Incorporating human fatigue and recovery into the learning–forgetting process

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Abstract: The available learning and forgetting models do not consider the physical loading that performing a task requires. In some situations, physical loading results in workers’ fatigue on the job that is followed by rest breaks to alleviate it. The aim of this paper is to present the "learning-forgetting-fatigue-recovery model" (LFFRM) that addresses possible issues relating to workers’ capabilities and restrictions in manufacturing environments. Numerical examples are solved to address some research questions regarding the model optimization and its constraints. The main results show that incorporating learning into a production process decreases fatigue and improves the performance of the system. Worker’s fatigue, on the other hand, increases production time and decreases production output. A recovery break must be of enough length to alleviate some of the accumulated fatigue. However, longer recovery times extend the lead time and deteriorate the production performance due to forgetting.

Keywords: Learning; forgetting; production break; fatigue; recovery

1. Introduction

The “Learning Curve” (LC) has been an interesting subject to many researchers and practitioners including industrial engineers and psychologists since the work of Wright (1936), and Hovland (1951). Since then, several learning curve models were developed that have different forms (e.g., power exponential, S-curve). The Wright's learning curve has been the prominent model because of its simplicity and ability to fit well a wide range of learning data (Yelle, 1979; Badiru, 1992; Jaber, 2006).

Globalization and competition have changed the market environment and imposed pressures on manufacturing firms to deliver new and quality products at competitive prices more frequently, requiring these firms to be responsive, efficient and flexible. To cope with these changes, manufacturing firms started acquiring flexible workforce approaches. These can alleviate the detrimental effects of bottlenecks resulting from machine breakdowns, product type changes, or external demand changes. They can also reduce work-in-process inventory levels, lead-times, and improve customer service performance (Park & Bobrowski, 1989). However, flexibility
comes at the cost of the workforce partially or fully forgetting the knowledge acquired for one
task while performing different tasks (Jaber et al., 2003; Inman et al., 2004). Automotive
manufacturers are an example where significant amounts of money have been spent on training
(learning) and retraining (relearning following forgetting) workers (Park & Bobrowski, 1989;
Omar et al., 2011). This enticed many researchers to investigate how learning and forgetting
interacts, and to develop models that capture these phenomena (Bailey, 1989; Jaber & Bonney,
1997a; Kleiner et al., 2011).

In environments that emphasize flexibility, forgetting, which may be thought of as the loss of
access to knowledge or procedures, occurs either because a worker has been away from a given
task for some time or because he/she confuses similar knowledge or skills (Bjork & Bjork, 2011). In
the industrial engineering literature, it has been documented that the length of a
production break has a direct effect on the degree to which humans forget (Anderlohr, 1969;
Cochran, 1973) and it may occur over relatively short periods of time (MacLeod & Macrae,
2001; Anderson, 2003). Attempts to mathematically model the effects of forgetting in the
industrial engineering literature are as early as the works of Hoffman (1968) and Adler and
Nanda (1974). Modelling the forgetting curve falls in one of three groups: mathematical,
experimental, and empirical. The works of Carlson and Row (1976), Globerson and Levin
(1987), Elmaghraby (1990), Jaber and Bonney (1996), Jaber and Kher (2002), Sikström and
Jaber (2002), and Jaber et al. (2003) fall in the first group. These models were developed based
on valid assumptions, but were not generally tested (although a few were tested later) against
experimental or empirical data. The works of Bailey (1989), Globerson et al. (1989), and Hewitt
et al. (1992) fall in the second group, where forgetting was modelled using data collected from
laboratory experiments performed by students as surrogates of workers in manufacturing
environments. Nembhard (2000) and Nembhard and Uzumeri (2000a) cautioned against using
these models as they use single rather than multiple breaks when modelling forgetting. The
studies by Badiru (1995) and Nembhard and Uzumeri (2000a) fall in the third and final group
who modelled forgetting using empirical data collected from industrial settings. In recent years,
some of the models of the first group were tested against empirical data (Nembhard & Osothsilp,
2001), where the “learn forget curve model” (LFCM) was found to be a promising model (Jaber
& Bonney, 1997a; Jaber et al., 2003; Jaber & Sikström, 2004a, 2004b). The models of the three
groups were most likely investigated in environments where a worker may be subjected to
fatigue on the job. Although the above studies did not mention fatigue, this does not necessarily
mean that fatigue did not occur. There is evidence in the literature that repetitive human daily
activities deplete an individual’s resources leading to fatigue (Winwood et al., 2005; Sonnentag
& Zijlstra, 2006). It is reasonable therefore to assume that workers in an industrial setting are
subjected to fatigue on the job. For example, Grosse and Glock (2013) in their experimental
study observed the presence of workers learning and fatigue in order picking systems.

Fatigue is multidimensional. Tiredness and lack of energy (Barker & Nussbaum, 2011), physical
exertion (Barker & Nussbaum, 2011), physical discomfort (Yoshitake, 1978), lack of motivation
(De Vries et al., 2003), and sleepiness (Smith et al., 2005; Theorell-Haglöw et al., 2006) have been distinguished as dimensions of fatigue (Åhsberg, 2000). Fatigue is a common result of work (Winwood et al., 2005) and leads to performance problems. It has detrimental effects on judgment, omission of results, indifference to essentials, decreased efficiency and productivity, and higher error rates and quality problems (Rohmert, 1987; Åhsberg, 1998; Chaffin et al., 2006; ElMaraghy et al., 2008). When fatigue becomes chronic or excessive, it reduces a person’s quality of life and can contribute to work-related disorders (Åhsberg, 1998; Frank, 2000; Asfaw et al., 2011; Bevilacqua et al., 2012). Rest breaks help alleviate body fatigue and recover a worker to his/her normal strength and capacity.

In a flexible workforce schedule, workers alternate between different tasks that require various workloads. When a worker performs a task, muscle force capacity is reduced by time up to a threshold value (maximum endurance time) due to the muscle fatigue (Ma et al., 2009). This fatigue is either alleviated by a rest break or by the worker moving to perform another task that has less and different physical loading allowing for some recovery. However, some of the learning acquired during performance is lost when the worker has been away from a task for a period of time. Although fatigue has been considered widely in the ergonomic literature, it has not received much attention in the Operation Research literature. There is no study available that demonstrates the mathematical relation between worker fatigue and production outputs such as production time and volume.

Conflicting system objectives arise when the effects of learning-forgetting and fatigue-recovery are combined. On one hand, the system productivity improves as workers move down on their learning curves by producing more per unit time; although their ability to work for long periods of time is constrained by muscular fatigue. On the other hand, a rest break or transferring a worker to a different task alleviates fatigue and recovers a worker’s physical condition to a comfort state (Jaber & Neumann, 2010); however, it may impede the worker’s productivity because of forgetting (Jaber et al., 2003). There is no available model in the learning-forgetting literature that accounts for human fatigue and recovery. The aim of this paper is to present the "learning-forgetting-fatigue-recovery model" (LFFRM) that addresses possible issues relating to workers’ capabilities and restrictions in manufacturing environments. The rest of this paper is organised as follows. The next section presents the mathematics of the “learn forget curve model (LFCM)”, followed by a brief introduction to human fatigue and recovery models. Next, the mathematics of the integrated LFFRM is developed. This is followed by an experimental design section to identify which input parameters affect the system’s output the most. To complement the study, numerical examples are solved, with their results discussed, to address several research questions. The purpose of this section is to set the grounds for future researches. The last section is for summary and conclusions.
2. Mathematics of the LFCM

The following notations will be used in this paper:

Notations

- $i$: a subscript identifying the batch number
- $n_i$: number of repetitions (units produced) in batch $i$
- $u_i$: experience measured in number of units remembered at the beginning of batch $i$
- $\tau_i$: length of the interruption period or break
- $B$: time needed for total forgetting to occur
- $MET$: Maximum Endurance Time at a given force level
- $MVC$: Maximum Voluntary Contraction; maximum force a muscle can generate voluntarily
- $f_{MVC}$: fraction of maximum voluntary contraction ($MVC$)
- $\lambda$: fatigue exponent, specifying fatigue rate
- $\mu$: recovery exponent, specifying recovery rate
- $\delta$: fatigue alleviation factor which determines how much fatigue needs to be alleviated
- $\theta$: experience transfer factor, representing the transfer of experience between working cycles
- $s_i$: number of items that would have been produced if no interruption occurred
- $b$: learning exponent, defining the rate of learning
- $f$: forgetting exponent, defining the rate of forgetting
- $T_i$: time to produce the first unit
- $t_{n_i}$: time to produce $n_i$ number of items in batch $i$

Wright’s (1936) learning curve suggests that the time to perform a task decreases by a constant percentage each time the cumulative output doubles, and it is represented by

$$T_x = T_1 x^{-b}$$

(1)

where $T_1$ is the time to produce the $x^{th}$ unit, $T_1$ is the time to produce the first unit, $x$ is the cumulative production, and $b$ is the learning exponent ($0 \leq b < 1$), where $b = -\log(LR)/\log(2)$ and LR is the learning rate measured in percentage ($0 < LR < 1$; e.g. $LR = 0.8$ or 80%). The forgetting phenomenon has been considered to be a mirror image (Globerson et al., 1989; Jaber et al., 2003) of the learning curve and it is represented by

$$\hat{T}_x = \hat{T}_1 x^f$$

(2)
, where \( \hat{T}_x \) is the time for the \( x^{th} \) unit of lost experience of the forgetting curve, \( \hat{T}_i \) is the intercept of the forgetting curve, \( x \) is the amount of output that would have been accumulated if interruption did not occur and \( f \) is the forgetting exponent. Figure 1 depicts the learning-forgetting process.

**Figure 1.** The behaviour of the learning-forgetting process over time

Using Eqs.(1) and (2), Jaber and Bonney (1996) developed the “learn forget curve model” (LFCM) which is represented by

\[
T_n = T_1(u_i + n_i)^{-b}
\]

, where \( T_n \) is the time to produce the \( n^{th} \) unit in cycle \( i \), \( u_i \) is the experience measured in the number of units remembered at the beginning of cycle \( i \), and \( n_i \) is the number of repetitions (units produced) in cycle \( i \). The term \( u_i \) is determined from the work of Jaber and Bonney (1996) as

\[
u_{i+1} = (u_i + n_i)^{1+f_i/b}(u_i + n_i + s_i)^{-f_i/b}
\]

, where \( u_i = 0 \), \( s_i \) is the number of units of a product that could have been produced in cycle \( i \) if production interruption did not occur. The terms \( s_i \) and \( f_i \) are given respectively as

\[
s_i = \left[ \frac{1-b}{T_1} \tau_i + (u_i + n_i)^{1-b} \right]^{1/(1-b)} - (u_i + n_i)
\]

\[
f_i = \frac{b(1-b)\log(u_i + n_i)}{\log(1+B/t(u_i + n_i))}
\]
, where \( \tau_i \) is the length of the interruption period that has occurred after producing batch \( i \), \( B \) is the time to which total forgetting occurs (\( \tau_i \leq B \)), and \( t(u_t + n_t) \) is the time to perform (produce) \( u_t + n_t \) repetitions (units) continuously on the learning curve. Assuming that total forgetting can occur is not necessarily unrealistic given that it was observed in the studies of Anderlohr (1969), McKenna and Glendon (1985), Globerson et al. (1998).

3. Fatigue and recovery models

Fatigue can take many forms such as mental fatigue, lack of alertness, specific muscular fatigue, or general body fatigue (Åhsberg, 1998). The effect of fatigue on performance has been documented across many industrial and service sectors, e.g., the loss of throughput (Wang & Hu, 2010), error increasing (Misawa et al., 1984; Kopardekar & Mital, 1994), worker’s dissatisfaction (Saijo et al., 2008), performance decrement (Mital et al., 1991; Goode, 2003; Leung et al., 2006; Barker & Nussbaum, 2011), and injuries and accidents (Schuster & Rhodes, 1985; Dinges, 1995; Kristal-Boneh et al., 1996; Muggleton et al., 1999; Burke & Fiksenbaum, 2008). However, a mathematical model that relates fatigue and production outcomes has not been introduced. In this paper, we focus on muscular fatigue which is defined as the inability of the body muscles to sustain a specific posture or force level required to perform a task (Ma et al., 2009). A model will be developed to quantify the relation between worker’s muscular fatigue and production outputs.

Maximum Endurance Time (MET) is the duration for which a specific body posture (or muscular effort) can be sustained (applied) by a worker before his/her capability limits are reached (Niebel et al., 1999; El ahrache et al., 2006). It is a function of the level of the force being applied, e.g., \( f_{MVC} \). This force is usually a fraction of the muscle’s Maximum Voluntary Contraction (MVC) when performing a specific task (El ahrache et al., 2006), e.g., \( f_{MVC} = \%MVC \). Readers may refer to El ahrache et al. (2006) or Ma et al. (2009) for a good review of a number of empirical models that predict MET. The available models are either of exponential forms, e.g., \( MET = \beta_0 \times e^{-\beta_{MVC}} \) (Rose et al., 1992), or power forms, e.g., \( MET = \alpha_0 \times f_{MVC}^\alpha \) (Sjøgaard, 1985), where \( \alpha_0, \alpha, \beta_0 \) and \( \beta \) are model-specific parameters and MET is measured in minutes. These MET models are limited in the sense that they predict a fatigue end-point under a given \( f_{MVC} \) load but provide no indication to the shape of the fatigue accumulation function or depict the fatigue state over the course of task execution.

Although some researchers asserted that fatigue accumulates exponentially with time (Lindstrom et al., 1977; Bechtold & Summers, 1988; Konz, 1998b), none have empirically or experimentally showed that it does. This remains an open research question to be addressed in the Human physiology literature. On the contrary, more attention was paid by researchers in ergonomics to the form of the recovery function, which Konz also suggests is exponential (Konz, 1998b), with
maximum benefit in the earlier phases of the recovery period. Readers may refer to El ahrache and Imbeau (2009) for a review of recovery models. As suggested by Konz (1998b), this paper assumes that fatigue and recovery are of the following forms respectively

\[ F(t) = 1 - e^{-\lambda t}, \quad (7) \]

\[ R(\tau_i) = F(t)e^{-\mu \tau_i}, \quad (8) \]

where \( F(t) \) is the fatigue accumulation at time \( t \leq MET \), and \( R(\tau_i) \) is the residual fatigue after a rest break of length \( \tau_i \geq 0 \); \( R = 0 \) represents complete recovery (no residual fatigue) and \( R = 1 \) represents no recovery (maximum fatigue). In Eqs.(7) and (8) \( \lambda \) and \( \mu \) are fatigue and recovery parameters, respectively. These parameters control the speed of fatigue accumulation and recovery alleviation; i.e., a low value of \( \lambda \) (\( \mu \)) means slow fatigue (recovery) whereas a high value means a fast one. Figure 2 illustrates the behaviour of a fatigue-recovery process.

\[ \text{Fatigue Level} \]

\[ \text{Fatigue} \quad \text{Recovery} \quad \text{Fatigue} \quad \text{Recovery} \]

\[ \text{work} \quad \text{rest} \quad \text{work} \quad \text{rest} \]

\[ \text{Time} \]

**Figure 2.** The behaviour of the fatigue and recovery process over repeated work-rest cycle

Note that Eq.(7) advocates that fatigue accumulates over \( t \) from an initial value of zero, meaning that full recovery was attained in the previous rest break. In practice, rest breaks separating work cycles are usually short and do not permit full recovery to occur. The residual fatigue, \( R(\tau_i) \), carried forward into cycle \( i+1 \) and Eq.(7) will be re-written as

\[ F_{i+1}(t) = R(\tau_i) + (1 - R(\tau_i))(1 - e^{-\lambda(t_{i+1} - t_i)}) \quad (9) \]
where \( t_n \) is the production time of the cycle \( i \) and \( t_i \) is determined by projecting the value of \( R(\tau_i) \) on the fatigue curve as

\[
t_i = -\ln(1 - R(\tau_i))/\lambda
\]  

(10)

A dynamic fatigue model (Ma et al., 2009) will be used to consider different maximum endurance times (\( MET \)) for different production batches as a result of fatigue accumulation. This assumption is based on the hypothesis that the maximum force a subject can exert declines as the muscle’s capacity diminishes because of fatigue (Ma et al., 2009). With this assumption, the endurance time decreases as more batches are processed. Therefore, \( MET \) for a given batch varies between two extreme values, which are when the body is fresh or completely recovered from previously accumulated fatigue and when the body is in the state of complete tiredness. Thus, and for simplicity, it is assumed that \( f_{MVC} = i\lambda \), in which \( i \) is the batch number and \( \lambda \) is the fatigue rate. \( f_{MVC} \) is equal to \( \lambda \) for the first batch, \( 2\lambda \) for the second batch and so on.

4. The learning-forgetting-fatigue-recovery model (LFFRM)

The LFCM in Section 3 favours a policy of longer production cycles with short breaks as this improves productivity. However, an ergonomic policy, aimed at minimizing injury risks, will be to minimize a worker’s fatigue through short production cycles and rest breaks of lengths that guarantee quick recovery. Here, the LFFRM is a constrained LFCM model. The production time in cycle \( i \) cannot exceed \( MET \). From Eq.(1) we have

\[
t_{n_i} = \int_{u_i}^{n_i + u_i} T_e dx = \int_{u_i}^{n_i + u_i} T_i x^{-b} dt = \frac{T_i}{1 - b} \left( (n_i + u_i)^{-b} - u_i^{-b} \right)
\]  

(11)

where \( t_{n_i} \) is the production time in cycle \( i \) which should be equal or less than \( MET \)

\[
t_{n_i} \leq MET
\]  

(12)

from which the following constraint is determined

\[
0 < n_i \leq \left( \frac{1 - b}{T_i} \beta_0 e^{-\beta f_{MVC}} + u_i^{-b} \right)^{1/(1-b)} - u_i, \text{ or } 0 < n_i \leq n_i^{U_i}
\]  

(13)

Ideally, the length of a rest break would not exceed the time for total forgetting, i.e., \( \tau_i \leq B \). However, the break time could either be the time a worker spends performing another task (producing a different product) during which some of the strained body muscles are relaxed and recovered or, it could be a real rest break. The length of a break is usually governed by
scheduling output constraints that managers have to weigh carefully against the workers’ welfare and the system’s productivity. Therefore, the length of a break should be less than \( B \), but long enough to alleviate a significant amount of the worker’s fatigue. It is assumed that management would always try to balance between the workers’ welfare and system’s productivity. Therefore, management would like to alleviate at least \( \delta \% \) of a worker’s accumulated fatigue and ensure that \( \theta \% \) of the experience is transferred from one batch to the next. If a manager wants to alleviate \( \delta \% \) of the worker’s fatigue then from Eq.(8), \( R(\tau_i) = \delta \times F(t) = F(t)e^{-\mu \tau_i} \), implying that \( \tau_i \geq -\ln(\delta)/\mu \). On the other hand, a manager has to sustain an acceptable level of productivity by retaining a specific level of learning to be transferred between cycles, e.g., at least \( \theta \% \) of the worker’s experience should be retained, then from Eqs.(4), (5) and (8), we have

\[
-ln(\delta)/\mu \leq \tau_i \leq \frac{T_1}{1-b} \left\{ \frac{\theta}{(u_i + n_i)^{\tau_i/b}} \right\} - \left( u_i + n_i \right)^{-b}, \text{ or } \tau_i^L \leq \tau_i \leq \tau_i^U
\]

(14)

where \( \theta \) is the specific level of learning transferred between cycles. The derivation of Eq.(14) is shown in the Appendix.

The average processing time (performance), \( Z \), is computed by dividing the total processing time by the total number of units produced as

\[
Z = \frac{\sum_{i=1}^{N} t_{ni}}{\sum_{i=1}^{N} n_i}
\]

(15)

The productivity of the system is highest when Eq.(15) records its lowest value. Therefore, the above model could be written as Minimize \( Z \); subject to the following constraints: (1) \( 0 < n_i \leq n_i^{U} \), and (2) \( \tau_i^L \leq \tau_i \leq \tau_i^U \).

5. Experimental design

In this section, we examine the effects of the independent parameters of the model developed in Section 4 on the performance (output) of the system. The independent parameters are: learning rate, batch size, time for total forgetting, fatigue rate, and recovery rate. The outputs of the system (dependent variables) are the average processing time, the average fatigue level, the production volume, and the average length of a rest break. These variables are set as follow:
The independent variables:

- The learning rate is set to 3 levels: slow (90%), moderate (80%), and fast (70%), corresponding to $b=0.152$, $b=0.322$ and $b=0.515$, respectively (Dar-El et al., 1995).
- The number of batches, into which a production lot is split, is $N=5$ and 10.
- Total forgetting time is set to 2 levels: $B=2000$ and 4000 units of time.
- The fatigue accumulation index is set to 3 levels: slow, moderate, and fast, corresponding to $\lambda=0.01$, $\lambda=0.03$ and $\lambda=0.05$, respectively.
- The recovery speed index is set to 3 levels: slow, moderate, and fast, which are corresponding to $\mu=0.03$, $\mu=0.05$ and $\mu=0.07$ respectively.

The last two parameters, fatigue accumulation index and recovery speed index, were determined using a pilot test. With the assumption of 80% maximum fatigue per unit of time, the fatigue index was obtained as 1.6094. With the assumption of 80% recovery, the recovery index per unit of break time is also obtained as 1.6094. The optimal problem was examined with these measures but did not yield any solution. These values decreased until the above set of $\lambda$ and $\mu$ values was found that yield optimal solutions.

The dependent variables:

- $Z$: Average processing time of a unit, which is computed from Eq.(15), is the performance measure of the system.
- $F$: Average accumulated fatigue from production of $N$ batches, which is computed from Eqs.(7)-(9).
- $Q$: Production volume as restricted by Eq.(13) where $Q = \sum_{i=1}^{N} n_i$.
- $\tau$: Average length of a rest break between batches, where the number of breaks is equal to the number of batches minus one, is computed as $\tau = \sum_{i=1}^{N} \tau_i$, where $\tau_i$ is restricted by Eq.(14).

Other parameters:

- The time required to produce the first item is set to 1 unit of time, $T_1 = 1$.
- The parameters for MET calculation, $\beta_0= 4.16$ and $\beta = 7.96$, are taken from the study of Rose et al. (1992).
- Fatigue alleviation factor ($\delta$) and the experience transfer factor ($\theta$) are set at 50%, $\delta = \theta = 0.5$.

A full factorial design study of 108 cases ($3 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 108$) is performed. The optimization of each case (example) is performed using EXCEL SOLVER enhanced with Visual Basic codes, where the objective function given in Eq.(15) is minimised subject to Eqs. (13) and (14). An ANOVA statistical analysis was performed using SPSS software. The results of the study are summarised in Table 1.
The analysis of variance was performed to identify the most important parameters for every response. The results of Table 1 show that:

i. The learning exponent \( b \) has significant effects on the system performance \( Z \) and the production volume \( Q \) at level of \( p < 0.0005 \).

ii. The number of batches \( N \) has significant effects on the production volume \( Q \) and the average fatigue \( F \) at level of \( p < 0.05 \).

iii. The fatigue growth parameter \( \lambda \) has significant effects on \( Q \) and \( F \) at levels of \( p < 0.05 \) and \( p < 0.0005 \), respectively.

iv. The total forgetting time have not been recognized to have important effects on the problem outputs, we will use \( B = 2000 \) units of time for the total forgetting time from now on.

Given the above input parameters, the best and worst cases of the system performance are summarised in Table 2:

Table 2. Best and worst cases of system performance

<table>
<thead>
<tr>
<th>( b )</th>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>( B )</th>
<th>( Z ) (( N = 5 ))</th>
<th>( Z ) (( N = 10 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best case</td>
<td>0.515</td>
<td>0.01</td>
<td>0.07</td>
<td>4000</td>
<td>0.250</td>
</tr>
<tr>
<td>Worst case</td>
<td>0.152</td>
<td>0.05</td>
<td>0.03</td>
<td>2000</td>
<td>0.828</td>
</tr>
</tbody>
</table>

While the most effective parameters of the LFFRM model were identified, the shape and nature of these effects are not known. In the next section, further analyses will be done to help gain additional insights into the behaviour of the LFFRM regarding the independent variables. In the next section we will demonstrate how the input parameters, i.e., learning rate, number of batches, fatigue rate, and recovery rate, influence the outcomes of the production process. The recovery rate has not been identified as an effective parameter in the ANOVA analysis however; it is
useful to investigate this factor since it has large impact on the lead time of the production process.

6. Numerical Results

The behaviour of the learning-forgetting-fatigue-recovery model (LFFRM) developed in Section 4 is further investigated using several numerical examples. The numerical results are discussed to highlight the managerial implications and insights of the developed model. For this purpose, the results presented in this section are those of the optimal solutions. That is, the optimal solution is determined by minimising Eq.(15), \( Z \), subject to the Eq.(13), \( 0 < n_i \leq n_i^U \), and Eq.(14), \( \tau_i^L \leq \tau_i \leq \tau_i^U \), where \( i = 1, ..., N \). The input parameters such as time to produce the first item (\( T_1 \)), total time for forgetting (\( B \)), MET parameters (\( \beta_0, \beta \)), fatigue alleviation factor (\( \delta \)), and the experience transfer factor (\( \theta \)) are used as set in Section 5. The number of batches is set to \( N = 10 \). The purpose of this section is to extend upon the results of the ANOVA analysis described in Section 5, where the effects of the learning rate, the number of batches, the fatigue rate, and the recovery rate on the LFFRM are studied.

6.1. What effect does the learning rate have on the production process?

In order to investigate the behaviour of the process regarding the learning rate, in addition to the input parameters and their values given in Section 5 (\( T_1 = 1 \), \( B = 2000 \), \( \theta = 50\% \), and \( \delta = 50\% \)), the following input parameters are assumed: (1) Three learning rates are selected as will be described later, (2) fatigue and recovery rates are set to medium levels, i.e., \( \lambda = 0.03 \) and \( \mu = 0.05 \), and (3) ten batches, \( N = 10 \), are considered for the process. The mathematical programming model described by Eqs.(13), (14) and (15) is optimised for these input parameters. The optimal average length of a beak in-between adjacent batches for the three learning rates was found to be \( \tau^* = 6 \). The reason that \( \tau \) is not changed with the learning rate is because it depends on \( \delta \), the fatigue alleviation factor, and on \( \mu \), the recovery exponent, as per Eq.(14), which restricts the break length to a value irrespective of the learning rate. Figure 3 shows that, as learning becomes faster (\( b = 0.515 \)), the system’s performance, \( Z \), improves and becomes flatter. This suggests a range of values over which \( Z \) is relatively stable which may provide managers with more flexibility to meet their obligations for their workers’ welfare while also meeting their productivity targets. Other results are summarised in Table 3.
Figure 3. The behaviour of the system’s optimal performance due to different learning rates.

Table 3. The effect of learning rate on the process outputs when $\lambda=0.03$ and $\mu=0.05$

<table>
<thead>
<tr>
<th>Learning rates</th>
<th>$b = 0.152$</th>
<th>$b = 0.322$</th>
<th>$b = 0.515$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production volume, $Q$</td>
<td>59</td>
<td>112</td>
<td>340</td>
</tr>
<tr>
<td>Total production time, $\sum_{i=1}^{N} t_{ni}$</td>
<td>39.64</td>
<td>40.82</td>
<td>42.19</td>
</tr>
<tr>
<td>Average fatigue (%)</td>
<td>17.44</td>
<td>17.87</td>
<td>18.39</td>
</tr>
<tr>
<td>Average production time per unit, $Z$</td>
<td>0.67</td>
<td>0.36</td>
<td>0.12</td>
</tr>
</tbody>
</table>

The results in Table 3 show that as learning becomes faster ($b$ increases from 0.152 to 0.515), the production volume increases by about 82% (from 59 to 340). The average production time per unit, or $Z$, decreases (improves) correspondingly from 0.67 to 0.12 as shown in Figure 3. Average fatigue does not change considerably with the learning rate as has been suggested previously by ANOVA analysis. The results suggest that a worker with fast learning rate experiences slower fatigue accumulation and improves the system’s performance (smaller $Z$ values). This further suggests that investing to improve workers learning rates may be a good strategy for processes that are highly labour intensive.

6.2. What effects does the number of batches have on the production process?

In order to investigate the behaviour of the process regarding the number of batches, in addition to the input parameters and their values given in Section 5 ($T_1=1$, $B=2000$, $\theta=50\%$, and $\delta=50\%$), the following input parameters are assumed: (1) The learning rate, fatigue rate, and recovery rate are set at their medium values as $b=0.322$, $\lambda=0.03$, and $\mu=0.05$, respectively, and (2) two batch numbers are considered, which are $N=5$ and $N=10$. The mathematical programming model described by Eqs.(13), (14) and (15) is optimised for these input parameters. The results in Table
4 show that the production volume and subsequently the total production time increase as the number of batches \(N\) increases from 5 to 10. The optimal beak length for the both plans was found to be \(\tau^* = 6\). However, the average fatigue decreases by about 16\% and the average performance \((Z)\) improves by about 12\%. It may be concluded from these results that producing in more batches improves workers’ welfare as more rest breaks are provided reducing the average fatigue level, which positively impacts \(Z\).

<table>
<thead>
<tr>
<th>Batch Size</th>
<th>(N = 5)</th>
<th>(N=10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production volume, (Q)</td>
<td>65</td>
<td>112</td>
</tr>
<tr>
<td>Total production time, (\sum_{i=1}^{N} t_{n_i})</td>
<td>26.74</td>
<td>40.81</td>
</tr>
<tr>
<td>Average length of the rest break, (\tau)</td>
<td>6.02</td>
<td>6.02</td>
</tr>
<tr>
<td>Average fatigue (%)</td>
<td>21.26</td>
<td>17.87</td>
</tr>
<tr>
<td>Average production time per unit, (Z)</td>
<td>0.41</td>
<td>0.36</td>
</tr>
</tbody>
</table>

### 6.3. What effects does the fatigue rate have on the production process?

In order to investigate the behaviour of the process regarding the fatigue rate, in addition to the input parameters and their values given in Section 5 (\(T_1=1, B=2000, \theta=50\%, \text{ and } \delta=50\%\)), the following input parameters are assumed here: (1) Three different fatigue rates are selected for slow (\(\lambda = 0.01\)), medium (\(\lambda = 0.03\)) and fast (\(\lambda = 0.05\)) fatigue accumulation, (2) learning and recovery rates are set at their medium values as \(b = 0.322\) and \(\mu = 0.05\), and (3) \(N =10\), are considered for the process. The mathematical programming model described by Eqs.(13), (14) and (15) is optimised for these input parameters. The results are summarised in Table 5.

<table>
<thead>
<tr>
<th>Fatigue index</th>
<th>(\lambda = 0.01)</th>
<th>(\lambda = 0.03)</th>
<th>(\lambda = 0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production volume, (Q)</td>
<td>209</td>
<td>112</td>
<td>65</td>
</tr>
<tr>
<td>Total production time, (\sum_{i=1}^{N} t_{n_i})</td>
<td>62</td>
<td>40</td>
<td>28</td>
</tr>
<tr>
<td>Average fatigue (%)</td>
<td>9</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>Average production time per unit, (Z)</td>
<td>0.3</td>
<td>0.36</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Results show that the fatigue exponent does not affect the behaviour of the system’s performance \((Z)\) because the numerator and denominator of Eq.(15) both decline with more fatigue. The optimum length of the rest break was found to be \(\tau^* = 6\) for all cases. Table 5 illustrates that with more fatigue accumulation (from 9\% to 28\%), the production volume \(Q\) decreases by 68\%. The model suggests that the production actually declines as a direct result of worker fatigue. Figure 4 suggests that physically difficult tasks (large \(\lambda\) value) negatively affect the production output (cumulative number of items produced reduces) because a fatigued worker tends to produce less, either in smaller batches or shorter production runs.
6.4. What effects does the recovery rate have on the production process?

In order to investigate the behaviour of the process regarding the recovery rate, in addition to the input parameters and their values given in Section 5 ($T_1=1$, $B=2000$, $\theta=50\%$, and $\delta=50\%$), the following input parameters are assumed: (1) Three different recovery rates are selected for slow ($\mu=0.03$), medium ($\mu=0.05$) and fast ($\mu=0.07$), (2) learning and fatigue rates are set at their medium values as $b=0.322$ and $\lambda=0.05$, respectively, and (3) $N=10$, are considered for the production process. The mathematical programming model described by Eqs. (13), (14) and (15) is optimised for these input parameters. The results are summarised in Table 6, which show that as recovery from fatigue becomes faster, $\mu$ increases from 0.03 to 0.07, production volume $Q$ increases from 102 to 118 and the total production time decreases from 130 to 79 resulting in an improvement in the system’s performance $Z$ of about 13\% (from 0.4 to 0.35), and reducing the length of a rest break by 60\% (from 10 to 4).

Table 6. The effect of recovery rate on the production process when $b=0.322$ and $\lambda=0.03$

<table>
<thead>
<tr>
<th>Recovery index</th>
<th>$\mu=0.03$</th>
<th>$\mu=0.05$</th>
<th>$\mu=0.07$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production volume, $Q$</td>
<td>102</td>
<td>112</td>
<td>118</td>
</tr>
<tr>
<td>Total production time, $\sum_{i=1}^{N} t_{n_i}$</td>
<td>130</td>
<td>95</td>
<td>79</td>
</tr>
<tr>
<td>Average length of the rest break, $\tau$</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Average production time per unit, $Z$</td>
<td>0.4</td>
<td>0.36</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Lead time is defined as the sum of the production times for all 10 batches and the lengths of the 9 production breaks. As recovery speed increases, the lengths of the production breaks are affected
considerably. For a slow recovery speed, i.e. $\mu = 0.03$, the optimal break length obtained as $\tau^* = 10$ units of time, for a medium recovery speed, $\mu = 0.05$, $\tau^* = 6$, and for a fast recovery speed, $\mu = 0.07$, $\tau^* = 4$, which results in about 47% reduction in the lead time in the fast recovery case. The average production time is not changing considerably with the speed of recovery since in this case the production is not constrained by the fatigue level. Although ANOVA analysis did not suggest studying the rate of recovery, the results of Table 5 indicate an increase in the lead time due to slow recovery. It may be concluded from the results that according to the LFFRM model, slow recovery deteriorates the process lead time which has not been considered in the unit production time, $Z$, or in ANOVA factorial analysis of Section 5. The effect of recovery rate on the average process time (12.5%) and production volume (13.5%) are not considerable which is in accordance with the results of ANOVA analysis.

7. LFFRM constraints and feasible solutions

This section elaborates the importance of the constraints in the LFFRM modelling. We may find solutions for the LFFRM model without the constraints, but they are not feasible solutions. The solutions of an optimization problem usually lie in a more restricted range regarding to the general solutions. By studying the constraints of the LFFRM model, we will test the solutions and their feasibility. More specifically, we need to demonstrate how much the results would differ if the constraints are not considered and how this impacts the problem. The LFFRM constraints arise from worker’s maximum endurance time (upper limit of production in Eq.(13)), the necessary level for fatigue alleviation (lower limit of break length in Eq.(14)) and maximum forgetting (upper limit of the break time in Eq.(14)).

In addition to the input parameters whose values are given in Section 5 ($T_1=1$, $B=2000$, $\theta=50\%$, and $\delta=50\%$), other parameters are set as follows: (1) Two batch numbers are considered, $N=5$ and $N=10$, (2) total production volume is set to $Q=200$ items, i.e., 200 items are produced in 5 or 10 batches with $n_i=20$ and $n_i=40$ items per batch, respectively, and (3) three break times of $\tau=1.5$, $\tau=6$, and $\tau=12$ are considered for the production process. Eqs.(1) to (15) are solved without considering constraints Eq.(13) and (14). Further, we study the effects of 3 learning rates (slow, medium, and fast), 3 fatigue rates (slow, medium, and fast), and 3 recovery rates (slow, medium, and fast) on the outputs. When studying the effect of one parameter, we set the other parameters to their medium values. Results are in accordance with our previous observations indicating that fatigue is reduced when: learning is fast ($b=0.515$), production occurs in small more frequent batches ($N=10$), longer break times ($\tau=18$), and faster recovery from fatigue ($\mu=0.07$). This indicates that although some of the solutions produced when the constraints are ignored are infeasible, the model’s behaviour remains the same as when the constraints are imposed.

To consider the limitations of fatigue allowances which set the upper and lower bounds on the production quantity produced in each batch and the length of the rest, we consider the case where
The optimal solution occurs when $Q=37$ units, four rest breaks of $\tau=6$ units of time each, $F=20.8\%$, and $Z=0.76$. Excluding the constraints in Eqs.(13) and (14) we have $Q=200$ units, four rest breaks of $\tau=1.5$ units of time each, $F=64.6\%$, and $Z=0.586$. Comparing these cases, it is clear that solving the problem without the fatigue constraint, will yield better results regarding the production, i.e., more production ($Q\uparrow$) in less time ($Z\downarrow$), but poor worker welfare as average fatigue increases ($F\uparrow$), i.e., $64.6\%$ versus $20.8\%$. Furthermore, the optimization regarding the required alleviating of $50\%$ of the worker’s fatigue will yield a rest break of $\tau=6$ units of time in length ($\tau\uparrow$). This further means that having $\tau=1.5$ units of time is not possible unless workers welfare is ignored.

8. Discussion and concluding remarks

In this paper, a learning-forgetting-fatigue-recovery model (LFFRM) has been developed and exemplified through numerical examples. Our contribution was to develop a model which integrates the “learn forget curve model” (LFCM) with a fatigue-recovery model since these two phenomena occur simultaneously during the production process and production breaks. This model is capable of capturing the quantitative effects of workers’ abilities, i.e., improvement through learning and replenishment of the resources by recovery, and restrictions, i.e., losing experience by forgetting and capabilities due to physical fatigue, on the outputs of a production process. An interesting aspect of this research is that learning and recovery and forgetting and fatigue have opposite impacts on the performance, which requires trade-offs to optimise the system’s performance.

The effect of learning on the model has then been investigated and it was shown that, as learning increases, the system’s performance improves, which is consistent with the previous studies (Benkard, 2000; Syverson, 2010). A further insight from the LFFRM model was that smaller batch sizes are recommended as they result in less fatigue. The benefits of reduced fatigue observed during the fast learning scenarios suggest that technologies and/or training programs may quicken the learning process of the workers. It has been shown that more production results in longer production times and, subsequently, more fatigue. In the LFFRM model the lengths of breaks are optimized such that enough fatigue is alleviated and not much experience is lost because of forgetting. Similar to previous studies, e.g. Tucker (2003), the model recommends shorter frequent breaks to increase the production output.

LFFRM model demonstrated that:

i) Faster learning rates help to reduce fatigue. It was concluded that learning will cause the process to have more production with the same amount of fatigue. It may suggest for managers to consider investing in training programs as this meets the two managerial objectives, the welfare of the workers and the productivity of the system.
ii) Fatigued workers are counterproductive. This finding has been observed by Mital et al. (1991) who showed that the worker’s performance declines because of fatigue. Fatigued workers also have higher likelihood of making errors and being injured (Kopardekar & Mital, 1994; Dinges, 1995; Frank, 2000; Dionisio, 2010; Asfaw et al., 2011; Bevilacqua et al., 2012). Therefore direct and indirect results of fatigue may cause the process to be more costly (Ricci et al., 2007). If the length of a break is not enough to alleviate the fatigue, or if the fatigue rate is high, management could consider to invest in learning to lessen the fatigue, designing less fatiguing jobs, or allowing for longer breaks. This suggests that investing in training may be an improvement strategy for processes that are highly labour demanding either by training the workers, or by designing work tasks in ways that foster learning. For instance, designing material kits that are implicit instructive for assembly sequence (Medbo, 2003), encoding inferences into routines, e.g., forms, rules, procedures and strategies (Levitt & March, 1988), adopting new technologies (Shafer et al., 2001), changing the product design, tools, equipment, and works methods (Konz & Johnson, 2000), using knowledge diversity (Kellogg, 2009), and using cross-trained employees at numerous work stations (Finch & Luebbe, 1995) have been mentioned as ways to increase the learning rates. Another way to confront fatigue is to reduce the fatigue rate. Fatigue rates might be modified by controlling the workload levels for operators through reduced physical forces, improved worker postures, and reduced durations of force application (James & William, 1993; Yeow & Nath Sen, 2003).

iii) As the recovery speed increases, the limitation on batch number is relieved and more units can be produced with less average production time (better performance). As a result, when the worker recovers faster from the accumulated fatigue, lead time shrinks. Recovery rate may be considered as a personal attribute. The authors were not able to find any industry related study for improving the recovery rates. However, there is literature available in the Physiology field, e.g., the study of Tessitore et al. (2007) which suggests dry aerobic exercises and electrostimulation help soccer players to recover faster and to avoid damage of their muscles.

In order to study the limitations that arise from worker’s capabilities, we studied the effect of the constraints of the LFFRM model. The LFFRM constraints arise from the worker’s maximum endurance time (fatigue), the necessary level for fatigue alleviation and maximum forgetting. Relieving the upper bound of the production volume constraint and lower bound of the break time constraint revealed that the real production process would not achieve the initial goals that have been set for it because of these constraints. The literature of the productivity loss due to health problems (Punnett & Wegman, 2004; Meerdinger et al., 2005; Escoffizzo, 2008) may support the results of the LFFRM modelling. Worker’s capabilities and limitations are crucial
issues that must be considered in designing the future manufacturing systems (Keyserling & Chaffin, 1986).

The LFFRM which is presented in this study provides a basis for further research into the process improvement and also workforce scheduling however it has some limitations. First of all, only physical fatigue has been modeled in this version of LFFRM. Secondly, we only considered the effect of fatigue on productivity. Other aspects of the problem such as the effect of fatigue on a worker’s health or product quality have not been studied. Thirdly, the product variety has not been investigated and the process was assumed to produce only one type of product. Finally, we did not provide solutions for increasing learning and recovery rates or decreasing the fatigue rate; however, some resolutions have been provided in the literature as discussed above.

A future work of this study could consider a process with dynamic scheduling for break times and their frequency. We studied LFFRM for processing of a fixed number of batches and breaks. A dynamic program is being developed to have enough flexibility for scheduling an optimized process for varying number and frequency of batches and break times. Also, the fatigue level in the model must be adjusted according to the empirical data. If so, break times and frequencies can be scheduled upon actual data. The developed models like the LFCM could be investigated in the context of inventory management or worker’s cross-training and deployment in dual resource constraint systems. These extensions are on the authors’ research list.

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Appendix

To derive Eq.(14), we have $0.5(u_i + n_i) \geq (u_i + n_i)^{1+f_i/b}(u_i + n_i + s_i)^{-f_i/b}$, from which we have

$$s_i \leq \left\{ \frac{0.5(u_i + n_i)}{(u_i + n_i)^{1+f_i/b}} \right\}^{-b/f_i} - u_i - n_i \Rightarrow \left[ \frac{1-b}{T_i} \tau_i + (u_i + n_i)^{1-b} \right]^{-1/(1-b)} - u_i - n_i$$

$$\leq \left\{ \frac{0.5(u_i + n_i)}{(u_i + n_i)^{1+f_i/b}} \right\}^{-b/f_i} - u_i - n_i \Rightarrow \left[ \frac{1-b}{T_i} \tau_i + (u_i + n_i)^{1-b} \right]^{-1/(1-b)} \leq \left\{ \frac{0.5(u_i + n_i)}{(u_i + n_i)^{1+f_i/b}} \right\}^{-b/f_i}$$

$$\frac{1-b}{T_i} \tau_i + (u_i + n_i)^{1-b} \leq \left\{ \frac{0.5(u_i + n_i)}{(u_i + n_i)^{1+f_i/b}} \right\}^{-b(1-b)/f_i} \Rightarrow$$

$$\tau_i \leq \frac{T_i}{1-b} \left\{ \frac{0.5(u_i + n_i)}{(u_i + n_i)^{1+f_i/b}} \right\}^{-b(1-b)/f_i} - (u_i + n_i)^{-b}$$
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