Performance Analysis of Incremental Selection Decode-and-Forward Relaying over Rayleigh Fading Channels

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Abstract—Selection decode-and-forward (DF) relaying and incremental relaying were introduced as efficient relaying protocols to increase capacity and performance of cooperative diversity networks. This paper proposes an incremental relaying in conjunction with selection DF and provides its performance in terms of outage probability and bit error probability. Our proposed protocol offers a good trade off between performance and channel resource with an appropriate given threshold. Simulations are performed to confirm our theoretical analysis.

I. INTRODUCTION

Recently, cooperative communications has gained a great interest in research community (see, e.g., [1-8]). The fundamental idea of cooperative networks is that several single-antenna relays help the source transmit the information to the destination. In [5], Laneman proposed two cooperative protocols with different processing technique at the relays: Amplify-and-Forward and Decode-and-Forward and proved that they can provide the spatial diversity to the system.

In most recent publications on the cooperative diversity networks, a distributed relay selection is proposed for two hop Amplify-and-Forward (AF) or Decode-and-Forward (DF) systems, where the selected criterion is the best instantaneous SNR composed of the SNR across the two-hops [2, 3, 6]. Although, these protocols can achieve full diversity, they lead to a certain loss in the channel resource because the best relay among available relays repeats all the time, making inefficient use of the degrees of freedom of the channels. In [5, 7, 8], incremental relaying with one relay based on AF was introduced as an efficient protocol in term of system capacity in which it suggests the use of limited feedback from the destination, e.g., a single bit indicating the success or failure of the direct transmission, so that the relay knows when to forward what it receives from the source. Such a system makes more efficient use of the channel recourses, because the relay will forward the information only when it is necessary. With an appropriate threshold, the relay repeats rarely and the source can utilize the most of degrees of freedom.

Motivated by all of the above, in this paper, we focus on combining incremental relaying with selection DF relaying [3] and evaluating its performance. In particular, the closed-forms for outage probability and bit error probability for $M$-ary square quadrature amplitude modulation ($M$-QAM) of the incremental relaying cooperative diversity network equipped with DF relays in both independent identically distributed (i.i.d) and independent but not identically distributed (i.n.d.) Rayleigh fading channels are derived.

The rest of this paper is organized as follows. In section II, we introduce the model under study and describe the proposed protocol. Section III shows the formulas allowing for evaluation of the outage probability, the average BER of the system. In Section IV, we contrast the simulations and the results yielded by theory. Finally, the paper is closed in section V.

II. SYSTEM MODEL

We consider the wireless relay network consisting of one source (S), $N$ relays $R_i$ with $i = 1, \ldots, N$ and one destination (D) as illustrated in Fig. 1. Each node is equipped with single antenna and operates in half-duplex mode. The communication occurs in two phases. In the first phrase, the source broadcasts the information to $N$ relays as well as the destination. Based on the quality of the received signal at the destination, the destination decides whether it should request the help from the relays or not. If the source-destination SNR is sufficiently high, the feedback indicates success of the direct transmission and the relays keep silent. Otherwise, if the SNR of the direct transmission at the destination is less than the pre-determined threshold, $\gamma_{th}$, the destination will need assistance from the best relay among $N$ available relays in the second phrase. The selected criterion to choose the best relay is based on the SNRs across the two hops of each relay. During the second phrase transmission, selection diversity is applied, i.e., only the best relay is selected for forwarding the information to the destination. Finally, the destination combines two signals by using selection combining technique, which only selects the best signal out of two replicas for further processing and neglects the remaining ones. The benefit of using SC as opposed to maximum ratio combining (MRC) is reduced hardware complexity at the destination. In addition, it also reduces the computational costs and may even lead to a better performance than MRC, because channels with very low SNR can not accurately estimated and contribute much noise in practice.
It is assumed that every channel between the nodes experiences slow, flat, Rayleigh fading. Due to Rayleigh fading, the channel powers, denoted by $\alpha_0 = |h_{SD}|^2$, $\alpha_{1,i} = |h_{SR_i}|^2$ and $\alpha_{2,i} = |h_{R_iD}|^2$ where $i = 1, \ldots, N$ are independent and exponential random variables whose means are $\lambda_0$, $\lambda_{1,i}$ and $\lambda_{2,i}$, respectively. Let us define $\gamma_0 = \rho \alpha_0$, $\gamma_{1,i} = \rho \alpha_{1,i}$ and $\gamma_{2,i} = \rho \alpha_{2,i}$ as the instantaneous SNR for the links $S \rightarrow D$, $S \rightarrow R_i$ and $R_i \rightarrow D$, respectively, where $\rho$ is the average transmit signal-to-noise (SNRs) for the source and the relays.

III. PERFORMANCE ANALYSIS

A. PDF OF END-TO-END SNR

There are two events happening at the destination depending on the quality of the source-destination SNR: 1) it satisfies the threshold and relays do nothing, 2) it does not satisfy the threshold and the destination requests the help from the relays to send another replica of the source signal. Assuming that the source-destination SNR is not satisfied the threshold, the destination will need assistance from the relays. With fixed decode-and-forward relaying [5], it is noticed that due to the imperfect detection at the relays, it maybe forward incorrectly decode signals to the destination. Hence, similarly as in [4], for any modulation scheme, the dual hop $S \rightarrow R_i$ $\rightarrow D$ channel can be modeled as an equivalent signal hop whose output SNR $\gamma_i$ can be tightly approximated in the high SNR regime as follows

$$\gamma_i = \min(\gamma_{1,i}, \gamma_{2,i})$$

(1)

For the relay selection scheme [1], the signal with largest SNR is selected to forward the source information toward the destination. Then, the instantaneous SNR at the output relay selection combiner can be given by

$$\beta = \max_{i = 1, \ldots, N} \gamma_i$$

(2)

Since $\gamma_{1,i}$ and $\gamma_{2,i}$ are exponentially distributed random variables with hazard rates $1/\gamma_{1,i} = 1/\rho \lambda_{1,i}$ and $1/\gamma_{2,i} = 1/\rho \lambda_{2,i}$, respectively. From (1), it follows from the fact that the minimum of two independent exponential random variables is again an exponential random variable with a hazard rate equals to the sum of the two hazard rates [9], i.e., $\gamma_i^{-1} = \gamma_{1,i}^{-1} + \gamma_{2,i}^{-1}$. Under the assumption that two hops are subject to independent fading, order statistics gives the cumulative distribution function (CDF) of $\beta$ as:

$$F_\beta(\gamma) = \Pr(\gamma_1 < \gamma, \ldots, \gamma_N < \gamma) = \prod_{i=1}^{N} F_{\gamma_i}$$

(3)

Hence, the joint pdf of $\beta$ is given by differentiating (3) with respect to $\gamma$ [10].

$$f_\beta(\gamma) = \frac{\partial}{\partial \gamma} F_\beta(\gamma)$$

$$= \sum_{i=1}^{N} \left[ \frac{1}{\gamma_i} e^{-\frac{\gamma}{\gamma_i}} \prod_{j=1, j \neq i}^{N} \left(1 - e^{-\frac{\gamma}{\gamma_j}} \right) \right]$$

(4)

where $\omega_i = \sum_{l=1}^{i} \gamma_l^{-1}$.

Since selection combining is exploited at the destination, the signal with largest SNR is always selected. Then the instantaneous SNR at the output of the destination selection combiner can be given by

$$\gamma = \max(\gamma_0 | \gamma_0 < \gamma_{th}, \beta)$$

(5)

In addition, in a flat Rayleigh fading channel, the conditional pdfs of $\gamma_0 | \gamma_0 < \gamma_{th}$ and $\gamma_0 | \gamma_0 \geq \gamma_{th}$ can be obtained by using conditional probability, respectively, as follows [9]:

$$f_{\gamma_0 | \gamma_0 \geq \gamma_{th}}(\gamma) = \left\{ \begin{array}{ll}
0 & , \gamma < \gamma_{th} \\
\frac{e^{-\gamma_{th}/\gamma_0}}{\gamma_0} e^{-\gamma/\gamma_0} & , \gamma \geq \gamma_{th}
\end{array} \right.$$ 

(6)

$$f_{\gamma_0 | \gamma_0 < \gamma_{th}}(\gamma) = \left\{ \begin{array}{ll}
\frac{1}{(1-e^{-\gamma_{th}/\gamma_0})\gamma_0} e^{-\gamma/\gamma_0} & , \gamma < \gamma_{th} \\
0 & , \gamma \geq \gamma_{th}
\end{array} \right.$$ 

(7)

where $\gamma_0 = E[\gamma_0] = \rho \lambda_0$.

Hence, from (4), (7) and (5), $f_\gamma(\gamma)$ can be easily determined as eq. (8) shown at the top of the next page. It is noticed that the form of (8) has a mathematical tractable form which is convenient to derive the closed-form expressions for outage probability and average BER of the system.

B. OUTAGE PROBABILITY

In incremental relaying, the destination will request the assistance from the best relay if the SNR of the direct link at the destination is less than the pre-determined threshold, $\gamma_{th}$. Hence, the outage probability of the system can be determined as

$$P_o = \Pr(\gamma_0 < \gamma_{th}) \Pr(\gamma < \gamma_{th})$$

$$= \Pr(\gamma_0 < \gamma_{th}) \int_{0}^{\gamma_{th}} f_\gamma(d\gamma)$$

(9)
$$f_\gamma(\gamma) = \begin{cases} \frac{1}{1-e^{-\gamma/\gamma_0}} \sum_{i=1}^{N} (-1)^{i-1} \sum_{n_1,...,n_i=1}^{N} \left[ \frac{1}{70} e^{-\frac{\gamma}{\gamma_0}} + \omega_i e^{-\gamma \omega_i} - \left( \omega_i + \frac{1}{70} \right) e^{-\gamma (\omega_i + \frac{1}{70})} \right] , & \gamma < \gamma_{th} \\ \sum_{i=1}^{N} (-1)^{i-1} \sum_{n_1,...,n_i=1}^{N} \omega_i e^{-\gamma \omega_i} , & \gamma \geq \gamma_{th} \end{cases} \quad (8)$$

Substituting (8) into (9) and interchanging the integral and summation, we obtain:

$$P_o = \sum_{i=1}^{N} (-1)^{i-1} \sum_{n_1,...,n_i=1}^{N} \left[ 1 - e^{-\frac{2\gamma}{\gamma_0}} - e^{-\gamma \omega_i} + e^{-\gamma (\omega_i + \frac{1}{70})} \right] \quad (10)$$

C. Bit Error Probability

The average bit error probability of the incremental relaying system can be derived as follows

$$P_b = \Pr (\gamma_0 \geq \gamma_{th}) P_D^1 + \Pr (\gamma_0 < \gamma_{th}) P_D^2$$

$$= \left[ 1 - \Pr (\gamma_0 < \gamma_{th}) \right] P_D^1 + \Pr (\gamma_0 < \gamma_{th}) P_D^2 \quad (11)$$

where $P_D^2$ is the average bit error probability at the destination given that $\gamma_0 \geq \gamma_{th}$, $P_D^2$ denotes the average bit error probability of the system given that $\gamma_0 < \gamma_{th}$.

Under the assumption that $\gamma_0$ follows the Rayleigh distribution, therefore, $\Pr (\gamma_0 < \gamma_{th})$ can be easily derived as

$$\Pr (\gamma_0 < \gamma_{th}) = \int_{0}^{\gamma_{th}} f_{\gamma}(\gamma) d\gamma = \int_{0}^{\gamma_{th}} \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} d\gamma$$

$$= 1 - e^{-\gamma_{th}/\gamma_0} \quad (12)$$

The conditional bit error probability over Rayleigh fading channels for $M$-ary square quadrature amplitude ($M$-QAM) modulation ($M = 4^m, m = 1, 2, \ldots$) with Gray mapping can be given as [11]

$$P_D^1 = \int_{0}^{\infty} \frac{1}{\sqrt{M}} \log_2 \sqrt{M} v_j \left[ \frac{\phi_k^j}{\sqrt{M} \log_2 \sqrt{M}} \right] \text{erfc} \left( \sqrt{\frac{\gamma_k}{\gamma_0}} \right) d\gamma \quad (13)$$

where $v_j = (1 - 2^{-j}) \sqrt{M} - 1$, $\phi_k = \frac{M+1}{M-2} \frac{1}{M-1}$, $\phi_k = \frac{1}{M-1} \left[ \frac{k+1}{\sqrt{M}} - 1 \right]$. Furthermore, we define $[.]$ and erfc(.) as the floor and complementary error function, respectively. Substituting (6) into (13) and taking the integral with respect to $\gamma$, we achieve the conditional bit error probability $P_D^1$ as follows:

$$P_D^1 = e^{-\gamma_{th}/\gamma_0} \int_{\gamma_{th}}^{\infty} \log_2 \sqrt{M} v_j \left[ \frac{\phi_k^j}{\sqrt{M} \log_2 \sqrt{M}} \times \text{erfc} \left( \sqrt{\frac{\gamma_k}{\gamma_0}} \right) \right] d\gamma \quad (14)$$

where $I(.)$ is defined as follows [12]:

$$I(a,b,c,\gamma_{th}) = \int_{\gamma_{th}}^{\infty} \frac{1}{c} e^{-\frac{\gamma}{c}} d\gamma \quad (15)$$

$$= a \left[ e^{-\frac{2\gamma}{c}} \text{erfc} \left( \sqrt{\frac{\gamma_{th}}{c}} \right) - \sqrt{\frac{bc}{1+bc}} \text{erfc} \left( \sqrt{\frac{2\gamma}{c}} \left( 1 + bc \right) \right) \right] \quad (17)$$

Similarly, from (8), $P_D^2$ can be derived as eq. (16) shown at the top of the next page and $J(.)$ can be solved in closed-form as [12].

$$J(a,b,c,\gamma_{th}) = \int_{0}^{\gamma_{th}} \frac{1}{c} e^{-\frac{\gamma}{c}} d\gamma \quad (17)$$

where erfc(.) denotes the error function.

Substituting (12), (14) and (16) into (11), we can obtain a closed-form expression for the bit error probability of the DF incremental relaying cooperative system over Rayleigh fading channels.

At high SNR regime, i.e., $\gamma_i \rightarrow \infty$, and applying $1 - e^{-x} \approx 0$ and $e^{-x} \approx 0$, (11) can be approximated as follows:

$$P_b \approx \left[ 1 - \Pr (\gamma_0 < \gamma_{th}) \right] P_D^1 \quad (18)$$

From (18), it can be seen that the performance of the system at high SNR regime will converge asymptotically to that of direct transmission given that $\gamma_0 \geq \gamma_{th}$ and will not depend on the number of relays in the network. In addition, it is straightforward to see that our proposed protocol results in a better performance as compared to direct transmission.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we provide some simulation results of the system performance and verify these results with our derived formulas. Figs. 2, 3 show the outage probability of the proposed protocol with different numbers of cooperative nodes and with different values of threshold. The results are based on the assumption that $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ are set to be uniformly distributed between 0 and 1. As the number of relays increases from 1 to 5, the outage probability improves, as expected. In Fig. 4, we investigate the relationship between bit error rate and number of relays with a fixed threshold.
were examined. Again, we assume that and independent but not identically distributed (i.n.d.) channels of modulation. In Fig. 7, the performance of the proposed we study the performance of the system with different levels direct transmission and the selection DF relaying. In Fig. 6, we also compare two expressions for average BER of square -QAM, i.e. eq. (11) and eq. (18). It can be seen that the results obtained from the approximation expression are tight with that of the closed-form one in high region of SNR. However, there is a big gap between two curses with lower SNRs because the condition for the approximation in eq. (18) has not been satisfied. Fig. 5 also shows that at high SNR, the bit error rates for incremental relaying scheme with different values of tend to be parallel with direct transmission as mentioned in (18). From Fig. 5, it is obvious that the error performance of the proposed protocol lies between that of the direct transmission and the selection DF relaying. In Fig. 6, we study the performance of the system with different levels of modulation. In Fig. 7, the performance of the proposed protocol in both independent and identically distributed (i.i.d.) and independent but not identically distributed (i.n.d.) channels were examined. Again, we assume that , and is set to be 3 and to be uniformly distributed between 0 and 1 for the i.i.d. and i.n.d. case, respectively. It can be seen that our analytical results and the simulation results are in excellent agreement.

V. CONCLUSION

We have proposed a combination of incremental relaying with selection DF relaying that allows overcoming the disadvantage of the selection DF relaying in wasting the channel resource. The proposed protocol exploited both advantages offered by incremental relaying and selection DF relaying. By choosing an appropriate threshold, our proposed protocol could provide a good trade-off between performance and channel resource. In addition, the closed-form expressions for outage probability and average bit error rate for square -QAM have been derived. Various performance evaluation results have been also presented verifying the analysis. Simulation results are in excellent agreement with the derived expressions. Our analysis reveals an interesting result for this relaying protocol: performance of the system with an appropriate value of threshold at high SNR regime does not depend on the number of relays on the network.

\[
P_D = \sum_{j=1}^{\infty} \sum_{k=0}^{M} \frac{\phi_j^k}{\sqrt{M \log_2 M}} \text{erfc} \left( \sqrt{\frac{2}{\gamma \log_2 M}} \right) = \sum_{j=1}^{\infty} \sum_{k=0}^{M} \frac{\phi_j^k}{\sqrt{M \log_2 M}} \text{erfc} \left( \sqrt{\frac{2}{\gamma \log_2 M}} \right) = \sum_{j=1}^{\infty} \sum_{k=0}^{M} \frac{\phi_j^k}{\sqrt{M \log_2 M}} \text{erfc} \left( \sqrt{\frac{2}{\gamma \log_2 M}} \right)
\]

It can be seen that the improvement of the bit error rate will be proportional to the number of relays in low SNR regime. However, in high SNR regime, BERs converge to a limit and that limit tends to be parallel with that of direct transmission (DT). We can conclude that with the incremental relaying, the system only achieves diversity gain in low SNR regime not in all range of SNR. This is due to the fact that at high SNR regime, the source-destination link is with high enough reliability for decoding process; hence, the source rarely requests the help from the relays. In Fig. 4, we also compare two expressions for average BER of square -QAM, i.e. eq. (11) and eq. (18). It can be seen that the results obtained from the approximation expression are tight with that of the closed-form one in high region of SNR. However, there is a big gap between two curses with lower SNRs because the condition for the approximation in eq. (18) has not been satisfied. Fig. 5 also shows that at high SNR, the bit error rates for incremental relaying scheme with different values of tend to be parallel with direct transmission as mentioned in (18). From Fig. 5, it is obvious that the error performance of the proposed protocol lies between that of the direct transmission and the selection DF relaying. In Fig. 6, we study the performance of the system with different levels of modulation. In Fig. 7, the performance of the proposed protocol in both independent and identically distributed (i.i.d.) and independent but not identically distributed (i.n.d.) channels were examined. Again, we assume that , and is set to be 3 and to be uniformly distributed between 0 and 1 for the i.i.d. and i.n.d. case, respectively. It can be seen that our analytical results and the simulation results are in excellent agreement.

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Fig. 4. Bit error rate (4-QAM) for incremental relaying with relay selection with different number of relays.

Fig. 5. Bit error rate for 16-QAM for incremental relaying with relay selection with different values of $\gamma_{th}$.

Fig. 6. Bit error rate for incremental relaying with relay selection with different levels of modulation.

Fig. 7. Bit error rate for incremental relaying with relay selection over i.i.d. and i.n.d. Rayleigh fading channels.

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