-2216	
	Conventional Sequence (CS)	K	K	K	K	K	K	R	
	Conventional Annual Costs (CC)	-2476	-2690	-2928	-3196	-3495	-3831	-17006	
	Annual Benefit (AB)	**-6100**	854	948	389	1587	1776	14790	
Scenario II	OS	R	K	R	K	R	K	K	18906
	OC	-5245	2044	1121	1997	1063	1946	1683	
	CS	K	K	K	K	K	K	R	
	CC	855	543	206	-154	-546	-970	-14230	
	AB	**-6100**	1502	915	2151	1609	2916	15913	
Scenario III	OS	R	R	R	K	R	K	K	23495
	OC	-5245	2363	2335	2773	1816	2722	2435	
	CS	K	K	K	K	K	K	R	
	CC	855	543	206	-154	-546	-970	-14230	
	AB	**-6100**	1820	2129	2927	2362	3692	16665	

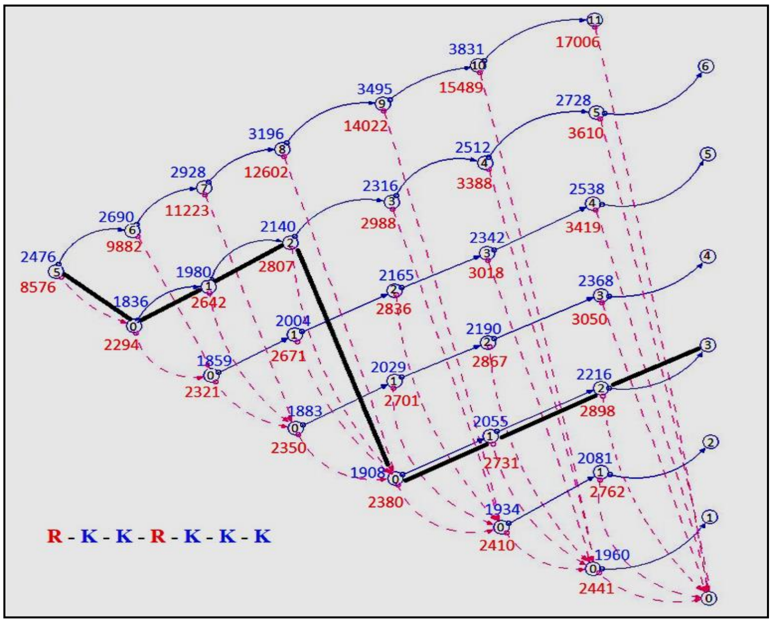


Fig. 3. Decision tree of scenario I with keep-replace values.

Each node represents the age of the equipment starting from 5 years. Each arc represents the decision to either keep the equipment (solid arc) or replace the equipment (dashed arc). The remaining period of the equipment lifespan (7 years) is the design horizon. Figure 4(a) shows the cumulative costs of both the optimal sequence and the conventional sequence. The graph shows the optimal sequence with the lowest aggregate cost of -21,378 MU.

3.3. Scenario II: Equal Revenue for Defender and Challenger

In this scenario, there is no uninstallation cost and both defender and challenger equipment have equal revenues of 4000 MU with an annual decrement rate of 3%. We look for the highest benefit in this scenario. Table 1 shows the optimal sequence (R – K – R – K - R – K – K) with the optimal-sequence benefit value of 18,906 MU which is equal to 164% of the current acquisition cost. The cumulative returns of both the optimal sequence and the conventional sequence are shown in Fig. 4(b). The optimal sequence has the highest return of 4609 MU.

3.4. Scenario III: Higher Revenue for Challenger

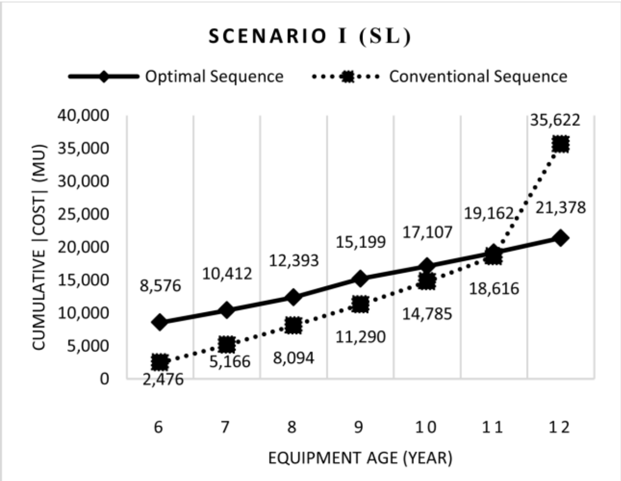
In this scenario, there is no uninstallation cost and the revenue of the challenger (4800 MU) is 20% more than the revenue of the defender (4000MU) where both have an annual decrement rate of 3%. We look for the highest benefit in this scenario. Table 1 shows the optimal sequence (R – R – R – K – R - K – K) with the benefit value of 23,495 MU which is equal to 204% of the current acquisition cost. The cumulative returns of both the optimal sequence and the conventional sequence are shown in Fig. 4c. The graph shows that the optimal sequence has the highest return of 9,198 MU.

3.5. Simulation Results for DB and DDB Depreciation Models

Figure 5 shows the simulation results for the three aforementioned scenarios under the declining-balance (DB) model. The optimal-sequence benefits for the three scenarios are 11,773 MU, 16,669 MU, and 21,302 MU. These benefits as percentages of the current acquisition cost are 102%, 145%, and 185%, respectively. Similarly, Fig. 6 shows the simulation results for the three aforementioned scenarios under the double-declining-balance (DDB) model. The optimal-sequence benefits for the three scenarios are 11,161 MU, 14,769 MU, and 19,090 MU. These benefits as percentages of the current acquisition cost are 97%, 128%, and 166%, respectively.

3.6. Effect of the Inflation Rate

Different inflation rates were used with different depreciation models for the three scenarios. Figure 7 shows that the optimal-sequence benefit changes with increasing inflation rate (1% to 10%) for the SL (with salvage value of 800 MU), DB (with 1/12 depreciation rate), DDB (with 2/12 depreciation rate) depreciation models for all scenarios. Indeed, the number of replacement actions decreases with increasing depreciation rate. For example, for Scenario I, the number of replacement actions are 30, 29 and 10 for the SL, DB and DDB models, respectively. In addition, while the benefit increases with increasing inflation rate for Scenario I, the benefits of Scenarios II and III decrease as the inflation rate increases. This pattern may be explained by the effect of the revenue in Scenarios II and III, or no revenue in Scenario I.



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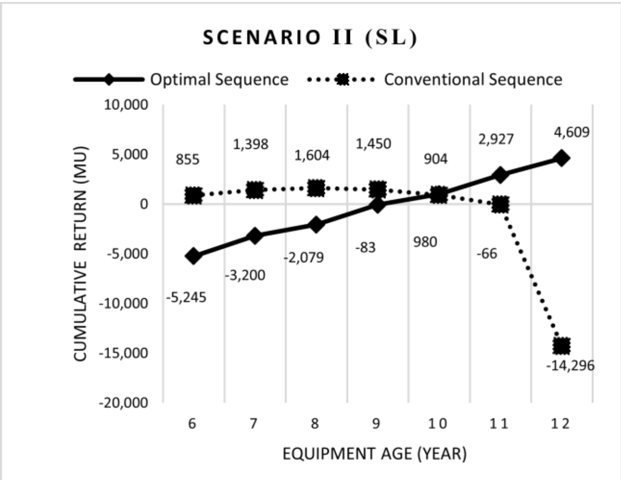

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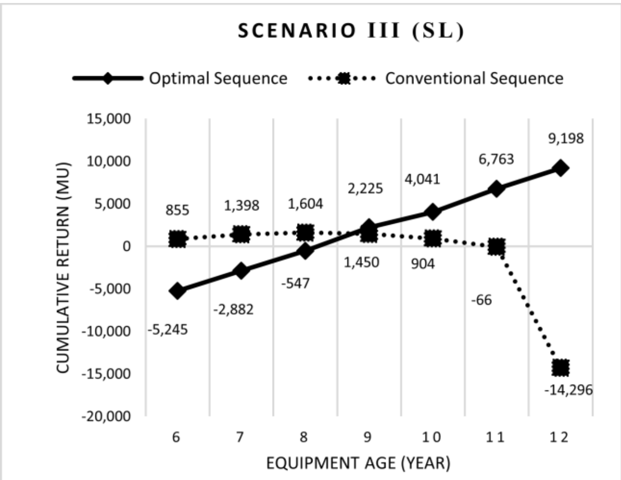
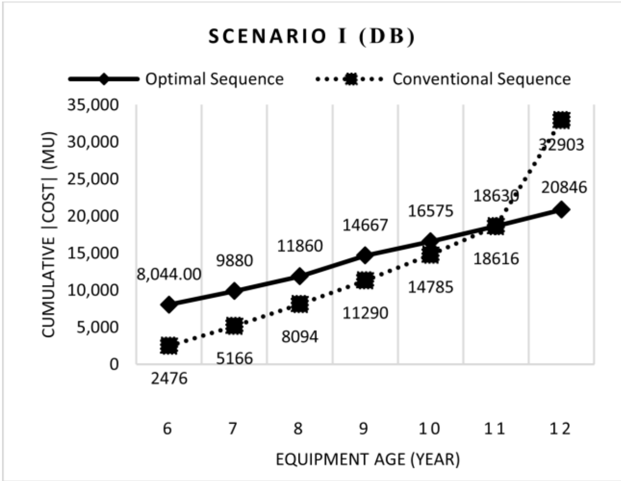


(c)

Fig. 4 Three scenarios for defender and challenger revenues in medical equipment replacement using the straight-line **(SL)** depreciation model. (a) Scenario I: No revenue for both defender and challenger. (b) Scenario II: Defender and challenger equipment generate the same revenue. (c) Scenario III: The challenger revenue is more than the defender revenue by 20%.



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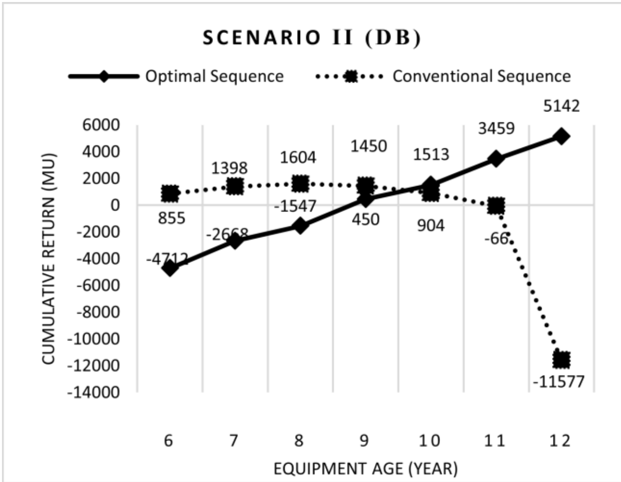

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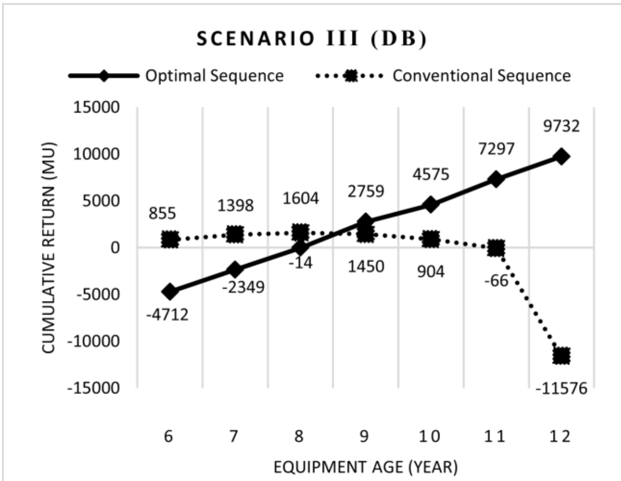

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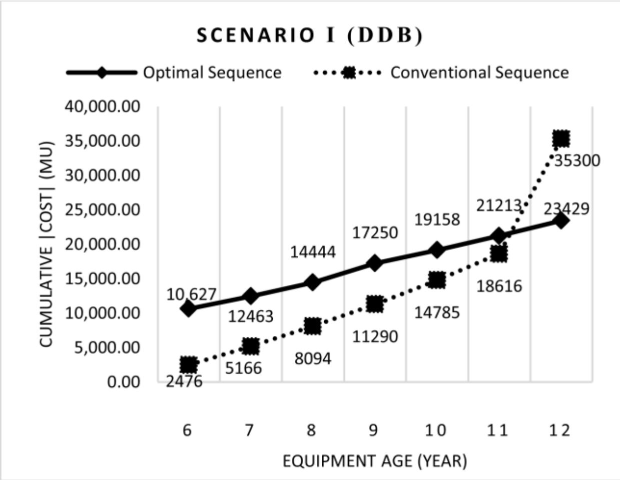
Fig. 5 Three scenarios for defender and challenger revenues in medical equipment replacement using the declining-balance (**DB**) depreciation model. (a) Scenario I: No revenue for both defender and challenger. (b) Scenario II: Defender and challenger equipment generate the same revenue. (c) Scenario III: The challenger revenue is more than the defender revenue by 20%.



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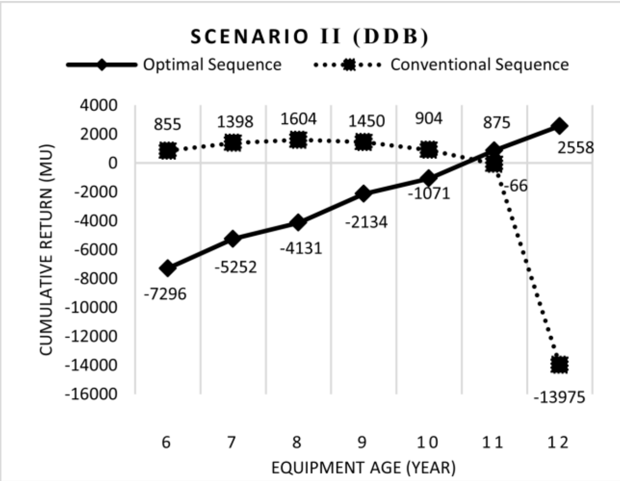

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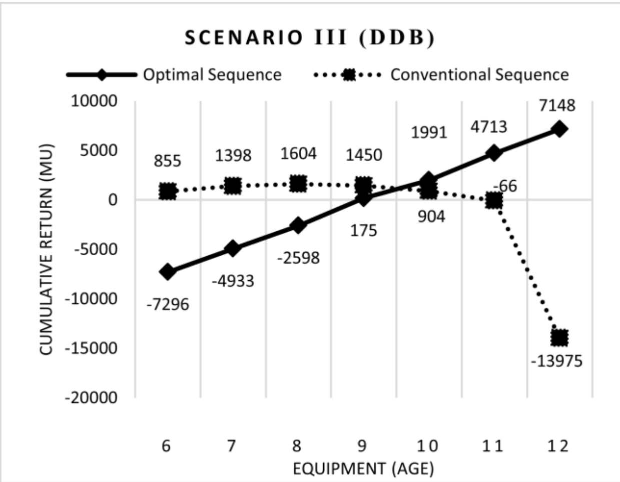

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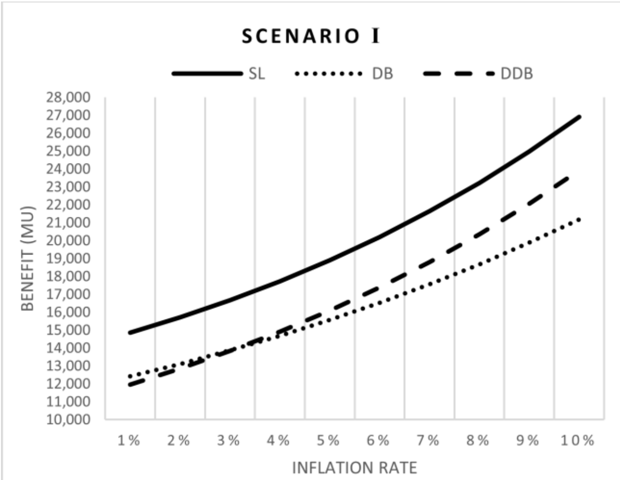
Fig. 6 Three scenarios for defender and challenger revenues in medical equipment replacement using the double declining-balance (DDB) depreciation model. (a) Scenario I: No revenue for both defender and challenger. (b) Scenario II: Defender and challenger equipment generate the same revenue. (c) Scenario III: The challenger revenue is more than the defender revenue by 20%.



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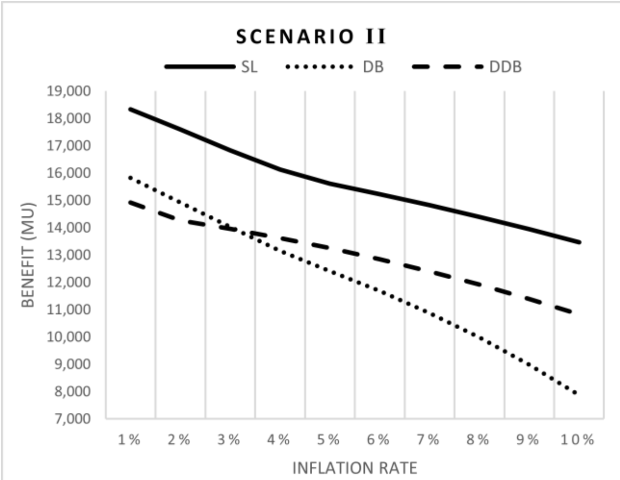

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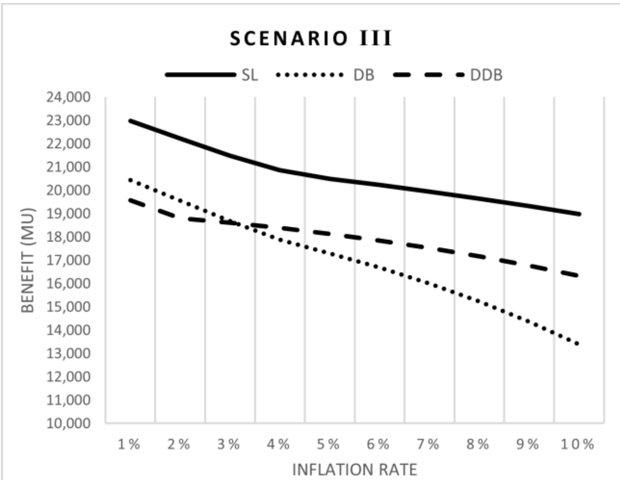

(c)

Fig. 7 Optimal-sequence benefit changes with different inflation rates for straight line (SL), declining balance (DB), and double declining balance (DDB) depreciation. (a) Scenario I: No revenue for both defender and challenger. (b) Scenario II: Defender and challenger equipment generate the same revenue. (c) Scenario III: The challenger revenue is more than the defender revenue by 20%.

3.7. Effect of the Challenger's Excess Revenue

For Scenario III and using the SL depreciation model, the cumulative returns of both the conventional sequence and the optimal sequence for increasing percentages of challenger revenue (compared to the defender revenue) are shown in Fig. 8. Expectedly, the optimal-sequence benefit increases in proportion to the increase in the challenger's excess revenue.

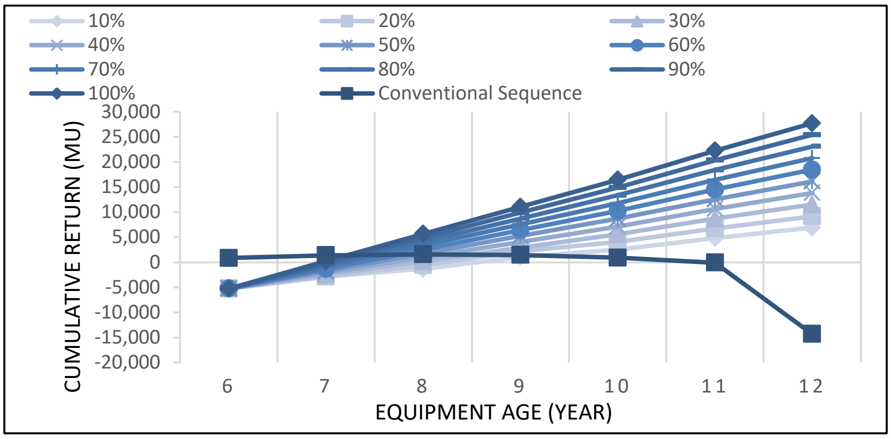

Fig. 8. Cumulative returns versus equipment age for Scenario III.

4. DISCUSSION

Due to the difficulty of obtaining maintenance and purchase data of the medical equipment, we resorted to assuming reasonable data in order to test the model and analyze its outcomes. We discuss the results under the SL depreciation model. In the first scenario, one finds the optimum replacement time when there is no revenue by calculating the lowest cost. The optimal sequence among 127 sequences Fig. 3 was to replace the equipment next year and use the new one for two years before replacing it with new equipment again and keeping this equipment for the remaining design period. In this case, the aggregate expected cost was decreased to the minimum level, where the final benefit equals 124% of the current acquisition cost Table1.

For the second scenario, the defender and the challenger have the same revenue. In this case, the optimal sequence was (R-K-R-K-R-K-K). Despite of the repeated replacement, Fig. 4b shows that the cost was increased at the beginning then the return was increased. In other words, the calculated cumulative return indicates that the final benefit value equals 18,906 MU which represents 164% of the current acquisition cost.

The last scenario discusses the replacement problem when the challenger revenue is more than that of the defender. This happens, for example, when the challenger has a newer technique, a higher performance, or a higher capacity and hence a higher revenue. To examine the effect of the relative increase in revenue of the challenger on the optimal sequence, we studied different increasing percentages of challenger revenue compared to the defender revenue Fig. 8. For example, when the challenger revenue was increased by 20%, the optimal sequence was (R-R-R-K-R-K-K) and the final benefit value reached 23,495 MU which is equal to 204% of the current acquisition cost.

Optimal-Sequence Benefit Analysis: The question here, is the optimal-sequence benefit (OSB) worthy of the extra investment in the optimal sequence?

To answer this question, let's use the concept of "Time Value of Money" and do some calculation for the Scenario I (Similar calculations can be done for the other scenarios). From Table 1, the investment (6100 MU) is the Present Value (PV) at year Y. Assuming a basic Interest Rate (IR) of 10%, we compute the Future Value (FV) for N number of periods where the investment will be held for 6 years at a compound interest rate as: $FV = PV\ (1 + IR)^N\ = 6100\ (1 + 0.10)^6\ = 10{,}807\ MU.$

Now by comparing FV with OSB, we can get the answer that the optimal-sequence earning (OSE) is: OSE = OSB – FV = 14,244 – 10,807 = 3,437 MU.

Alternatively, to obtain the same OSB (listed in the last column of Table 1), the IR should be at least 15%, 21%, and 25% in Scenarios I, II, and III, respectively. These hypothetical interest rates are quite high and unlikely. This shows that the optimal replacement strategy is more plausible and economical.

5. CONCLUSIONS

The decision process for medical equipment replacement or retention is affected by several factors of logistics, obsolescence, cost, and depreciation. In this paper, we analyzed the problem of optimal replacement of medical equipment using a deterministic dynamic programming approach. We investigated different realistic scenarios related to the revenues of the replacement or keeping options under different depreciation and inflation models. We found that the optimal sequences in all scenarios are clearly beneficial compared to the conventional sequence. We supported our conclusions through a hypothetical economic analysis. The application of the proposed approach in medical facilities can lead to effective budget management, and reduced maintenance and operation costs. In addition, the proposed approach keeps the medical facilities updated optimally. Finally, in our problem, the optimal replacement time is often before equipment reaches the end of its lifespan (depends on economic factor not technical obsolescence). However, a used medical equipment is of interest for small and medium health facilities especially that have not budget enough for new equipment. Also, it may be resold as a spare part. Therefore, our proposed model may have cost analysis by calculating or neglecting revenue from resold existing equipment. So, we can take the best decision.

DECLARATION OF CONFLICT OF INTERESTS

The authors have declared no conflict of interests.

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تحسين عملية استبدال الأجهزة الطبية باستخدام البرمجة الديناميكية الحتمية

يستخدم البحث البرمجة الديناميكية الحتمية متعددة المراحل لتحسين استبدال الأجهزة الطبية حيث تمثل نتيجة التسلسل (احتفاظ – استبدال) الحل الأمثل الذى يوضح ما إذا كان يجب الاحتفاظ بالجهاز (المدافع) أو استبداله بجهاز بديل (المنافس) أكثر اقتصاداً، وتم التحقق من ثلاثة سيناريوهات، الاول عدم وجود إيرادات للمدافع والمنافس، والثانى تساوى الإيرادات لكليهما، والثالث إيرادات المنافس أعلى من إيرادات المدافع وتم تعريف الفائدة لكل سيناريو على أنها الفرق بين أعلى العوائد (أقل التكاليف) للحل الأمثل وعوائد (تكاليف) الحل التقليدى وهو الاحتفاظ بالأجهزة حتى نهاية عمرها المتوقع وتظهر النتائج أن النسب المئوية لفوائد التسلسل الأمثل نسبة إلى تكلفة الاستحواذ الحالية للسيناريوهات الثلاثة هى ٢٠٤%، ١٦٤%، ١٢٤% على التوالى، كما تمت ملاحظة أن عدد إجراءات الاستبدال تزداد مع زيادة إيرادات المنافس وتتناقص مع زيادة معدل الإهلاك، وأخيرا، تمت دراسة تأثير معدل التضخم على فائدة التسلسل الأمثل.