Comparison of Input Shaping Techniques for Speed-Critical Multi-Mode Flexible Systems

William E. Singhose
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA, USA 02139
WESING@MIT.EDU

Lucy Y. Pao
ECE Department
University of Colorado
Boulder, CO, USA 80309-0425
PAO@COLORADO.EDU

Abstract
To achieve the highest level performance with computer controlled machines, the flexibility of the system must be taken into account. Feedforward methods are often the most desirable means of eliminating vibration. The fastest motions can be obtained by using the time-optimal flexible-body command. These commands depend on the system parameters, actuator limits, and the desired motion. Input shaping is another feedforward method which does not depend on actuator limits and motion parameters, only the system natural frequencies and damping ratios. Input shaping and time-optimal control are compared for use on multi-mode flexible systems. The two methods are compared in terms of move speed, robustness to modeling errors, transient deflection, and ease of implementation.

1. Introduction
Control of flexible structures requires the design of a stable feedback control system. In addition to stability, certain performance measures are also integrated into the design. For very rapid motions of a multi-mode flexible system, a satisfactory controller may prove difficult to design. Augmenting the feedback control system with a feedforward command shaping algorithm can facilitate the controller design or enable better performance. To attain the fastest possible motions, the time-optimal flexible-body command must be utilized. This command profile, which is the reference signal sent to the closed loop controller, depends on the system parameters, actuator limits, and motion parameters. Obtaining the time-optimal command for a multi-mode system generally requires the use of a nonlinear optimization program. The solutions obtained by such an optimization must be checked using the necessary and sufficient conditions provided by Pontryagin’s Maximum Principle [12]. For systems with rigid and flexible modes, the time-optimal command will be characterized by switches between full positive and full negative actuator effort.

Input shaping is a feedforward technique for reducing residual vibrations in computer controlled machines which does not depend on actuator limits or motion parameters [16]. Input shaping is implemented by convolving a sequence of impulses, an input shaper, with a desired system command to produce a shaped input that is then used to drive the system. The process of shaping a bang-bang input is demonstrated in Figure 1a. Instead of using the bang-bang input as the reference signal, the waveform resulting from the convolution is used as the command signal.

Input shapers can contain any number of impulses. The input shaper is designed by solving a set of equations that limit the dynamic response of the system. There are many types of constraint equations that will yield acceptable input shapers and several types of shapers have been determined in closed-form [16, 20, 22]. Their use only requires evaluating rather simple equations using estimates of the natural frequencies and damping ratios to obtain the amplitudes and time locations of the impulses which compose the input shaper.

Input shaping has been implemented on many different types of systems. It was used to improve the throughput of a wafer handling robot [13] and the performance of coordinate measuring machines [7, 14, 21]. Input shaping was a major component of an experiment in flexible system control which flew on the Space Shuttle Endeavor [22, 26, 27]. Input shaping has been proposed as a means of reducing residual vibrations in long reach manipulators [6, 9]. Shaping was combined with a post-maneuver damping controller to improve large angle slewing [3].

This paper compares input shaping to the time-optimal flexible-body control for rest-to-rest motion of flexible multi-mode systems. A brief review of input shaping will first be presented. Time-optimal flexible-body control will then be discussed and shown to be a special case of input shaping. The move speed, transient vibration amplitude, robustness to modeling errors, and ease of implementation of the two control methods will then be compared. This comparison provides useful reference information for practicing engineers who are developing or modifying a feedforward control system for a flexible machine.
2. Benchmark System

In order to compare the performance of input shaping and time-optimal control, the benchmark system shown in Figure 2 will be utilized. The model represents a system with two flexible modes and a rigid body mode. A force input, \( u(t) \), acts on mass \( m_1 \) with a maximum value of \( u_{\text{max}} \). The force-to-mass ratio (FM) is fixed by setting the total mass to one and then setting \( u_{\text{max}} \) equal to the desired value of FM. To simplify the comparison, only undamped systems will be considered here, so \( b_1 = b_2 = 0 \). The values of the masses and spring constants, \( k_1 \) and \( k_2 \), are chosen such that the low mode equals 1 Hz and the second mode equals \( r \) Hz, where \( r \) is the mode ratio.

3. Input Shaping for Multi-Mode Systems

The residual vibration of a linear system with natural frequency \( \omega \) and damping ratio \( \zeta \) can be expressed as the nondimensional ratio of residual vibration amplitude with shaping to that without shaping. This percentage vibration is given by [16]:

\[
V(\omega, \zeta) = e^{-\zeta \omega n} \sqrt{c^2 + s^2}
\]

where,

\[
C = \sum_{i=1}^{n} A_i e^{\zeta \omega n} \cos(\omega \sqrt{1 - \zeta^2 t_i})
\]

\[
S = \sum_{i=1}^{n} A_i e^{\zeta \omega n} \sin(\omega \sqrt{1 - \zeta^2 t_i}).
\]

\( A_i \) and \( t_i \) are the amplitudes and time locations of the shaper impulses, \( n \) is the number of impulses, and \( t_n \) is the time of the last impulse. Zero Vibration (ZV) shapers satisfy (1) with \( V \) set equal to zero at the modeling parameters, \( \omega_m \) and \( \zeta_m \) [16, 24]. For multi-mode systems, (1) is set equal to zero at the modeling parameters corresponding to each mode.

In addition to limiting residual vibration amplitude, most shaping formulations require some amount of robustness to modeling errors. The earliest form of robust shaping was obtained by differentiating (1) with respect to the frequency and then setting the resulting equation equal to zero [16]:

\[
0 = \frac{d}{d\omega} \left( e^{-\zeta \omega n} \sqrt{c^2 + s^2} \right)
\]

Zero Vibration and Derivative (ZVD) shapers satisfy the ZV constraints and the zero derivative constraint given by (2). Although more robust shaping algorithms have since been proposed [15, 20, 22, 23], this paper will investigate only the ZVD shaping algorithm.

A robust shaper can reduce residual vibration over some finite range of modeling errors. A shaper’s robustness is displayed graphically by a sensitivity curve: a plot of residual vibration amplitude versus frequency, i.e., (1) plotted as a function of \( \omega \). A sensitivity curve reveals how much residual vibration will exist when there is an error in the estimation of the system frequency. The difference in robustness between ZV and ZVD shapers is best described by the sensitivity curves shown in Figure 3. The horizontal axis is the nondimensional frequency, \( \omega_{\text{actual}}/\omega_{\text{model}} \). The vertical axis is the residual vibration amplitude when shaping is used divided by the amplitude of residual vibration when a step input is used.

The ZV shaper is very sensitive to modeling errors; small errors in the modeling frequency lead to significant residual vibration. The ZVD shaper has considerably more robustness to modeling errors, which is evident by noting that the width of the ZVD curve is much larger than the width of the ZV curve. To compare robustness quantitatively, the width of the sensitivity curve can be measured at some acceptable level of residual vibration. This width, measured as a nondimensional frequency range, is called the insensitivity. The 5% insensitivities (the width at 5%) have been labeled in Figure 3.
The time-optimal flexible-body control for many types of systems is characterized by full actuator effort and rapid switching of the command between full positive and full negative \([I, 8, 10, 17, 18, 25]\). This type of command profile can be interpreted as a step input convolved with a special type of input shaper \([19]\). The shaper has the form:

\[
A(t) = \begin{bmatrix}
1 & -2 & 2 & -2 & \ldots & -2 & 1 \\
0 & t_2 & t_3 & t_4 & \ldots & t_{n-1} & t_n
\end{bmatrix}
\]

(3)

Figure 1b shows the process of using input shaping to generate the time-optimal flexible-body command. Note that the time-optimal flexible-body control will always produce a faster response than standard input shaping. Simultaneous (direct) shapers lead to faster rise times. This paper will present results for both convolved and direct ZVD shapers.

4. Time-Optimal Flexible-Body Control

The time-optimal flexible-body control for many types of systems is characterized by full actuator effort and rapid switching of the command between full positive and full negative \([1, 8, 10, 17, 18, 25]\). This type of command profile can be interpreted as a step input convolved with a special type of input shaper \([19]\). The shaper has the form:

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The problem of constructing the time-optimal flexible-body command reduces to finding the impulse time locations, \(t_2-tn\) (\(t_1\) is always set equal to zero). These values can be determined using the residual vibration and robustness constraints given in (1) and (2) and constraints on the rigid-body motion. For example, the rigid-body motion of the system shown in Figure 2 is governed by:

\[
\ddot{x} = \frac{u(t)}{M}
\]

(4)

where \(x\) is the displacement of the mass center and \(M=m_1+m_2+m_3\) is the total system mass. Integrating (4) with respect to time results in the mass center velocity:

\[
v_d = \int_0^{t_n} \frac{u(t)}{M} \, dt
\]

(5)

where \(v_d\) is the velocity at the end of the command profile. By integrating once again, an expression for the position as a function of the impulse times can be obtained:

\[
x_d = \int_0^{t_n} \int_0^{t_n} \frac{u(t)}{M} \, dt^2
\]

(6)

where \(x_d\) is the desired move distance. A constraint equation is obtained from (6) by entering the desired move distance on the left side. While many solutions satisfy the constraint equations, (due to the transcendental nature of the vibration constraints) the shortest one is the desired time-optimal shaper. The time-optimal flexible-body command subject to the constraint of zero residual vibration will be referred to as the TO ZV command.

To obtain robustness to modeling errors, the ZVD constraints can be used with the rigid-body constraints to design a robust time-optimal command. The time-optimal ZVD shaped command for the system shown in Figure 2 is equivalent to the time-optimal control (subject only to actuator constraints) of the system with double flexible modes at the same frequencies and dampings as the modes in the original system \([11]\).

The timing of the command switches for the ZV and ZVD time-optimal commands have been shown to have several interesting properties \([10, 17, 18]\). For multi-mode systems, the time-optimal command can change in a complicated manner when one of the design parameters is varied. When the solutions are plotted as a function of move distance, the solution can have various degrees of complexity depending on the mode ratio. For integer values of \(r\), the solution space is the least complicated. To illustrate the most general features, a non integer value of \(r\) must be used. Figure 4 shows the impulse time locations (switch times) of the TO ZVD command when \(x_d\) is
varied, while r = 4.4 and FM = 1. The solution is well behaved over large ranges of the desired move distance. However, for certain ranges, both the number of impulses and their time locations change rapidly.

5. Comparison of Control Methods

To compare input shaping and time-optimal flexible-body control, applications of rest-to-rest slewing will be examined. To perform rest-to-rest motion with input shaping the unshaped command must accomplish rest-to-rest motion of the rigid body. The time-optimal control for rest-to-rest motion of a rigid body is bang-bang with the switch occurring at mid-maneuver. The performance obtained using a shaped bang-bang command will be compared to the response using the time-optimal flexible-body commands. The commands generated by shaping the bang-bang command will be labeled as "Shaper Name" BB. For example, using a convolved ZVD shaper on a bang-bang command yields a convolved ZVD BB command. Note that the shaped command is not a bang-bang signal. (See again Figure 1a.)

The three shaped command profiles that will be compared are the convolved ZVD BB, the direct ZVD BB, and the TO ZVD. The non-robust ZV shaping methods are not considered here. The three types of commands are compared in terms of maneuver speed, robustness to modeling errors, transient deflection, and ease of implementation. The system of Figure 2 with r = 4.4 will be used.

5.1. Maneuver Speed

Figure 5 shows the move duration for the three command profiles as a function of move distance. The time-optimal ZVD command is shown with a solid line, the direct ZVD BB is shown with a dotted line and convolved ZVD BB command is shown with a dash-dot line. The TO ZVD command is, of course, the fastest command. The TO ZVD ranges from 0.6 to 1.0 seconds faster than the direct ZVD BB. Note that the increase in move duration follows a repeating cycle; it does not continually increase. Therefore, the percentage increase in move duration decreases as the move distance is increased. The direct ZVD BB is faster than the convolved ZVD BB command by approximately 0.13 sec.

5.2. Robustness to Modeling Errors

To quantitatively compare the robustness to modeling errors, the 5% insensitivity for each of the commands was calculated as a function of move distance. (Please see again Figure 3 for the meaning of 5% insensitivity.) For multi-mode systems there is a sensitivity associated with each mode. Figure 6 shows the sensitivity curve for the TO ZVD command when r = 4.4. In this case, the frequency axis is not normalized because there are two modeling frequencies. Note that the vibration goes to zero and the curve has zero slope at both 1 Hz and 4.4 Hz. The insensitivity for the second mode is obtained just as for the first mode – the width of the curve at 4.4 Hz is measured and the resulting frequency range is divided by 4.4 Hz to get the nondimensional insensitivity.
Figures 7a and 7b show the 5% insensitivities for the first and second modes. The input shaped commands are considerably more robust than the time-optimal commands. Input shaping (convolved or direct) gives an average of nearly 4 times more robustness than the time-optimal commands to errors in the first mode (Figure 7a). The direct shapers give an average of 3.7 times the robustness to second-mode errors, while the convolved shapers offer more than an order of magnitude improvement in second-mode robustness as compared to the TO ZVD commands (Figure 7b). Note that the direct and convolved shapers give approximately the same robustness to errors in the first mode, but differ largely in second-mode robustness, with the convolved shapers being much more robust. This effect has been previously noted [2, 5]. By comparing Figures 7a and 7b it can be seen that the first mode robustness is usually better than second mode robustness.

5.3. Transient Deflection

Although the shaping methods under consideration yield zero residual vibration when the model is perfect, there is deflection during the motion. Decreasing vibration during the move can increase life span and improve trajectory following. Figure 8 shows the deflection of the model shown in Figure 2 when a time-optimal ZVD command is used to slew the system 3.0 units. The deflection between m1 and m2, D1, is defined as x2-x1; the deflection D2 is x3-x2.

Figure 9 shows the maximum deflection as a function of the move distance. The time-optimal commands result in significantly more deflection than the shaped bang-bang commands. The direct and convolved shapers yield approximately the same transient deflection. Note that maximum deflection results are highly dependent on the system parameters. A system with a low mode of 1 Hz and a second mode of r Hz can be obtained using a variety of values for the masses and springs constants. In each case, the deflection would differ. However, the same general trends shown in Figure 9 would occur. Namely, the convolved ZVD shaper tends to cause the least transient deflection, while the TO ZVD command causes the most deflection. Furthermore, the maximum deflection with shaping reaches a constant value after some minimum move distance, while the deflection with time-optimal control varies in a complicated manner.

5.4. Ease of Implementation

Input shaping is a more general purpose technique than the time-optimal flexible-body control because once a shaper is designed, it can be used with any unshaped input; however, the switch times in the time-optimal flexible-body command must be computed for every desired move distance. The switch times vary with move distance in a complicated manner as was shown in Figure 4. Furthermore, the time-optimal shapers must be used with a step input whose magnitude equals the maximum actuator effort. When using standard input shaping, the flexible dynamics are controlled with the shaper and the rigid-body dynamics are determined by the unshaped input. To use time-optimal flexible-body control, the flexible and rigid-body dynamics must be controlled simultaneously.

It is difficult to generate a quantitative measure to compare the ease of implementation of the two control methods. However, based on our experience, input shaping is significantly easier to implement than time-optimal control. Analytical or curve fit formulas exist for all input shapers discussed and many others not discussed. Hence, input shaping can be implemented in real time, with any unshaped command signal.

On the contrary, the time-optimal control must be computed using an optimization for every move distance. While tables of optimal switch times can be stored and interpolated for desired move distances, this process is more cumbersome and interpolation will generally lead to some error in the actual switch times used (i.e., the interpolated switch times will not satisfy the optimal
constraint equations exactly. These errors may lead to even lower robustness than that displayed in Figure 7.

6. Conclusions

Multi-mode input shaping has been compared with robust time-optimal flexible-body control. The methods were compared in terms of speed, robustness to modeling errors, transient deflection, and ease of implementation. While time-optimal commands yield the fastest responses, the percentage increase in move duration with input shaping decreases as move distance increases. Input shaping provides vastly increased robustness to modeling errors, while being much easier to implement. Furthermore, input shaping generates significantly less transient deflection than does time-optimal control. Given the numerous advantages provided by input shaping, it provides an attractive alternative to time-optimal flexible-body control.

References


