Subspace-Based Blind Adaptive Multiuser Detection Using Kalman Filter

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Abstract: A new blind adaptive multiuser detection scheme based on a hybrid of Kalman filter and subspace estimation is proposed. It is shown that the detector can be expressed as an anchored signal in the signal subspace and the coefficients can be estimated by the Kalman filter using only the signature waveform and the timing of the desired user. The resulting subspace-based algorithm brings the benefit of lower computational complexity than the full-rank approach, and the theoretical analysis indicates that the proposed algorithm is also superior in convergence performance. The adaptive implementation in dynamic environment such as variable number of users is obtained by seamlessly integrating a subspace tracking algorithm. The new proposed subspace-based method is effective in AWGN channels as well as in slowly time-varying Rayleigh fading channels. Moreover, the proposed detector is much more robust against the signalling waveform mismatch and inaccurate knowledge of the amplitude of the desired signal than the full-rank one, as demonstrated by computer simulations.

Index Terms: Blind multiuser detection, subspace estimation, subspace tracking, reduced-rank, Kalman filter, CDMA, Doppler fading, robust detection, signature mismatch.
1. Introduction

Blind adaptive multiuser detection technologies are of special interest in the mitigation of multiple-access interference (MAI) in the code-division multiple-access (CDMA) systems as only the a priori knowledge of the signature waveform and the timing of the desired user are needed [1]. In recent years, a significant amount of research on blind multiuser detection has been reported in literature [1-13]. In [1], Honig et al derived a blind version of the minimum-mean-square-error (MMSE) detector by using constrained mean-output-energy (MOE) as the cost function. By minimising the sum of exponentially weighted output energy, Poor and Wang [2] proposed a recursive least-squares (RLS) blind adaptive detector which shows faster convergence speed than the least-mean-square (LMS) detector. In [3], Wang and Poor demonstrated that the linear decorrelating detector and MMSE detector could be blindly expressed in closed forms by the eigencomponents in the signal subspace and the adaptive implementation was realized by employing adaptive subspace estimation technology. By exploiting the property that the optimum MOE detector exists in the signal subspace, Roy [4] proposed a subspace-based MOE detector which exhibited better convergence performance compared with the blind MOE detector in [1].

Kalman filter is well known in the optimal statistical estimation and control theory [14]. In [15] Kalman filter was used for signal detection and channel estimation in an asynchronous wireless communication system and showed performance gain over MMSE and RLS detectors. However this approach is not a blind detection strategy in the sense that it needs to know the signature waveforms and the time delays of all users. Recently, Zhang and Wei [5] proposed a simple and effective state space model for the multiuser detection problem in a stationary or slowly fading channel and employed Kalman filter as the adaptive algorithm. Compared with the LMS approach in [1] and the RLS approach in [2], this detector demonstrates lower steady-state excess output energy in adaptation. Interestingly, though the state space model for the Kalman filter was devised under a time-invariant assumption, the resulting algorithm could work well in slowly time-varying environment. In this paper, motivated by the signal subspace concept in [3], we propose a modified version of this blind adaptive multiuser detector by modelling the detector as a vector in the signal subspace and employing a Kalman filter philosophy similar to that in [5] to derive the coefficients adaptively. As we shall see, compared with the full-rank approach in [5], despite some similarity in the state-space model, this new subspace-based multiuser detector has some
significantly important merits. Firstly it has lower computational complexity and faster convergence rate in terms of signal-to-interference-plus-noise-ratio (SINR). Secondly and importantly, in the case of signature waveform mismatch which could cause serious cancellation of the desired signal, it still works well while the full-rank detector suffers rapid performance degradation. Thirdly, it is less conditioned on some system parameters such as the desired users’ signal amplitude than the full-rank method, thus it is a blind detection method in a more strict sense. Additionally, the detection effectiveness is maintained both in additive Gaussian noise channels and in slowly time-varying Rayleigh fading channels. In a dynamical system where users can enter and leave at random, the structure of the signal subspace is also time-varying. In this case, a subspace tracking algorithm is seamlessly integrated into the proposed detector to track the changes and provide an online estimation of the signal subspace.

2. Signal Model

Consider a baseband synchronous direct-sequence (DS) CDMA system with \( K \) users and employing binary phase-shift keying (BPSK) modulation for transmission of information over a slowly time-varying multipath frequency non-selective fading channel. In the \( n \)-th symbol, the received signal is the superposition of all \( K \) users’ signals plus the background additive white Gaussian noise (AWGN)

\[
r(t) = \sum_{k=1}^{K} A_k b_k(n) \sum_{l=1}^{L} h_{k,l}s_k(t-nT-(l-1)T_c) + \sigma z(t)
\]

where \( T \) and \( T_c \) are the symbol duration and chip duration, \( T = NT_c \) (\( N \) is the processing gain), \( A_k \) and \( b_k(n) \in \{+1,-1\} \) are, respectively, the received amplitude and the \( n \)-th information bit of the \( k \)-th user, \( h_{k,l} \) stands for the channel gain for the \( l \)-th path of the \( k \)-th user, \( L \) is the number of resolvable paths for each user, \( \{s_k(t); t \in [0,T]\} \) denotes the signalling waveform of the \( k \)-th user and is assumed to be supported only on the interval \([0,T]\), and \( z(t) \) is white Gaussian noise with unit power spectral density and \( \sigma \) is the standard deviation. The user signalling waveforms take form

\[
s_k(t) = \sum_{i=0}^{N-1} \alpha_k^i \varphi(t-iT_c), \quad t \in [0,T]
\]

where \( \{\alpha_k^i\}_{i=0}^{N-1} \) is a \( \pm 1 \)-valued spreading sequence assigned to the \( k \)-th user, and \( \varphi(t) \) is a normalized chip waveform of duration \( T_c \), \( \int_0^{T_c} \|\varphi(t)\|^2 dt = 1 \).
The single-path signal model considered in literature can be deemed as a simplified case of (1) by defining $L=1$, $h_{k,1} = 1, \forall k$

$$r(t) = \sum_{k=1}^{K} A_k b_k(n) s_k(t - nT) + \sigma z(t)$$

The signal model in discrete-time format is obtained by processing (2) through a chip-matched filter followed by chip rate sampling, which yields an $N \times 1$ vector within the $n$th symbol duration

$$r(n) = \sum_{k=1}^{K} A_k b_k(n) s_k + \sigma z(n)$$

where $s_k = (1/N)[\alpha_k^0 \alpha_k^1 \cdots \alpha_k^{N-1}]^T$, $z(n)$ is a white Gaussian noise vector with mean $0$ and covariance matrix $I_N$ which denotes the $N \times N$ identity matrix. Without loss of generality, it is assumed that $\{b_k\}_{k=1}^{K}$ are independent equiprobable random variables, and the signature waveform vectors $\{s_k\}_{k=1}^{K}$ of the $K$ users are mutually linearly independent and each has unit energy, i.e., $\|s_k\|^2 = 1$, for $k=1,2\ldots K$.

We note that the proposed blind adaptive detector in this paper is developed and analysed under the AWGN channel as in (3). However, the performance of the proposed detector in receiving signals distorted by the multipath channel as in (1) is also studied by simulation to examine its robustness against signature waveform mismatch, which can cause signal cancellation and is a common concern in blind linear multiuser detector design [1,3]. We indicate that the multipath channel adopted in this paper is of frequency non-selective fading nature, therefore, the inter-symbol interference (ISI) is negligible and the channel equalization is not necessary, however, this multipath fading could cause interchip interference (ICI) and hence is one of many reasons that cause signature waveform mismatch [1,3]. In all simulations, no channel knowledge is assumed known or estimated. The implementation details will be shown in the simulation section.

3. Subspace-based blind multiuser detector using Kalman filter

3.1 Subspace and multiuser detection in signal subspace

Based on the signal model (3) and the associated independent assumptions, the autocorrelation matrix of the received signal $r(n)$ can be expressed as
\[ R = E[r(n)r(n)^T] = \sum_{k=1}^{K} A_k s_k^2 + \sigma^2 I_N = SAS^T + \sigma^2 I_N \]  \hfill (4)

where \( S = [s_1 \ s_2 \ldots \ s_K] \) denotes the signature matrix, and \( A = \text{diag}(A_1^2 \ A_2^2 \ldots \ A_K^2) \) denotes the diagonal matrix of the signal amplitudes. On the other hand, applying an eigendecomposition to the matrix \( R \) yields

\[ R = UAU^T = U_s A_s U_s^T + U_n A_n U_n^T \]  \hfill (5)

where \( U = [U_s \ U_n] \), \( A = \text{diag}(A_1 \ A_2 \ldots \ A_K) \) contains the \( K \) largest eigenvalues of \( R \) in descending order, and \( U_s \in \mathbb{R}^{N \times K} \) contains the corresponding orthonormal eigenvectors. \( A_s = \sigma^2 I_{N-K} \) contains another \( N-K \) eigenvalues of \( R \) and \( U_n \in \mathbb{R}^{N \times N-K} \) contains the corresponding \( N-K \) orthonormal eigenvectors. The column vectors of \( U_s \) and \( U_n \) span two orthogonal subspaces, namely, the signal subspace and the noise subspace with \( U_s^T U_n = 0 \). It can be seen that the range spaces spanned by \( S \) and \( U_s \) are identical, i.e. \( \text{range}(S) = \text{range}(U_s) \).

Assuming that user 1 is the desired user and has detecting vector \( c_1 \), then the signal-to-interference-plus-noise-ratio (SINR) in the soft output is

\[ \text{SINR} \triangleq \frac{E[|c_1^T A_1 b s_1|^2]}{E[|c_1^T (r - A_1 b s_1)|^2]} \]  \hfill (6)

the signal bit is determined by

\[ \hat{b}_1 = \text{sign}(c_1^T r) = \text{sign}(c_1^T A_1 b s_1 + c_1^T \sum_{i=2}^{K} A_i b_i s_i + c_1^T \sigma z) \]  \hfill (7)

In (7), it can be seen that both the desired signal \( A_1 b s_1 \) and the interference \( \sum_{i=2}^{K} A_i b_i s_i \) lie in the range space of \( S \), thus it is reasonable to seek the detector \( c_1 \) in the signal subspace to recover the desired signal and suppress the interference. In fact, if the detector is designed in the full-rank space, it can be formulated as \( c_1 = U_s w_s + U_n w_n \), where \( w_s \) and \( w_n \) are the weight vectors for the signal subspace and the noise subspace, respectively. However, only the subset \( U_s w_s \) which is in the signal subspace plays the effective role in suppressing the MAI while the remaining subset \( U_n w_n \) which is in the noise
subspace does not affect the signal components of the detector output but increases the computational complexity and slows down the convergence.

Motivated by the above observations, we propose a detector with anchored structure as follows

\[ \mathbf{c}_i = \mathbf{s}_i + \mathbf{s}_{\text{null}} \mathbf{w}_i \quad \text{subject to } \mathbf{c}_i^T \mathbf{s}_i = 1 \]  

(8)

where the column vectors of the matrix \( [\mathbf{s}_i \quad \mathbf{s}_{\text{null}}] \) compose the signal subspace basis set and \( \mathbf{w}_i \in \mathbb{R}^K \) is a weight vector. \( \mathbf{s}_{\text{null}} \in \mathbb{R}^{N \times K-1} \), its columns span the null space of \( \mathbf{s}_i \), i.e. \( \mathbf{s}_i^T \mathbf{s}_{\text{null}} = \mathbf{0} \), and they are orthonormal, i.e. \( \mathbf{s}_{\text{null}}^T \mathbf{s}_{\text{null}} = \mathbf{I}_{K-1} \). Since \( \mathbf{s}_i \) is assumed to be known and \( \mathbf{s}_{\text{null}} \) can be obtained, for example, by applying eigenvalue decomposition (EVD) to the autocorrelation matrix \( \mathbf{R} \), the problem then becomes how to find the optimal weight vector \( \mathbf{w}_i \) that minimises the MAI. We demonstrate that this vector can be estimated by using the Kalman filter method.

3.2 Estimation using Kalman filter

Kalman filter is a state-space approach for parameter estimation in dynamic systems. The general format of the state-space model for a linear system can be mathematically written as

\[ \mathbf{x}(n) = \Phi(n)\mathbf{x}(n-1) + \mathbf{v}(n-1) \]  

(9)

\[ \mathbf{y}(n) = \mathbf{H}(n)\mathbf{x}(n) + \mathbf{u}(n) \]  

(10)

where \( \mathbf{x}(n) \) is a \( N \times 1 \) state vector containing system variables that may not be directly measurable and \( \mathbf{y}(n) \) is a \( M \times 1 \) measurement vector which represents the noisy observations. \( \Phi(n) \), \( \mathbf{H}(n) \), \( \mathbf{v}(n-1) \) and \( \mathbf{u}(n) \) are the \( N \times N \) state transition matrix, \( M \times N \) measurement matrix, \( N \times 1 \) process noise vector and \( M \times 1 \) measurement noise vector, respectively, and all of them are allowed to be time-varying. \( \mathbf{v}(n-1) \) and \( \mathbf{u}(n) \) are mutually independent Gaussian processes with covariance of \( \mathbf{Q}_{n-1} \) and \( \mathbf{R}_n \), respectively.

Considering the detector (8) in the state-space framework, we can view \( \mathbf{w}_i \) as the state vector of the system and hence transform the signal detection problem into a state estimation problem.

A. Multiuser Detection in Time-Invariant or Slowly Time-Varying System
In a time-invariant or slowly time-varying system, there is sufficient time for the adaptive detector to converge to the optimum state and stay nearly constant, i.e. $c_{\text{opt}}(n) = c_{\text{opt}}(n-1)$. Therefore, on using (8), the state transition equation is written as:

$$w_{\text{opt}}(n) = w_{\text{opt}}(n-1) \quad (11)$$

$w_{\text{opt}}(n)$ is the weight vector corresponding to the optimum detector at time $n$. Correlating the received signal with the optimum detector produces the soft output

$$e_{\text{opt}}(n) \triangleq c_{\text{opt}}^T(n)r(n) = s^T_i r(n) + r^T(n)s_{\text{null}} w_{\text{opt}}(n) \quad (12)$$

Define

$$y(n) \triangleq s^T_i r(n) \quad \text{and} \quad H^T(n) \triangleq -r^T(n)s_{\text{null}}$$

rearranging (12) yields

$$y(n) = H^T(n)w_{\text{opt}}(n) + e_{\text{opt}}(n) \quad (13)$$

Equations (11) and (13) form the state-space model with $e_{\text{opt}}(n)$ to be the measurement noise. To apply the Kalman filtering recursion, some statistics of the random variant $e_{\text{opt}}(n)$ need to be determined, namely, $\mu$ and $\varphi_{\text{min}}$, which are given by

$$\mu = E[e_{\text{opt}}(n)] = E[c_{\text{opt}}^T(n)r(n)] = c_{\text{opt}}^T(n)E[r(n)] = 0 \quad (14)$$

and

$$\varphi_{\text{min}} = \text{cov}[e_{\text{opt}}(n)] = E[[e_{\text{opt}}(n) - \mu]^2] = E[e_{\text{opt}}^2(n)]
= \text{MOE}(c_{\text{opt}}(n))
= A_1^2 + \text{MSE}(c_{\text{opt}}(n))
= A_1^2 + \epsilon_{\text{min}} \quad (15)$$

where $\text{MOE}(c_i(n)) \triangleq E[|c_i^T(n)r(n)|^2]$ is the mean-output-energy, $\text{MSE}(c_i(n)) \triangleq E[|c_i^T(n)r(n) - A_j b_j|^2]$ is the mean-square-error, and $A_j$ is the signal amplitude of user 1. When $c_i(n)$ achieves the optimum quantity, $\text{MOE}(c_{\text{opt}}(n))$ and $\text{MSE}(c_{\text{opt}}(n))$ take the minimum values, namely, $\varphi_{\text{min}}$ and $\epsilon_{\text{min}}$, respectively. The minimum-mean-output-energy and the minimum-mean-square-error for the detector in (8) are, $\varphi_{\text{min}} = 1/(s_i^2 R s_i)$ and $\epsilon_{\text{min}} = 1/(s_i^2 R s_i) - A_j^2$, respectively.

Taking advantage of the model in (11) and (13) and utilising the recursion in (11)-(15), we summarise the proposed reduce-rank multiuser detection algorithm in Table 1.
**Table 1**: Subspace-based Kalman filtering estimation for blind multiuser detection in time-invariant or slowly time-varying system

**Signal subspace estimation:**

Compute autocorrelation matrix \( \mathbf{R} \) for a batch of \( J \) symbols

\[
\mathbf{R} = \frac{1}{J} \sum_{j=1}^{J} \mathbf{r}_j \mathbf{r}_j^T
\]  
(16)

Perform eigenvalue decomposition of \( \mathbf{R} \)

\[
\mathbf{R} = \mathbf{U} \mathbf{A} \mathbf{U}^T = \mathbf{U}_s \mathbf{A}_s \mathbf{U}_s^T + \mathbf{U}_u \mathbf{A}_u \mathbf{U}_u^T
\]  
(17)

Form matrix

\[
\mathbf{Z} = [\mathbf{s}_1 \quad \mathbf{u}_1 \quad \ldots \quad \mathbf{u}_{K-1}]
\]  
(18)

where \( \mathbf{u}_i \) is the \( i \)th column vector of \( \mathbf{U}_s \), \( \forall i = 1, \ldots, K-1 \)

Apply Gram-Schmidt method on \( \mathbf{Z} \) to obtain an orthonormal matrix

\[
\mathbf{Y} = [\mathbf{s}_1 \quad \mathbf{y}_1 \quad \ldots \quad \mathbf{y}_{K-1}]
\]  
(19)

Let

\[
\mathbf{s}_{\text{null}} = [\mathbf{y}_1 \quad \ldots \quad \mathbf{y}_{K-1}]
\]  
(20)

be the null signal subspace of \( \mathbf{s}_1 \).

**Kalman filtering estimation in symbol-rate adaptation:**

For \( n=1,2,\ldots \)

**Riccati equations:**

\[
\mathbf{k}_n = \mathbf{P}_{s,n} \mathbf{H}(n)[\mathbf{H}^T(n)\mathbf{P}_{s,n} \mathbf{H}(n) + \varphi_{\text{null}}]^{-1}
\]  
(21)

\[
\mathbf{P}_s = [\mathbf{I} - \mathbf{k}_n \mathbf{H}^T(n)]\mathbf{P}_{s,n-1}
\]  
(22)

**State estimate and update equations:**

\[
\hat{\mathbf{w}}_{\text{opt}}(n) = \hat{\mathbf{w}}_{\text{opt}}(n-1) + \mathbf{k}_n [\mathbf{y}(n) - \mathbf{H}^T(n)\hat{\mathbf{w}}_{\text{opt}}(n-1)]
\]  
(23)

**Signal detection:**

\[
\hat{\mathbf{c}}_{\text{opt}}(n) = \mathbf{s}_1 + \mathbf{s}_{\text{null}} \hat{\mathbf{w}}_{\text{opt}}(n)
\]  
(24)

\[
\hat{b}(n) = \text{sign}(\hat{\mathbf{c}}_{\text{opt}}^T(n)\mathbf{r}(n))
\]  
(25)

**End**

Compared with [5], the state vector and measurement matrix in the state-space model (Eq.(11) and (13)) have reduced ranks, and so do all the involved vectors and matrices in the equations (21)-(23) of the new
algorithm. The computational complexities of several existing algorithms are compared in term of the number of multiplications and additions per adaptation iteration as shown in Table 2.

Table 2: Comparison of Computational Complexity.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Computational Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-rank LMS algorithm [1]</td>
<td>$O(N)$</td>
</tr>
<tr>
<td>Subspace LMS algorithm [4]</td>
<td>$O(NK)$</td>
</tr>
<tr>
<td>RLS algorithm [2]</td>
<td>$O(N^2)$</td>
</tr>
<tr>
<td>Subspace adaptive decorrelating detector [3]</td>
<td>$O(NK^2)$</td>
</tr>
<tr>
<td>Proposed subspace Kalman filtering algorithm</td>
<td>$O(NK)$</td>
</tr>
</tbody>
</table>

It can be seen that the complexity of the new proposed subspace Kalman filtering detector is much lower than those in [2] and [5]. This becomes significantly important in practical systems where $N$ is usually large.

B. Multiuser Detection in a Dynamic Environment with Variable Number of Users

In a dynamic environment, when the users enter and leave randomly, the signal subspace rank and basis will change. To maintain the effectiveness of the proposed detector in the new subspace, we need a mechanism to determine or detect the changes when they happen and then resize the matrix $s_{null}$ and weight vector $w$, in (8) accordingly, then the Kalman filtering adaptation and the signal detection will be operated in the new signal subspace. In this paper, the projection approximation subspace tracking with deflation (PASTd) algorithm [3, 17, 18] is seamlessly integrated into the proposed subspace-based Kalman filter to track the varying subspace. Based on the estimated eigenvalues, the rank of the signal subspace, or the number of active users can be estimated by using information theoretic criteria such as Akaike information criteria (AIC). The advantages of PASTd include almost guaranteed global convergence to the signal eigenvectors and eigenvalues and the rank tracking capability. Table 3 briefly summarizes the proposed blind adaptive subspace-based Kalman filter integrated with PASTd algorithm in pseudo-code. The key feature of this proposed detection algorithm is that the number of users
becomes a time-varying variable, $K(n)$, and when the number of users changes, the structure of the subspace detector needs to be adjusted and optimized. The subspace adaptation in the dynamic environment adds a slight further computational complexity ($O(NK)$) due to the rank and subspace tracking. Because the number of users, denoted by $K$, is time-varying in a dynamic system, the computational complexity is also time-varying.

Table 3: Subspace-based Kalman filtering estimation for blind multiuser detection in dynamic environment with variable number of users

<table>
<thead>
<tr>
<th>FOR $n=1,2,…$ (symbol iteration)</th>
</tr>
</thead>
<tbody>
<tr>
<td>/<em>PASTd algorithm:</em>/</td>
</tr>
<tr>
<td>Updating the eigenvectors and eigenvalues of the signal subspace ${\mathbf{u}<em>l, \lambda_l}</em>{l=1}^{K(n-1)}$;</td>
</tr>
<tr>
<td>Using AIC criteria to update the rank of the signal subspace $K(n)$;</td>
</tr>
<tr>
<td>/* Subspace-based Kalman filter:*/</td>
</tr>
<tr>
<td>IF $K(n) \neq K(n-1)$</td>
</tr>
<tr>
<td>resize $\mathbf{s}_{\text{null}} = [\mathbf{u}<em>1 \ldots \mathbf{u}</em>{K(n)}]$ and make it orthogonal to $\mathbf{s}_i$;</td>
</tr>
<tr>
<td>resize the length of $\mathbf{w}_i$ to $K(n)$ and reinitialize $\mathbf{w}_i$;</td>
</tr>
<tr>
<td>implement Kalman filter algorithm and signal detection according to equations (21)-(24);</td>
</tr>
<tr>
<td>END</td>
</tr>
</tbody>
</table>

4. Convergence Analysis

Kalman filter can be optimal in various senses [15], for instance, minimum-mean-square-error (MMSE), maximum a posteriori (MAP), or some other appropriate measure. When the noise vectors $\mathbf{v}(n)$ and $\mathbf{u}(n)$ in (9) and (10) are assumed to be individually and mutually uncorrelated with correlation matrices as

$$E\{\mathbf{v}(i)\mathbf{v}^T(j)\} = \mathbf{Q}_i \delta_{ij}$$
$$E\{\mathbf{u}(i)\mathbf{u}^T(j)\} = \mathbf{R}_i \delta_{ij}$$
$$E\{\mathbf{v}(i)\mathbf{u}^T(j)\} = 0$$
where $\delta_j$ is the Kronecker delta function, the Kalman filter then gives the MMSE estimate of the state. In our state-space model in (11) and (13), since $v(n) = 0$ and $u(n) = e_{\text{opt}}(n)$, this exactly satisfies the above conditions and thus the Kalman filter proposed in this paper is optimal in the sense of MMSE. We now analyse the steady-state SIR performance when $n \to \infty$.

**Theorem 1:** For a stationary system, subspace-based Kalman filtering detector converges faster than the full-rank Kalman filtering detector and both approach the optimum signal-to-interference-plus-noise-ratio (SINR) value when $n \to \infty$.

**Proof:** See Appendix.

5. Simulation Results

We provide four simulation examples to evaluate the performance of the proposed subspace-based Kalman filtering algorithm in various scenarios and compare it with some other existing algorithms in literature. In all simulations, we assume a synchronous DS-CDMA system and use Gold codes as the signature codes (processing gain $N=31$). In measuring the MAI in the soft output of the detector, the time-averaged SINR is used rather than that in (6):

$$\text{SINR}(n) = 10\log \frac{\sum_{m=1}^{M}(c_{m}^{T}s_{1})^2}{\sum_{m=1}^{M}(c_{m}^{T}(r_{m}(n) - h_{m}(n)s_{1}))^2} \text{ (dB)}$$

(26)

where $M$ is the number of independent runs. The subscript $m$ indicates that the associated variable depends on the particular run. In this paper, $M$ is set to be 500.

In all experiments, except explicitly stated, the ambient noise variance $\sigma^2$ takes value 0.01 and $\varphi_{\text{min}}$ in Kalman filtering algorithms takes $\varphi_{\text{min}} = 1$. The signal energies are measured in decibels relative to the ambient noise variance $\sigma^2$. User 1 is assumed to be the desired user with unit energy $A_{1}^{2} = 1$, equivalent to $\text{SNR}=10\log(A_{1}^{2} / \sigma^2) = 20\text{dB}$. In example 1 to example 3, there are five 30dB, three 40dB and one 50dB simultaneous interfering users (thus $K=10$).

**Example 1:** Performance of the proposed detector in a non-fading channel. We assume a non-fading channel modelled by (3) and study the convergence performance of the new proposed subspace-based Kalman filtering detector in Table 1, the LMS-MOE detector [1], the RLS detector [2], the subspace-
based adaptive decorrelating detector [3] and the full-rank Kalman filtering detector [5]. The step size is a crucial parameter affecting the convergence behaviour of stochastic algorithms. In this paper, the step size for the subspace-based LMS-MOE algorithm is chosen to be $\mu = 0.0003$ to satisfy the following stability condition of convergence [5]

$$\mu < \frac{2}{\sum_{i=1}^{K} A_i^2 + N\sigma^2}$$

(27)

In applying the RLS algorithm, the forgetting factor is set to be $\beta = 0.997$, and the initial value of the inverse of the correlation matrix is given by $R^{-1}(0) = \delta^{-1}I$, $\delta = 0.01$. In implementing the PASTd algorithm, the initial estimates of the eigencomponents are obtained by applying an eigenvalue decomposition (EVD) to the autocorrelation matrix of the first 50 data vectors, and the forgetting factor is $\beta = 0.997$. $s_{i\text{null}}$ is obtained by applying an EVD to the first 50 data vectors in each run and orthonormalised with respect to $s_i$ by using the Gram-Schmidt method.

The time-averaged SINR versus iteration numbers for the five algorithms is plotted in figure 1(a). It is seen that the performance of both the subspace-based and the full-rank Kalman filtering algorithms is almost identical and both outperform the rest. When $n$ is sufficiently large, both Kalman filtering algorithms approach SNR=20dB. This means that the MAI in the SINR has been eliminated almost completely. However, we note that the subspace-based algorithm achieves this near-optimum performance at a significantly reduced computational complexity compared with the full-rank algorithm. Figure 1(b) compares the same algorithms but at lower SNR in the input signal. The ambient noise variance $\sigma^2$ is set to be 0.04 and all user signal amplitudes are kept unchanged. Thus the first user is 8dB, and there are five 18-dB, three 28-dB and one 38-dB interfering users. The result shows that at worse SNR, all algorithms converge to lower SINR but the convergence rate is not much affected, and the proposed detector still outperforms the rest.

**Example 2:** Performance of the proposed detector in the slowly time-varying Rayleigh fading channel.

We assume a single-path Rayleigh-fading channel with a Doppler frequency of 22 Hz, which is obtained based on Clarke's model [17]. The signal model is expressed by
where $h_k$ is the channel coefficient for the $k$-th user and is a random variable following Rayleigh distribution. This signal model can be obtained by simplifying (1) by letting $L=1$. The convergence curves for the proposed detector, the full-rank Kalman filter and the subspace-based blind adaptive decorrelating detector are plotted in figure 2. The results show that all algorithms can track the slow channel fluctuation, and the subspace-based Kalman filtering detector still yields better performance than the other two detectors.

Example 3: Performance of the proposed detector in the case of signature waveform mismatch. In both example 1 and 2, we assume that the original signature waveform $s_k$ which is known to the desired user is exactly the effective signature waveform. However, in practice, this may not always be true because the original signature waveform may have been distorted when the signals traverse through a multipath channel while the receiver still assumes the original signature waveform as its nominal. In this case, there will be a signature waveform mismatch between the assumed signature waveform and the true effective one, which can cause serious cancellation of the desired signal [1].

Consider the case that the signature waveform mismatch is caused by a multipath frequency non-selective fading channel. Suppose that $K$ users are synchronously transmitting through this multipath channel. The continuous-time signal model is as presented in (1). The discrete-time format of the chip-matched filter output within one symbol duration is

$$ r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma z(t), \quad t \in [0,T] \quad (28) $$

where $h_k$ is the channel coefficient for the $k$-th user and is a random variable following Rayleigh distribution. This signal model can be obtained by simplifying (1) by letting $L=1$. The convergence curves for the proposed detector, the full-rank Kalman filter and the subspace-based blind adaptive decorrelating detector are plotted in figure 2. The results show that all algorithms can track the slow channel fluctuation, and the subspace-based Kalman filtering detector still yields better performance than the other two detectors.

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$$ r(t) = \sum_{k=1}^{K} A_k b_k s_k + \sigma n \quad (29) $$

where $\hat{s}_k = [h_k * s_k]_{\text{pr}}$, $h_k = [h_{k,1}, ..., h_{k,L}]^T$, (*) denotes convolution operation and $[\cdot]_{\text{pr}}$ denotes a pruning operation to take only the first $N$ taps of the vector and neglect the rest. This pruning operation is equivalent to neglecting the signal energy leaking into the neighbouring symbols, which is reasonable because when symbol interval is much longer than the multipath delay spread, the ISI is negligible [18].

We can observe that due to the interchip interference (ICI), in the received signal, the actual spreading waveform is the distorted waveform $\hat{s}_k$ rather than the original $s_k$, however, the receiver still uses the original spreading waveform for detection as indicated in (8).

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In this example, the number of resolvable path is \( L=3 \). Figure 3 shows the time-averaged SINR versus the iteration number in the presence of spreading waveform mismatch at different deterioration levels. In figure 3(a), \( h = [0.790 \ 0.105 \ 0.105]^T, k = 1...10 \) and \( <s_i \hat{s}_i> = 0.9884 \), and in figure 3(b), \( h = [0.400 \ 0.350 \ 0.250]^T, k = 1...10 \) and \( <s_i \hat{s}_i> = 0.6562 \). Those channel parameters are not known to the receiver. In both cases, the subspace-based Kalman filtering detector steadily converges to a stable state after some number of iterations, whereas the full-rank approach does not converge but degrades quickly. Furthermore, by comparing figure 3(a) and 3(b), we find that under worst waveform mismatch, the subspace-based detector suffers moderate performance loss in that the steady-state SINR is lower. This is the penalty paid for not knowing the channel, fortunately it is very mild compared with the performance loss in the full-rank detector.

Furthermore, figure 4 illustrates the performance of the proposed detector and the full-rank detector under various \( \varphi_{\text{min}} \) values and signature waveform mismatch \( <s_i \hat{s}_i> = 0.9884 \). It is shown that when there is signature waveform mismatch, \( \varphi_{\text{min}} \) has significant impact on the performance of the full-rank detector but little on the proposed one. \( \varphi_{\text{min}} \) is related to the minimum mean square error \( \varepsilon_{\text{min}} \), i.e. \( \varphi_{\text{min}} = A_i^2 + \varepsilon_{\text{min}} \) (equation (15)). In practical situations, \( \varepsilon_{\text{min}} \) is usually required to be very small compared with user’s signal energy to maintain a certain level of SNR, thus \( \varphi_{\text{min}} \approx A_i^2 \). Therefore, the requirement to know \( \varphi_{\text{min}} \) is actually the requirement to know \( A_i \). However, \( A_i \) is not known in blind multiuser detection. The results in figure 4 demonstrate that the proposed method is more robust against inaccurate knowledge of the signal amplitude of the desired user than the full-rank one, therefore, the proposed detector is a blind algorithm in a stricter sense.

*Example 4:* Performance of the proposed detector in dynamic environment with variable number of users. At the beginning (\( n=0 \)), the system configurations are one 20dB user (the desired user), five 30dB users, three 40dB users and one 50dB user. At \( n=600 \), three 40dB interfering users enter the system. At \( n=1200 \), four 40dB and one 50dB interfering users leave the system. Both the subspace-based Kalman filter and the blind linear decorrelating detector use PASTd algorithm to track the rank and signal subspace with the forgetting factor \( \beta = 0.997 \). Figure 5 and 6 illustrate the tracking performance of the
detectors. The only difference in the algorithms for the two figures is that, in figure 5, whenever new
users enter or existing users leave the system, the proposed detector and the blind linear decorrelating
detector adjust the ranks of the signal subspace-related matrices and vectors and reinitialize them,
whereas in figure 6, only when new users enter the system, these ranks are increased and the matrices
and vector are reinitialized, and when existing users leave the system, the ranks are not decreased and
the matrices and vectors are not reinitialized. The methodological difference in dealing with users’
leaving leads to the performance difference for the proposed detector and the blind LDD in the two
figures. It is also seen that the full-rank Kalman filter tracks faster than the subspace approaches due to
the reason that in the subspace tracking strategy, the subspace-based algorithm includes two adaptation
phases, that is, adaptive subspace tracking and then adaptive signal detection, whereas the full-rank
detector includes only one adaptation phase.

6. Conclusion

In this paper, a new blind adaptive multiuser detector based on the hybrid of subspace-based method and
Kalman filtering estimation has been developed. It is near-far resistant and has lower computational
complexity and better convergence performance compared with existing strategies in literature. It is
effective in both AWGN channel and slowly time-varying Rayleigh fading channel. Furthermore, the
proposed subspace approach is significantly more robust against the signature waveform mismatch
caused by the multipath fading channel than the full-rank method by nulling out the noise subspace
component of the mismatched signature waveform. It is also a blind detection method in a stricter sense
because it is less conditioned on the knowledge of the signal amplitude of the desired user. Adaptation in
the dynamic environment with variable number of users is enabled by seamlessly integrating a subspace
tracking methodology at the cost of slight increment in computational complexity. Although the
proposed detector is more robust against signature waveform mismatch than the full-rank detector, it
still suffers some moderate SINR performance loss. Future research will consider the use of channel
estimation technique to mitigate this performance loss.

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8. References


9. Appendix

Proof of theorem 1

Using the constraint $c_i^T s_i = 1$, SINR in (6) can be written as

$$SINR(n) = \frac{E[|A h(n)c_i^T s_i|^2]}{E[|c_i^T r(n) - A h(n)c_i^T s_i|^2]}$$

$$= \frac{A_i^2}{E[|c_i^T r(n) - A h|^2]}$$

$$= \frac{A_i^2}{MSE(n)}$$

(30)

It is seen that the detector approaches the maximum (optimum) SINR when reaching the minimum MSE. Since $MOE(n) = MSE(n) + A_i^2$, the minimisation of $MSE(n)$ and minimisation of $MOE(n)$ result in identical solution. Thus we can investigate $SINR(n)$ with respect to $MOE(n)$ as follows:

$$SINR(n) = \frac{A_i^2}{MOE(n) - A_i^2} = \frac{A_i^2}{\phi(n) - A_i^2}$$

(31)

where $\phi(n) = MOE(n)$.

It is known in [5] that for sufficiently large $n$, the $\phi(n)$ for the full-rank Kalman filtering algorithm can be approximated by

$$\phi_{\text{fullKF}}(n) \approx tr\left(\frac{\phi_{\text{min}}}{\phi_{\text{min}}} (I + n R_{mb})^{-1} R_{mb} \right) + \phi_{\text{min}}$$

(32)

where $R_{mb} = E[HH^T]$ is the autocorrelation matrix of $(N-1) \times (N-1)$ dimension with rank $(N-1)$ and $tr(\cdot)$ denotes trace operation. However, for the subspace-based Kalman filtering algorithm, we have
\[ \varphi_{\text{fullKF}}(n) \equiv \text{tr}\{\varphi_{\text{mm}}(I+nR_{sl})^{-1}R_{sl}\} + \varphi_{\text{mm}} \]  

(33)

where \( R_{sl} = E[HH^T] \) is the autocorrelation matrix of \((K-1) \times (K-1)\) dimension with rank \((K-1)\) which is smaller than \((N-1)\). In both (32) and (33), \( \varphi_{\text{mm}} = 1/(s_i^T R^{-1} s_i) \) is the minimum MOE achieved when the detector is optimal where \( R = E[r^T r] \). When \( n \) is sufficiently large, \( \varphi_{\text{mm}}I + nR_{ji} \equiv nR_{ji} \) and \( \varphi_{\text{mm}}I + nR_{sl} \equiv nR_{sl} \), and substituting both into (32) and (33), respectively, yields

\[ \varphi_{\text{fullKF}}(n) \equiv \text{tr}\{\varphi_{\text{mm}}(nR_{ji})^{-1}R_{ji}\} + \varphi_{\text{mm}} \]

\[ = \text{tr}\{\varphi_{\text{mm}}n^{-1}R_{ji}^{-1}R_{ji}\} + \varphi_{\text{mm}} \]

\[ = \varphi_{\text{mm}}(N-1)/n + \varphi_{\text{mm}} \]  

(34)

and

\[ \varphi_{\text{subKF}}(n) \equiv \text{tr}\{\varphi_{\text{mm}}(nR_{sl})^{-1}R_{sl}\} + \varphi_{\text{mm}} \]

\[ = \text{tr}\{\varphi_{\text{mm}}n^{-1}R_{sl}^{-1}R_{sl}\} + \varphi_{\text{mm}} \]

\[ = \varphi_{\text{mm}}(K-1)/n + \varphi_{\text{mm}} \]  

(35)

Therefore,

\[ \omega(n) \equiv \varphi_{\text{fullKF}}(n) - \varphi_{\text{subKF}}(n) \equiv \varphi_{\text{mm}}(N-K)/n > 0 \]  

(36)

thus

\[ \varphi_{\text{fullKF}}(n) > \varphi_{\text{subKF}}(n) \]  

(37)

\[ \varphi_{\text{fullKF}}(n) - A^2_i > \varphi_{\text{subKF}}(n) - A^2_i \]  

(38)

\[ \text{SINR}_{\text{fullKF}}(n) < \text{SINR}_{\text{subKF}}(n) \]  

(39)

where \( \text{SINR}_{\text{fullKF}}(n) \) and \( \text{SINR}_{\text{subKF}}(n) \) denote SINR of the full-rank Kalman filtering algorithm and the subspace-based Kalman filtering algorithm at time \( n \), respectively. We have utilised (31) in deriving (39) where it is shown that the subspace-based approach outperforms the full-rank approach in terms of convergence rate.

Applying the infinite limit to (34) and (35) leads to

\[ \varphi(\infty) = \lim_{n \to \infty} \varphi_{\text{fullKF}}(n) = \lim_{n \to \infty} \varphi_{\text{subKF}}(n) = \varphi_{\text{mm}} \]  

(40)

that is, the steady-state MOE achieves the optimum value. By using (31), we conclude that the steady-state SINR achieves the optimal value given by

\[ \text{SINR}(\infty) = \frac{A^2_i}{\varphi_{\text{mm}} - A^2_i} \]  

(41)
Figure 1: Time-averaged SINR (averaged out of 500 independent runs) versus number of iterations for the subspace-based Kalman filtering detector, the full-rank Kalman filtering detector, the LMS-MOE detector, the RLS blind detector and the subspace-based blind decorrelating detector. (a) in the input signal of the desired user, SNR = 20dB and, (b) SNR = 8dB.
Figure 2: Convergence in slowly time-varying Rayleigh fading channel.
Figure 3: Comparison of time-averaged SINR versus number of iterations between the subspace-based Kalman filter and the full-rank Kalman filter when there is signature waveform mismatch. (a) \( <\mathbf{\hat{s}}_i, \mathbf{s}_i> = 0.9884 \); (b) \( <\mathbf{\hat{s}}_i, \mathbf{s}_i> = 0.6562 \).
Figure 4: Convergence performance of the Kalman filtering detectors with inaccurate knowledge of \( \Phi_{\text{min}} \) (Phi_{\text{min}}) and signature waveform mismatch \( <\hat{\mathbf{s}}_1, \mathbf{s}_1> \approx 0.9884 \). (a) Full-rank Kalman filtering detector, (b) subspace-based Kalman filtering detector.
Figure 5: Tracking in the dynamic system with varying number of users. Whenever the number of users in the system changes, increasing or decreasing, the signal matrices in subspace-based Kalman filter and subspace-based LDD algorithms change ranks and are reinitialized.
Figure 6: tracking in dynamic environment. When new users enter the system, the signal subspace matrices in subspace-based Kalman filter and subspace-based LDD algorithms change ranks and are reinitialized, but when the users leave the system, the signal subspace matrices in subspace-based Kalman filter and subspace-based LDD algorithms are not reinitialized.