Discriminant Analysis of the two dimensional Gabor Features for face recognition

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Abstract –
In this paper, a new technique called Two Dimensional Gabor Fisher Discriminant (2DGFD) is derived and implemented for image representation and recognition. In our approach, the Gabor wavelets are used to extract facial features. The Principal Component Analysis (PCA) is applied directly on the Gabor transformed matrices to remove redundant information from the image rows and a new direct two dimensional Fisher Linear Discriminant (direct 2DFLD) method is derived in order to further remove redundant information and form a discriminant representation more suitable for face recognition. The conventional Gabor based methods transform the Gabor images into a high-dimensional feature vector. However, these methods lead to high computational complexity and memory requirements. Furthermore, it is difficult to analyse such high dimensional data accurately. The novel 2DGFD method was tested on face recognition using the ORL, Yale and Extended databases, where the images vary in illumination, expression, pose, and scale. In particular, the 2DGFD method achieves 98.0% face recognition accuracy when using 20×3 feature matrices for each Gabor output on the ORL database and 97.6% recognition accuracy compared to 91.8% and 91.6% for the 2DPCA and 2DFLD method on the Extended Yale database. The results show that the proposed 2DGFD method is computational more efficient than the Gabor Fisher Classifier method (GFC) by approximately 8 times on the ORL, 135 times on the Yale and 1.2801×10^8 times on the Extended Yale B datasets.

Index Terms: Biometrics, Gabor wavelets, Face Recognition, Feature extraction, Linear Discriminant Analysis, Principal Component Analysis (PCA).
1. INTRODUCTION

The human face plays a significant role in human communication and identification processes as humans identify faces within a fraction of a second. Therefore, it seems natural to use the face recognition for biometrics purposes. It is thus not surprising that in response to the increase in terrorist attacks and similar threats to global security, face recognition/verification has attracted attention in biometrics, computer vision and pattern recognition communities. However, building an automated face recognition system which can detect and identify faces in a scene with little or no effort is very challenging. The challenges are more profound when one considers the large variations in the visual stimulus due to illumination conditions, viewing directions or poses, facial expression, ageing, facial hair and glasses. A good face recognition methodology should consider representation as well as classification issues and a good representation method should introduce low dimensional features of the face object with enhanced discriminatory power. Furthermore, it should require minimum manual location of the local facial features suitable for face recognition. Since face recognition depends heavily on the choice of features used by the classifier, it is common to start with a given set of features and then attempt to derive an optimal subset (under some criteria) of features leading to high classification performances. Principal component analysis (PCA) is a popular technique used for both face representation and recognition. Kirby and Sirovich [1] showed that any face image can be efficiently represented along the eigenfaces (eigenvectors) coordinate space. Turk and Pentland [2] presented the well-known Eigenfaces method for face recognition in 1991 based on PCA. Since then, PCA has been widely investigated and has become one of the most successful approaches in face recognition [3-5]. Recently, PCA has been extended to two dimensional face recognition [6, 7].

Another popular method for face recognition and representation is the Fisher linear discriminant (FLD) [8], which maximizes the ratio of the trace of the between class scatter to the trace of the within class scatter matrix. However, the FLD method suffers from the singularity problem of the within class scatter matrix caused by the limited number of training samples in face recognition (the number of training samples is less than the dimension of
feature vectors). This problem is referred to as the small sample size (SSS) problem. To address this problem, PCA has been applied initially to reduce the dimensionality of the high dimensional vector space prior to the application of the FLD method. The PCA plus FLD approach (PCA + FLD) has received significant attention. In [9], PCA is first applied to remove the singularity of the within scatter matrix, the FLD method is then performed in the PCA subspace. The criterion for PCA and FLD may not be compatible with each other. The PCA step may remove dimensions that contain discriminate information required for face recognition and lead to low recognition accuracy. Chen et al. [10] proved that the eigenvectors corresponding to eigenvalues equal to zero or close to zero of the within scatter matrix contains the most discriminative information. Within this context Yu et al. [11] proposed the direct Linear Discriminant Analysis method by simultaneously diagonalizing the between scatter matrix first and then diagonalizing the within scatter matrix.

Recently, the results of the face recognition vendor test (FRVT) suggested that face recognition performances under varying illumination and pose still remains a big challenge. To overcome these limitations, a number of face recognition techniques apply the Gabor wavelets which model the receptive field profiles of cortical simple cells [12]. The Gabor wavelet representation, therefore, captures salient visual properties such as spatial localization, orientation selectivity and spatial frequency. In [13], the PCA method was applied on the Gabor transformed images and similarly in [14] the PCA + FLD method was applied on the Gabor wavelet outputs where superior performances were reported. However, these methods have the drawbacks of computational complexity and high dimensionality due to the transformation of the Gabor outputs into the high dimensional vectors. Usually five different scales and eight orientations Gabor wavelets are used in face recognition tasks, leading to forty Gabor outputs. For example, in standard face recognition techniques an image of size 112 × 92 is first transformed by the Gabor wavelet to produce 40 images of varying scales and orientation. The 40 Gabor outputs are transformed into 40 high dimensional vectors of size 10304 and each of the 40 high dimensional vectors are then concatenated to produce a larger feature vector. However, it is difficult to analyse such high dimensional
feature vectors directly because it is computationally very intensive to perform calculations in a high dimensional feature space. Therefore traditional techniques avoid this difficulty by downsampling each of the 40 vectors prior to concatenating them. For example, downsampling factors of 1, 2, 3 and 4 lead to the following dimensionalities for the concatenated feature vector 4.1216×10^5, 2.0608×10^5, 1.3739×10^5 and 1.0304×10^5. It is difficult to analyse such high dimensional vectors accurately due to high computation time and increased memory cost.

To circumvent the above problems, a new image representation and recognition technique called 2DGFD is derived and implemented for image feature extraction. The main contribution of this paper is that, rather than follow the conventional Gabor based techniques where the Gabor transformed images are integrated across different orientations and scales into a high dimensional vector and then apply a statistical based technique (e.g. Fisher Discriminant Analysis) on the high dimensional vector, it is better to apply a statistical based technique separately on each orientation and scale of the Gabor transformed image and then sum up the distance measures that are obtained on each of them in order to get the overall distance measure between any two face images. The novelty of the 2DGFD method can be summarised as follows:

- A direct 2DFLD method is derived which simultaneously considers the discriminant information both for the between scatter matrix and the within scatter matrix.
- As opposed to other Gabor based methods, the 2DGFD method is based on 2D Gabor transformed image matrices which incorporate spatial locality, scale and orientation matrices rather than 1D downsampled Gabor vectors. The 2D Gabor image matrices are not transformed into a high dimensional vector. As a consequence, it is easier to analyse smaller 2D Gabor matrices faster and more accurately in the 2DGFD method than high dimensional vectors.
- The Gabor wavelet transformed images resides in a space which contains redundant information not suitable for face recognition, the dimensionalities of the Gabor
outputs are firstly reduced, using the eigenvalue selectivity constraint of the PCA method along the image rows prior to the application of the direct 2DFLD method along the image columns.

- Lastly, the direct 2DFLD method is applied to the Gabor transformed images for the following reasons: the Gabor transformed face images possess characteristics of spatial locality, scale and orientation which can produce salient local features that are most suitable for face recognition. The direct 2DFLD method reduces redundancy and produces an image representation with enhanced discriminating features more suitable for face recognition.

2. **GABOR FEATURE ANALYSIS**

The Gabor wavelets capture the properties of spatial localization, orientation, spatial frequency and phase relationship. This seems to have similar characteristics to the 2D filter response profiles of the mammalian cortical simple cells, experiments conducted in cortical neurons [15, 16]. The Gabor wavelets have been found to be particularly suitable for image decomposition and representation when the goal is to find local and discriminating features. The experimental results in [12] have shown that the Gabor filter representation gave better performance for classifying facial expressions.

In this section, we outline the basics on Gabor wavelets, describe the Gabor feature representation of images and then show how it can be applied to for face recognition.

2.1 **Gabor Wavelets**

The Gabor wavelets (kernels, filters) can be defined as follows [12]:

\[
\varphi_{\mu,\nu}(z) = \frac{\|k_{\mu,\nu}\|}{\sigma^2} e^{-\frac{b_{\mu,\nu}^2}{2\sigma^2}} \left[ e^{i\mu z} - e^{\sigma^2} \right]
\]

where \( \mu \) and \( \nu \) define the orientation and scale of the Gabor kernels, \( z = (x, y) \). \( \| \) denotes the norm operator and the wave vector \( k_{\mu,\nu} \) is defined as:
\[ k_{\mu,\nu} = k_v e^{i\phi_v} \]  

where \( k_v = k_{\text{max}} / f^\nu \) and \( \phi_v = \pi \mu / 8 \). \( k_{\text{max}} \) is the maximum frequency and \( f \) is the spacing factor between kernels in the frequency domain [12].

The Gabor kernels in (1) are all similar since they can be generated from one filter, the mother wavelet, by scaling and rotation via the wave vector \( k_{\mu,\nu} \). Each kernel is a product of a Gaussian envelope and a complex plane wave while the first term in the square brackets in Eqn 1 determines the oscillatory part of the kernel and the second term compensates for the DC value. The effect of the DC term becomes negligible when the parameter \( \sigma \), that determines the ratio of the Gaussian window width to wavelength, has sufficiently large values. It is usual to use Gabor wavelets at five different scales, \( \nu \in \{0, \cdots, 4\} \) and eight orientations, \( \mu \in \{0, \cdots, 7\} \) with the following parameters: \( \sigma = 2\pi \), \( k_{\text{max}} = \pi / 2 \) and \( f = \sqrt{2} \) [16]. The kernels have strong characteristics of spatial locality and orientation [12], making them a suitable choice for image feature extraction for face classification.

### 2.2 Gabor Representation

The Gabor wavelet representation of an image is the convolution of the image with a family of Gabor kernels as defined by Eqn 1. Let \( I(x, y) \) be the gray level distribution of an image of size \( n_x \times n_y \), the convolution output of image \( I(x, y) \) and a Gabor kernel \( \varphi_{\mu,\nu}(x, y) \) is defined as follows:

\[ X_{\mu,\nu}(z) = I(z) \ast \varphi_{\mu,\nu}(z) \]  

(3)

where \( z = (x, y) \), \( \ast \) denote the convolution operator and \( X_{\mu,\nu}(x, y) \) is the convolution result corresponding to the Gabor kernel at orientation \( \mu \) and scale \( \nu \).

By applying the convolution theorem, we can derive each \( X_{\mu,\nu}(x, y) \) from Eqn 3 using the Fast Fourier Transform (FFT)

\[ \text{FFT}\left\{X_{\mu,\nu}(z)\right\} = \text{FFT}\left\{I(z)\right\} \times \text{FFT}\left\{\varphi_{\mu,\nu}(z)\right\} \]  

(4)
and

$$X_{\mu,\nu}(z) = FFT^{-1}\{FFT\{I(z)\} \times FFT\{\varphi_{\mu,\nu}(z)\}\} \tag{5}$$

where $FFT^{-1}$ denotes the inverse Fast Fourier Transform. In [14] it was shown that the convolution outputs of images have strong characteristics of spatial locality, scale, and orientation corresponding to those displayed by the Gabor wavelets. Such characteristics produce salient local features, such as the eyes, nose and mouth, that are suitable for visual event recognition. Since the magnitude of the convolution outputs shows the desirable characteristics necessary for efficient face recognition, we applied the magnitude (not the phase) in all our analysis, which is consistent with the application of Gabor representations in [17]. Since the convolution outputs consist of different local, scale and orientation features, it make sense to operate on them directly. The main idea of our proposed algorithm is to apply a discriminate analysis directly on the 2D convolution outputs of each image, rather than transform each output $X_{\mu,\nu}(x,y)$ into a vector before performing discriminate analysis.

3. PROPOSED ALGORITHM: DISCRIMINATE ANALYSIS OF THE GABOR FEATURES FOR FACE RECOGNITION

In this section, we describe our novel two dimensional Gabor Fisher Discriminant (2DGFD) method for face recognition. A new direct 2DFLD method will be derived which is more suitable for extracting the discriminant face features. The direct 2DFLD method is then applied on the Gabor wavelet outputs which are robust to variations due to illumination and pose.

3.1 New Direct Two Dimensional Fisher Linear Discriminant

Suppose there are $N$ training samples, $I_1, I_2, \cdots, I_N$, consisting of $K$ known pattern classes, denoted as $C_1, C_2, \cdots, C_K$, each of size $n_r \times n_c$. The aim is to find an $n_c \times 1$ transformation vector $\varphi$ such that image matrices within the $i^{th}$ class are close together while
making the $i^{th}$ and the $j^{th}$ class far apart as possible. The $j^{th}$ training sample in class $i$, $I_{ij}$, can be projected onto the vector $\phi$ by the following linear transformation [18]

$$\mathbf{b}_{ij} = I_{ij}\phi$$

(6)

The class separability of the transformed 2D matrices can be characterized by the Fisher criterion function [19] defined as

$$J(\phi) = \frac{\phi^T S_B \phi}{\phi^T S_w \phi}$$

(7)

where $\phi$ is a transformation vector, $S_B$ and $S_w$ are the between class scatter matrix and the within class scatter matrix defined [19] as

$$S_B = \frac{1}{N} \sum_{i=1}^{K} N_i (M_i - \mathbf{M})^T (M_i - \mathbf{M})$$

(8)

$$S_w = \frac{1}{N} \sum_{i=1}^{K} \sum_{j=1}^{N_i} (I_{ij} - M_i) (I_{ij} - M_i)^T$$

(9)

where $N_i$ is the number of training samples in class $i$, $M_i$ is the mean of the training samples in class $i$, $\mathbf{M}$ is the global mean of all training samples. The vectors $\phi$ that maximizes the Fisher criterion Eqn 7 are obtained by solving the generalized eigenvalue equation $S_B \phi = \delta S_w \phi$ for $\delta \neq 0$, where $\delta$ is the corresponding eigenvalue. The solution is usually obtained by applying the eigen-decomposition to the matrix $S_w^{-1}S_B$, if $S_w$ is non-singular [20]. However, our aim is to accurately compute the eigenvectors of $S_B$ and $S_w$ containing the most energy of the images. Suppose that the eigenvectors, $\mathbf{e}_i, i = 1, 2, \cdots, q$, corresponding to the largest eigenvalues of $S_w$, then the projection direction $\phi$ can be defined as

$$\phi = P\mathbf{e}_i, \text{ where } P = \begin{bmatrix} \mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_q \end{bmatrix}$$

(10)

where $\mathbf{p}_i, i = 1, 2, \cdots, q$ are the eigenvectors corresponding to the $q$ largest eigenvalues $\kappa_i, i = 1, 2, \cdots, q$ of $S_B$. Under this mapping, the Fisher criterion is converted into
\[ J(\phi) = \frac{e_i^T (P^T S_B P) e_i}{e_i^T (P^T S_w P) e_i} \]  

According to Eqn 11, we define the functions

\[ J(e_i) = \frac{e_i^T S_B^p e_i}{e_i^T S_w^p e_i} \]

where \( S_B^p = P^T S_B P \) and \( S_w^p = P^T S_w P \). It is easy to show that \( S_B^p \) is a diagonal matrix while \( S_w^p \) is not diagonalised. The eigenvectors \( e_i, i = 1, 2, \cdots, z \) are thus obtained by solving the eigenvalue equation \( S_w^p e_i = \omega_i e_i \) and selecting the eigenvectors corresponding to the largest eigenvalues which contains the most energy [18]. It must be noted that, the direct LDA method [11] and other Fisher based methods selects eigenvectors corresponding to the smallest eigenvalues of the within class scatter matrix rather than the largest eigenvalues which contain the most energy. Therefore the transformation in Eqn 6 can be decomposed into two transformations:

\[ A_y = I_y P \]  

and

\[ B_y = A_y \Gamma, \text{ where } \Gamma = [e_1, e_2, \cdots, e_z] \]

In summary, the proposed direct 2DFLD method is different from the convention Fisher based techniques such as the 2DPCA method , the 2DLDA method and two Dimensional Direct LDA method (2D-DLDA) [21]. Firstly, the new direct 2DFLD method simultaneously diagonalises the image between covariance matrix \( S_B \) and the image within covariance matrix \( S_w \). In standard methods such as 2D-DLDA which simultaneously diagonalises \( S_B \) and \( S_w \), the \( S_B \) matrix is usually whitened as part of the method. The proposed direct 2DFLD method avoids this whitening process because this whitening process can be shown to be redundant and also leads to increased computational complexity. Secondly, the simultaneous diagonalization of \( S_B \) and \( S_w \) is usually conducted by first selecting the projection directions which maximizes \( S_B \) in the transformed space and then the projection
directions which minimizes $S_w$ in the transformed space. In contrast to the proposed direct 2DFLD method, the projection directions of $S_w$ that contains the most energy are selected. As a consequence, image classes are transformed in such a way that they become as far apart as possible. Furthermore the distances between images of the same class are also maximised (the maximum variance of each class is returned) while keeping the distances between classes as far as possible. However the direct 2DFLD method is a natural extension of the classical direct LDA [11] and methods proposed by Li et al. and Xiong et al. [19, 20].

3.2 Two Dimensional Discriminant Gabor Feature Representation

Low dimensionality is important for learning and computation in tasks such as similarity judgment [22]. Since the Gabor outputs are two dimensional matrices, it is therefore necessary to reduce the dimensionalities in both directions (rows and columns). Principal component analysis is the preferred method in face recognition when the aim is dimension reduction and efficiency to represent the face image [2]. Therefore we apply the generalized total scatter criterion (PCA criterion) on the rows of $X_{\mu,\nu}(x, y)$ before applying the direct 2DFLD method as follows. The generalized total scatter criterion [6] can then be defined as

$$ J(\mathbf{v}) = \mathbf{v}^T \mathbf{C} \mathbf{v} $$

where $\mathbf{C}$ is an $n_r \times n_r$ covariance matrix applied directly on the image rows as

$$ \mathbf{C} = \frac{1}{\text{NOS}} \sum_{i,\mu,\nu} (X_{\mu,\nu}^i - \bar{X})^T (X_{\mu,\nu}^i - \bar{X}) $$

where $O$ and $S$ is the number of orientations and scales, $(\ )^T$ denotes the transpose and $\bar{X} = \frac{1}{\text{NOS}} \sum_{i,\mu,\nu} X_{\mu,\nu}^i$ is the global mean. The vector that maximizes Eqn 16 is the eigenvector $\mathbf{v}$ of $\mathbf{C}$ corresponding to the largest eigenvalue. In general, it is not enough to have only one projection direction. We usually need to select a set of projection directions, $V_d = [v_1, v_2, \ldots, v_d]$, which are orthonormal. In fact, the projection directions, $V_d = [v_1, v_2, \ldots, v_d]$, are the orthonormal eigenvectors of $\mathbf{C}$ corresponding to the first $d$
largest eigenvalues \( d \leq n_r \). From which the images can be efficiently represented along the rows using the \( d \) eigenvector as

\[
Y^i_{\mu,v} = V_d^T (X^i_{\mu,v} - \bar{X})
\]  

(17)

where \( Y^i_{\mu,v} \) is an \( d \times n_c \) feature matrix at orientation \( \mu \) and scale \( v \) for the \( i \)th image.

However, one should be aware that the PCA criterion takes into account the discriminating and non discriminating features, the non discriminating features may not lead to optimal performance for tasks such as face recognition. To address this obvious problem on the 2D image matrix \( Y^i_{\mu,v} \), we apply our proposed direct 2DFLD method on the image columns such that the image classes are well separated. From which the Fisher criterion can be rewritten as

\[
J(\mu) = \frac{\mu^T S^P_B \mu}{\mu^T S^P_w \mu}
\]  

(18)

where \( S^P_B \) & \( S^P_w \) are the between and within class scatter matrices of size \( n_c \times n_c \) defined as

\[
S^P_B = \frac{1}{NSO} \sum_{i=1}^{K} N_i SO (\bar{Y}_i - \bar{Y})^T (\bar{Y}_i - \bar{Y})
\]  

(19)

\[
S^P_w = \frac{1}{NOS} \sum_{i,j,\mu,v} (Y^i_{\mu,v} - \bar{Y}_i)^T (Y^i_{\mu,v} - \bar{Y}_i)
\]  

(20)

where \( \bar{Y}_i = \frac{1}{N_i SO} \sum_{\mu,v} Y^i_{\mu,v} \) is the mean of the \( i \)th class \( C_i \), \( \bar{Y} = \frac{1}{NOS} \sum_{\mu,v} Y^i_{\mu,v} \) is the global mean, both means takes into account the local, orientation and frequency property of the outputs \( Y^i_{\mu,v} \).

Therefore we first solve the eigenvalue equation \( S^P_B \beta = \Sigma \beta \) and select \( g \) eigenvectors

\[
\beta_g = [\beta_1, \beta_2, \cdots, \beta_g]
\]

corresponding to the \( g \) largest eigenvalues. From which the within class scatter matrix \( S^P_w \) can be transformed into the subspace for the between class scatter as

\[
\beta_g^T S^P_w \beta_g = S^B_w
\]  

(21)

where \( S^B_w \) is of size \( g \times g \) which can then be diagonalised as

\[
\gamma^T S^B_w \gamma = \eta
\]  

(22)
where $\gamma$ and $\eta$ are the eigenvector and eigenvalue matrices of $B^w$. The overall discriminant feature matrix can then be obtain by using $\gamma_q = [\gamma_1, \gamma_2, \cdots, \gamma_q]$ defined as

$$Z^i_{\mu,v} = Y^i_{\mu,v} \beta \gamma_q$$

(23)

In Eqn 23, the columns of the feature matrix $Z^i_{\mu,v}$ of size $d \times q$ contain the required discriminating information which is suitable for face feature classification. Figure 1 illustrates the flowchart of the proposed 2DGFD method.

### 3.3 Classification Method

After the transformation in Eqn 23, a feature matrix is obtained for each image. The nearest neighbour classifier is used for classification. The distance between two feature matrices varying in orientation and scale, $Z^n_{\mu,v} = [z^n_{\mu,v,1}, z^n_{\mu,v,2}, \cdots, z^n_{\mu,v,q}]$ and $Z^m_{\mu,v} = [z^m_{\mu,v,1}, z^m_{\mu,v,2}, \cdots, z^m_{\mu,v,q}]$, is defined as

$$\text{dis}(Z^n, Z^m) = \sum_{\mu,v,k} \|z^n_{\mu,v,k} - z^m_{\mu,v,k}\|_2$$

(24)

where $\|z^n_{\mu,v,k} - z^m_{\mu,v,k}\|_2$ denotes the Euclidean distance between the two discriminant column vectors $z^n_{\mu,v,k}$ and $z^m_{\mu,v,k}$. Let us suppose we have a test sample $Z^\text{test}_{\mu,v}$ and the face recognition system is operating in the identification mode (1 to N comparisons), if the minimum distance occurs at the face image $Z^w$, $\text{dis}(Z^\text{test}, Z^w) = \min_j \text{dis}(Z^\text{test}, Z^j)$, and $Z^w \in C_f$ (i.e. $Z^w$ is from the class $C_f$), then the resulting decision is that $Z^\text{test}$ is also from the class $C_f$, i.e. $Z^\text{test} \in C_f$.

In summary of the proposed 2DGFD method, the proposed technique operates directly on each of the 2D Gabor transformed images which vary in orientations and scales separately since each Gabor transformed image contains redundant information. This leads to the extraction of a set of feature matrices which vary in scales and orientations which contains very little redundant information. Therefore during image classification, image features in similar orientations and scales are compared and then summed up (Eqn 24). The proposed
2DGFD method is expected to achieve superior face recognition accuracy. This is because the proposed 2DGFD method takes into account that different discriminant features occurs at different frequencies and angles in an image.

### 3.4 Relationship with other Gabor based algorithms

In this section, we analyse other Gabor based methods and distinguish them from our new 2DGFD method for face recognition. Let us begin with the Gabor wavelet representation. In particular, we compare the proposed 2DGFD and the Gabor Fisher Classifier (GFC) (Gabor wavelet + PCA + LDA) method in terms of their Gabor wavelet representation, computational and memory requirements. In [14] the GFC method was proposed for face recognition. We first compare the Gabor representation for each method.

The GFC method encompasses different spatial frequencies (scales), spatial localities and orientations by concatenating all the different Gabor representations and derives an augmented feature vector as follows. Let the convolution outputs of an \( n_r \times n_c \) image and the Gabor wavelets be defined as in Eqn 3. The following steps are carried out:

1. Each \( n_r \times n_c \) output \( X_{\mu,\nu}(z) \) is transformed into a high dimensional vector \( o_{\mu,\nu}^p(z) \) of size \( n_r n_c \times 1 \).
2. Downsample each \( o_{\mu,\nu}^p(z) \) by a factor \( p \) to reduce the high dimensionality and normalize it to zero mean and unit variance.
3. Lastly, construct a vector output of \( o_{\mu,\nu}^p(z) \) by concatenating its rows (or columns) as

\[
\mathbf{h}_{\mu,\nu}^p = \begin{bmatrix} o_{(p)0,0}^p & o_{(p)0,1}^p & \cdots & o_{(p)0,7}^p \\
\end{bmatrix}^T
\]

(25)

where \( T \) is the transpose operator.

Figure 2 illustrates the flowchart of the GFC method. To our knowledge all the methods that apply the Gabor wavelet representation follows the described steps above. From the summary above we conclude the following:
• The image vectors \( o_{\mu,\nu}(z) \) of the convolution output usually lead to a high-dimensional image vector space, where it is difficult to analyze the images accurately.

• In face recognition, it is common practice to use eight orientations and five scales, \( o^\rho_{\mu,\nu} : \mu \in \{0, \ldots, 7\}, \nu \in \{0, \ldots, 4\} \). Therefore, in order to reduce the dimensionality of the final vector \( h^\rho_{\mu,\nu} \) to a manageable size, the downsampling factor of 64 has been suggested in face recognition [13] for images of size 128x128 on the analysis conducted on FERET face databases. However, the choice of the downsampling factor has a crucial effect on the face recognition performance, i.e., if one does not choose the downsampling factor properly through experiments, one will not achieve acceptable performances as reported in [13].

We then compare the computational requirement using the number of multiplications as a measurement of computational complexity. To calculate the transformation matrix, the GFC method must solve an \( N \times N \) eigenvalue problem, then a \( V_{PCA} \times V_{PCA} \) eigenvalue problem, and lastly a \( V_w \times V_w \) eigenvalue problem, where \( N \) is the size if the training set, \( V_{PCA} \) is the dimension of the PCA subspace (Eigenvectors corresponding to the largest eigenvalues, usually \( V_{PCA} = N - 1 \)), \( V_w \) is the dimension of the within covariance matrix subspace. In contrast, 2DGFD must solve an \( n_r \times n_r \) Eqn 16 eigenvalue problem, an \( n_c \times n_c \) Eqn 19 eigenvalue problem and a \( V_B \times V_B \) Eqn 22 eigenvalue problem, where \( V_B \) is the dimension of the subspace for the between class scatter matrix Eqn 22. Note, we assume that each image matrix is of size \( n_r \times n_c \). In [23] it is shown that the complexity of an \( N \times N \) eigenvalue problem is \( O(N^3) \), therefore the complexity of computing the GFC transformation matrix is \( O(N^3 + V_{PCA}^3 + V_w^3) \) which is much higher than that of 2DGFD method with complexity \( O(n_r^3 + n_c^3 + V_B^3) \). To illustrate the computations for the two methods, we use the ORL face database [24] containing 200 images from 40 subjects, each of size 112 x 92. From Table I, we can see that the computational complexity for GFC heavily depends on the number of
training sample $N$, which can be of high magnitude in real world applications. In contrast, the computational complexity for 2DGFD depends on the dimensions of the image matrix (rows $n_r$ and columns $n_c$) which are usually much smaller than $N$, $n_r << N$ and $n_c << N$. Therefore, the computational advantage of the proposed 2DGFD increases as $N$ increases. This is highlighted in Tables II & II on the analysis using the Yale and extended Yale B face database with 165 and 16 128 training samples. Table IV gives the ratio of the computational requirements for the 2DGFD method and the GFC. The 2DGFD method is more than 8 times (8:1) more efficient than the GFC method on the ORL database, more than 135 times (135:1) more efficient than the GFC method on the Yale database and lastly $1.2801 \times 10^8$ times ($1.2801 \times 10^8$:1) more efficient than the GFC on the Extended Yale B database. Table V shows the ratio of the memory requirements of the proposed 2DGFD method and the GFC method. The 2DGFD method has a lower memory requirement than the GFC method on the Extended Yale B database which contains more face images than both the ORL (200 images for training) and the Yale database (165 images). However, the 2DGFD method requires approximately 52 times (52:1) more memory space than the GFC method on the ORL database and it needs approximately 6 times (6:1) more memory storage than the GFC method on the Yale database. In addition, Tables I & II seem to suggest that the 2DGFD method requires more coefficients to represent an image. Table III shows that the maximum feature matrix size for the 2DGFD method is smaller than that of the GFC method as a larger training sample of 16 128 was used in the analysis. Each input face image will have a maximum $n_r \times n_c$ image representation with eight orientations and five scales. Consequently, each image will produce a populated set of 40 representations of maximum size if all the eigenvectors were used for projection. In face recognition tasks, since the energy of an image is concentrated on its eigenvectors corresponding to the largest eigenvalues we use these eigenvectors to represent the image [6].

4. RESULTS AND DISCUSSIONS
We assess the feasibility and performance of the 2DGFD method on the face recognition task, using three datasets: 1) an ORL dataset that contains 400 images corresponding to 40 subjects which are acquired under variable pose, scale and facial expression. 2) the Yale face database contains 165 images of 15 individuals with variations in lighting condition and facial expression. 3) lastly the Extended Yale B [25] face database containing 38 subjects under 9 poses and 64 illuminations. The effectiveness of the 2DGFD method is shown in terms of both absolute performance and comparative performance against some popular face recognition schemes such as the Gabor Principal Component Analysis [13], Gabor Fisher Classifier (GFC) method [14], two dimensional PCA method (2DPCA) [6] and the two dimensional FLD (2DFLD) method [19, 20] method. For consistency and fairness with the application of the Gabor wavelets to face recognition, we use eight orientations and five scales, $O_{\mu,\nu} : \mu \in \{0, \cdots, 7\}, \nu \in \{0, \cdots, 4\}$ in all our analysis.

4.1. ORL Face Database

The ORL face database (developed at the Olivetti Research Laboratory, Cambridge, U.K.) is composed of 400 images with ten different images for each of the 40 distinct subjects. The ORL images vary in pose, size, time and facial expression. The facial expressions include eyes open, eyes closed, smiling, not smiling and some images contain facial details such as glasses. Furthermore, some images were taken at varying angles of up to 20 degrees. The spatial and grey-level resolutions of the images are $112 \times 92$ and 256. All the 400 images from the ORL database are used to evaluate the face recognition performance of our 2DGFD method. Five images are randomly chosen from the ten images available for each subject for training, while the remaining five images (unseen during training) are used for testing.

The first analysis was performed on the dimensionality of the 2DGFD method because low dimensionality representation is important for learning [2]. We design a series of analyses for varying dimensions for the within class subspace matrix $S_w^B$ Eqn 22. Seven tests were performed with varying subspace dimensions for $S_w^B$ such that the transformation matrices,
\[ \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_d \] and \[ \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_q \], used to define the subspace for the within class scatter matrix were varied as \( d = q = 5, 10, 15, 20, 25, 30 \) and 35. The eigenvalue problem for \( \mathbf{S}_w^\beta \) was then solved and the eigenvectors used for projection Eqn 23. Table VI shows the top face recognition performances for varying subspace dimensions. The best recognition accuracy of 98.0% occurs when the subspace dimensions \( 20 \times 20 \) and \( 25 \times 25 \) for \( \mathbf{S}_w^\beta \) are used. The subspace dimensions \( 5 \times 5, 10 \times 10, 15 \times 15, 30 \times 30 \) and \( 35 \times 35 \) for \( \mathbf{S}_w^\beta \) achieved a top recognition accuracy of 97.5%. It is therefore reasonable to use dimensions of \( 20 \times 20 \) to define \( \mathbf{S}_w^\beta \) in all our analysis in order to reduce the computational complexity and memory requirements of the 2DGFD method.

A series of analyses were undertaken to compare the performance of 2DFLD and the proposed 2DGFD method under conditions where the training sample size is varied. Four tests were performed with a varying number of training samples. In the first test, two image samples per class were used for training and the remaining eight samples for testing. Similarly in the second test, three images per class for training and the remaining seven for testing. This process was repeated until the fourth test. Table VII presents the top recognition accuracy of 2DFLD and 2DGFD, which corresponds to different numbers of training samples. The 2DGFD method outperforms the 2DFLD method in all our analysis. Table VII further indicates that the dimension of the 2DGFD feature matrix \( (40 \times 20 \times 3) \) is always much higher than 2DFLD \( (112 \times q) \), where \( q \) is the number of eigenvectors corresponding to the \( q \) largest eigenvalues of the generalised eigenvalue problem. The Gabor wavelets at five different scales and eight orientations \( (40 \) varying outputs) are required to produce localised features varying in scale and orientation necessary for face recognition.

The third analysis was conducted on the performance of the one dimensional Gabor based methods such as the Gabor PCA [13] and the GFC method [14] and compared their performances to the proposed 2DGFD method. The augmented Gabor feature vectors, \( \mathbf{H}_{\mu\nu}^p \), were downsampled by \( p = 4 \) and 16 as suggested in [4, 13, 26]. Note, although the
downsampling factor of \( p = 64 \) was also suggested, it was not used in our analysis. This is because it results in a significant loss of information. For comparison purposes, the 2DGFD method is analyzed on the same downsampled images. After the images were downsampled by \( p = 4 \) and 16, the selected dimensionalities of the subspace for \( S^B_w \) were 5 × 5 and 20 × 20 in the 2DGFD method. We chose three eigenvectors corresponding to three largest eigenvalues after solving the eigenvalue problem Eqn 22 for the projection Eqn 23. The justifications of this selection is due to the results reported in Table VI, the top recognition accuracy occurred when three eigenvectors of \( S^B_w \) were used for projection. Similarly, we fixed the number of eigenvectors for Gabor PCA and GFC to \( N - 1 \), where \( N \) is the number of training samples. Figure 4 shows the recognition accuracy versus the downsampling factors \( p = 4 \) and 16. The results lead to the following conclusions: The matrix based analysis outperforms the vector based analysis, highlighting the merit of the 2DGFD method when compared to the Gabor PCA method and the GFC method. This is consistent with the results in [6], which reported that the 2DPCA outperformed the Eigenfaces method. In the proposed 2DGFD method, the dimensions of the extracted feature matrices can be seen as 600 (5 × 3 × 40) and 2400 (20 × 3 × 40) for images downsampled by \( p = 4 \) and \( p = 16 \). In contrast, the dimensionalities of the feature matrices for the Gabor PCA method and the GFC method are both 199 (\( N - 1 \times 1 \), where \( N = 200 \)). The high dimensionality of the proposed 2DGFD method contains more discriminant information necessary for correct face classification. In particular, the proposed 2DGFD method achieves 98.0% and 95.5% recognition accuracy, whereas the GFC method has recognition accuracies of 95.0% and 93.0 and finally the Gabor PCA method obtains 94.5% and 94.0% for \( p = 4 \) and 16. Lastly, downsampling the Gabor transformed images can lead to a decrease in face recognition accuracy. The optimal sampling rate that does not lead to a loss of discriminant information is not known. The discriminant information required for accurate recognition performances may be lost during the
downsampling process. Therefore, the results show that the proposed 2DGFD method is superior to the Gabor PCA method and GFC method.

4.2. Yale Face Database

The Yale face database developed at the Yale Center for Computational Vision and Control is used in this section to examine the performance when both facial expressions and illumination are varied. The database contains 165 facial images of 15 subjects. The face images were taken under different facial expression such as centre-light, with glasses, happy, left-light, without glasses, normal, right-light, sad, sleepy, surprised, and wink. All the images were used in our analysis. The face images cropped to a size of 32 × 32 by He et al. [27] are used in our analysis. Figure 5 shows some of the faces from the Yale database.

In this analysis we compare the performance of the proposed 2DGFD method to other recent two dimensional based face recognition methods such as two dimensional PCA (2DPCA) and 2DFLD. The first five images of each person are chosen for training, while the remaining six images are used for testing. Thus, we obtain a training set of 75 images and a testing set of 90 images. The subspace dimensions of 20×20 were used for the 2DGFC method due to the superior performance illustrated on the ORL face database. Figure 6 shows the recognition accuracy under a varying number of selected features (or feature vectors) for the 2DPCA method, the 2DFLD method and the 2DGFD method. The experimental results lead to the following conclusions: The classification performance for the proposed 2DGFD method is far superior to that achieved by the 2DPCA method and the 2DFLD method. The 2DGFD method achieves 78.9% recognition accuracy and the 2DFLD method performed the second best with an accuracy of 73.3%, while 2DPCA achieves the lowest recognition accuracy among the three methods with an accuracy of 72.2%. The superior performance of the 2DGFD method is due to integrating the Gabor wavelets and the direct 2DFLD method. The Gabor transformed face images which are localised, varying in scale and orientations are more robust to illumination changes. The direct 2DFLD method then eliminates redundant features and forms a discriminant representation in which these redundancies are reduced. It
can also be seen from Figure 6 that the recognition accuracy for 2DPCA and 2DFLD decreases on the Yale database as more features are added. The increase in dimensionality leads to the extraction of non discriminate features which are not suitable for face recognition purposes.

We then analysed the performance of 2DPCA, 2DFLD and the proposed 2DGFD method under conditions where the sample size is varied. Table VIII leads to the following observations: Firstly, the 2DGFD method consistently outperforms the 2DPCA method and 2DFLD method because the 2DGFD method takes into account the locality, scale and orientation properties of the face features. Secondly, using more samples in the training set increases the face recognition accuracy for the three methods. Thus the generalization capability to unseen images improves as more training samples are used, which is consistent with the findings on the ORL database. Lastly, the 2DFLD method performs lower than the 2DPCA method for smaller training set. The FLD based methods suffers from the limited number of training samples, however, the 2DGFD method seems to be less effected as it simultaneously diagonalises the image between scatter and image within scatter matrices. In addition, the 2DGFD method selects eigenvectors corresponding to the largest eigenvalues for both the image between scatter and image within scatter matrices which corresponds to the maximum energy of the images.

4.3. Extended Yale Face Database

In this section, we use the extended Yale Face Database B [25, 28] which contains 16 128 images from 38 subjects with 9 poses and 64 illuminations. In this analysis, we choose the frontal pose and use all the images under different illumination, thus we get 64 images for each person. The size of the cropped images is $32 \times 32$ shown in Figure 7. In our analysis, five images of each person are randomly chosen for training, while ten unseen images are used for testing. Thus, we obtain a training set of 190 and a testing set of 380 images. The
subspace dimensions of $20 \times 20$ for $S_w^B$ were used in the proposed 2DGFC method for consistency with the analysis on the ORL dataset and the Yale dataset.

We compare the recognition accuracies obtained using the 2DPCA method, the 2DFLD method and the proposed 2DGFD method as shown in Figure 8. The dimensions (horizontal) axis in Figure 8 refers to the number of image columns returned in each image after feature extraction. For example, the dimensions of 2, 3 and 4 from the graph is actually $40 \times 20 \times 2$, $40 \times 20 \times 3$ and $40 \times 20 \times 4$. This means that 40 images which vary in orientations and scales of sizes $20 \times 2$, $20 \times 3$ and $20 \times 4$ are produced for each face by the proposed 2DGFD method. Whereas the 2DPCA method and the 2DFLD method extracts feature matrices of sizes $32 \times 2$, $32 \times 3$ and $32 \times 4$ for the dimensions 2, 3, and 4 from Figure 8. The parameters that gave the best recognition accuracy on the Extended Yale B dataset are presented in Table IX. The results lead to the following conclusions. Firstly, the best recognition of the 2DGFD method is $97.6\%$, higher than the best accuracy of 2DPCA at $91.8\%$ and that of 2DFLD with an accuracy of $91.6\%$. The 2DGFD method is better than 2DPCA, 2DFLD in terms of recognition accuracy in all tasks. This trend is consistent for different databases and conditions i.e. varying training sample. Therefore we can conclude that the 2DGFD method is more robust than the 2DPCA method and the 2DFLD method to pose, expression and illumination. From Table IX, the dimensions of the feature matrix required to obtain the best recognition accuracy by the 2DPCA method, the 2DFLD method and the proposed 2DGFD method is $992$ ($32 \times 31$), $1024$ ($32 \times 32$) and $3200$ ($40 \times 20 \times 4$). The number of dimensions for the 2DGFD method larger than for the 2DPCA method and the 2DFLD method because the 2DGFD method treats each of the 40 different orientation and scale separately in order to extract discriminant face information. Secondly, the increase in dimensionality of the feature matrices for the 2DPCA method and the 2DFLD method leads to the extraction of more discriminate features which are suitable for face recognition purposes. This is not consistent with the results on the Yale database, which shows that the increasing the dimensionalities of the feature matrix for the 2DPCA method and the 2DFLD method leads to a decrease in
recognition accuracy. This inconsistency is due to the differences in the image variations of the Yale and Extended Yale database. The Yale database contains more variations in images of the same person such as illumination, pose and complex facial variations (facial wear, facial expression), whereas the images from the Extended Yale database vary in pose and illumination.

5. CONCLUSIONS

In this paper, a new two dimensional discriminant analysis of the Gabor features for image feature extraction and representation, two dimensional Gabor Fisher Discriminant (2DGFD), was derived and implemented. The 2DGFD method derives discriminant Gabor feature matrices from a set of 2D Gabor wavelet representations of face images incorporating different orientation and scale local features. The 2DGFD method combines the 2D Gabor wavelets and the new direct 2DFLD method because for the Gabor transformed face images are spatial localised, vary in scale and orientation similar to those displayed by the Gabor wavelets. Such properties produces salient local features such as the features around the eyes, the nose and the mouth, most suitable for face recognition. The new direct 2DFLD removes redundant features and efficiently represents the images. Therefore the 2DGFD method is more robust to variations such as pose, expressions and illumination. Furthermore, it is shown that the 2DGFD method is computationally more efficient when compared to other Gabor based face recognition methods. The performance of the 2DGFD method on face recognition was tested using three face benchmark databases, ORL, Yale and Extended Yale B. The 2DGFD method achieved a 97.6% recognition accuracy compared to 91.8% and 91.6% for the 2DPCA and 2DFLD method on the Extended Yale database. In addition, ratios of the computational requirements for the 2DGFD method and GFC method on the ORL, Yale and Extended Yale B are as follows: 1:8, 1:135 and 1:1.2801×10^8.

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Figure 1 The diagram of the proposed 2DGFD method: $I^1(z)$ and $I^N(z)$ are the 1st and $N$th face image; sets of Gabor transformed images, $X_{\mu,\nu}^1(z)$ and $X_{\mu,\nu}^N(z)$; $Y_{\mu,\nu}^1(z)$ and $Y_{\mu,\nu}^N(z)$ are sets of 2DPCA transformed images; $Z_{\mu,\nu}^1(z)$ and $Z_{\mu,\nu}^N(z)$ are sets of 2DGFD extracted features. Note, a total of 40 Gabor images were produced for each image, $X_{1,1}^i(z), X_{1,2}^i(z), \cdots, X_{8,8}^i(z)$.
Figure 2 The diagram of the GFC method: \(I^1(z)\) and \(I^N(z)\) represents the 1st and \(N\)th images; \(X_{\mu,\nu}^1(z)\) and \(X_{\mu,\nu}^N(z)\) are sets of Gabor transformed images; \(h_{\mu,\nu}^p\) and \(h_{\mu,\nu}^p\), concatenated vectors of size \(40n \times n_r\), \(C\) covariance matrix of size \(N \times N\); \(y^1\) and \(y^N\) PCA transformed image vectors of size \(1 \times N - 1\); size of \(S_B^p\) and \(S_w^p\) is \((N - 1) \times (N - 1)\)
and size of $S_w^n$ is $g \times g$; $z^1$ and $z^N$ are the feature vectors extracted by the GFC method of size $1 \times g$.

Figure 3. Example ORL images with spatial resolution $112 \times 92$.

Figure 4 Comparison of Gabor PCA [13], GCF [14] and 2DGFD on the ORL database for varying sampling factors.

Figure 5 Example of YALE images with spatial resolution $32 \times 32$. 
Figure 6 Comparison of the face recognition performance for 2DPCA, 2DFLD and the proposed 2DGFD on the Yale database.

Figure 7 Five images of one person from the Extended Yale B.
Figure 8 Comparison of the face recognition performances using the 2DPCA method, 2DFLD method and the 2DGFD method on the extended Yale B database.
### TABLE I
**Comparisons of Computational and Memory Requirements for GFC and 2DGFD on the ORL Database**

<table>
<thead>
<tr>
<th>Training Samples</th>
<th>GFC</th>
<th>2DGFD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 200$</td>
<td>$N = 200$</td>
</tr>
<tr>
<td>Maximum Computation</td>
<td>$O(N^3 + V_{PCA}^3 + V_{W}^3)$</td>
<td>$O(n_r^3 + n_c^3 + V_{B}^3)$</td>
</tr>
<tr>
<td></td>
<td>$(200^3 + 199^3 + 199^3) = 23761198$</td>
<td>$(112^3 + 92^3 + 92^3) = 2962304$</td>
</tr>
<tr>
<td>Maximum Feature Matrix</td>
<td>$((N-1)\times N)= 199\times 200$</td>
<td>$(n_r \times n_c \times N)=112\times 92\times 200$</td>
</tr>
</tbody>
</table>

### TABLE II
**Comparisons of Computational and Memory Requirements for GFC and 2DGFD on the Yale Database**

<table>
<thead>
<tr>
<th>Training Samples</th>
<th>GFC</th>
<th>2DGFD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 165$</td>
<td>$N = 165$</td>
</tr>
<tr>
<td>Maximum Computation</td>
<td>$O(N^3 + V_{PCA}^3 + V_{W}^3)$</td>
<td>$O(n_r^3 + n_c^3 + V_{B}^3)$</td>
</tr>
<tr>
<td></td>
<td>$(165^3 + 164^3 + 164^3) = 13314031$</td>
<td>$(32^3 + 32^3 + 32^3) = 98304$</td>
</tr>
<tr>
<td>Maximum Feature Matrix</td>
<td>$((N-1)\times N)= 164\times 165$</td>
<td>$(n_r \times n_c \times N)=32\times 32\times 165$</td>
</tr>
</tbody>
</table>

### TABLE III
**Comparisons of Computational and Memory Requirements for GFC and 2DGFD on the Extended Yale B Database**

<table>
<thead>
<tr>
<th>Training Samples</th>
<th>GFC</th>
<th>2DGFD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 16128$</td>
<td>$N = 16128$</td>
</tr>
<tr>
<td>Maximum Computation</td>
<td>$O(N^3 + V_{PCA}^3 + V_{W}^3)$</td>
<td>$O(n_r^3 + n_c^3 + V_{B}^3)$</td>
</tr>
<tr>
<td></td>
<td>$(16128^3 + 16127^3 + 16127^3) = 1.2584 \times 10^{13}$</td>
<td>$(32^3 + 32^3 + 32^3) = 98304$</td>
</tr>
<tr>
<td>Maximum Feature Matrix</td>
<td>$((N-1)\times N)= 41367\times 41368$</td>
<td>$(n_r \times n_c \times N)=32\times 32\times 16128$</td>
</tr>
</tbody>
</table>

### TABLE IV
**Comparisons of the Ratios of the Computational Efficiency for the GFC Method and 2DGFD Method on the ORL, Yale and Extended Yale B Databases**

<table>
<thead>
<tr>
<th>Database</th>
<th>GFC</th>
<th>2DGFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORL</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>YALE</td>
<td>135</td>
<td>1</td>
</tr>
<tr>
<td>EXTENDED YALE B</td>
<td>$1.2801 \times 10^8$</td>
<td>1</td>
</tr>
</tbody>
</table>
TABLE V

**Comparisons of the Ratios of the Memory Requirements for the GFC Method and 2DGFD Method on the ORL, Yale and Extended Yale B Databases**

<table>
<thead>
<tr>
<th>DATABASE</th>
<th>GFC</th>
<th>2DGFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORL</td>
<td>1</td>
<td>52</td>
</tr>
<tr>
<td>YALE</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>EXTENDED YALE B</td>
<td>104</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE VI

**Top Recognition Accuracy (%) of 2DGFD on the ORL for Varying Subspace Dimensions.**

<table>
<thead>
<tr>
<th>DIMENSIONS</th>
<th>40×5</th>
<th>40×10</th>
<th>40×15</th>
<th>40×20</th>
<th>40×25</th>
<th>40×30</th>
<th>40×35</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RECOGNITION ACCURACY (%)</td>
<td>97.5</td>
<td>97.5</td>
<td>97.5</td>
<td>98.0</td>
<td>98.0</td>
<td>97.5</td>
<td>97.5</td>
</tr>
</tbody>
</table>

Note that the dimensions of the feature matrices has five scales (frequency) and eight orientations leading to forty (40) outputs of size \(d \times q\), where \(d\) and \(q\) are the set of projection vectors for \(C\) in equation 16 and \(S^P_B\) in equation 19.

TABLE VII

**Comparison of the Top Recognition Accuracy (%) of 2DFLD and 2DGFD on the ORL using the Euclidean Similarity Measure (24).**

<table>
<thead>
<tr>
<th>TRAINING SAMPLES/CLASS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2DFLD</td>
<td>88.1 (112×2)</td>
<td>91.1 (112×4)</td>
<td>94.2 (112×5)</td>
<td>94.5 (112×5)</td>
</tr>
<tr>
<td></td>
<td>2DGFD</td>
<td>91.6 (40×20×3)</td>
<td>93.9 (40×20×3)</td>
<td>97.5 (40×20×3)</td>
<td>98.0 (40×20×3)</td>
</tr>
</tbody>
</table>

The values in parentheses denote the dimension of feature matrices for the best recognition accuracy. Note that the feature matrices for the 2DGFD method has five scales (frequency) and eight orientations leading to forty (40) variations of scale and orientations of sizes 20×3.

TABLE VIII

**Comparison of the Top Recognition Accuracy (%) of 2DPCA, 2DFLD and 2DGFD on the Yale Database.**

<table>
<thead>
<tr>
<th>TRAINING SAMPLES/CLASS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2DPCA</td>
<td>57.0 (32×4)</td>
<td>61.7 (32×4)</td>
<td>68.6 (32×4)</td>
<td>72.2 (32×4)</td>
</tr>
<tr>
<td></td>
<td>2DFLD</td>
<td>46.7 (32×2)</td>
<td>54.2 (32×2)</td>
<td>66.8 (32×3)</td>
<td>73.3 (32×3)</td>
</tr>
<tr>
<td></td>
<td>2DGFD</td>
<td>61.5 (40×20×4)</td>
<td>70.8 (40×20×3)</td>
<td>76.2 (40×20×8)</td>
<td>78.9 (40×20×3)</td>
</tr>
</tbody>
</table>

The values in parentheses denote the dimension of feature matrices for the best recognition accuracy.
TABLE IX  
RECOGNITION ACCURACY (%) OF 2DPCA, 2DFLD AND 2DGFD ON THE EXTENDED YALE B DATABASE

<table>
<thead>
<tr>
<th>METHOD</th>
<th>DIMENSIONS AT BEST ACCURACY</th>
<th>BEST RECOGNITION ACCURACY (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2DPCA</td>
<td>32×31</td>
<td>91.8</td>
</tr>
<tr>
<td>2DFLD</td>
<td>32×32</td>
<td>91.6</td>
</tr>
<tr>
<td>2DGFD</td>
<td>40×20×4</td>
<td>97.6</td>
</tr>
</tbody>
</table>