PAPR Reduction using PTS with Low Computational Complexity in Coherent Optical OFDM Systems

Huynh Vo Trung Dung, Hakjeon Bang, and Chang-Soo Park
School of Information and Communications
Gwangju Institute of Science and Technology (GIST)
1 Oryong-dong, Buk-Gu, Gwangju, 560-172, South Korea
E-mail: csp@gist.ac.kr

Seungil Myong
Optical Access Research Team
Electronics and Telecommunications Research Institute (ETRI)
161 Gajeong-dong, Yuseong-gu, Daejeon, 305-350, South Korea

Abstract—In optical communication systems for single-carrier transmission based on orthogonal frequency-division multiplexing (OFDM), high power signals result in fiber nonlinear effects such as self-phase modulation (SPM), which causes inter-symbol interference (ISI) between sub-carriers. To reduce peak-to-average power ratio (PAPR) at a transmitter of the optical OFDM system, a partial transmit sequence (PTS) technique can be applied, but it requires reduced computational complexity for practical implementation because the complexity in an optimum search over all combination of phase factors is exponentially increased with the number of sub-blocks. In this paper, a PTS method with low computational complexity is proposed by modifying an artificial bee colony (ABC) algorithm, which is an approach based on solution sets to solve combinatorial and numerical optimization problems. The effectiveness of the proposed method is confirmed by simulation results of complementary cumulative distribution function (CCDF) and BER in a coherent optical OFDM (CO-OFDM) system.

Index Terms—peak-to-average power ratio (PAPR); partial transmit sequence (PTS); computational complexity; coherent optical OFDM (CO-OFDM)

I. INTRODUCTION

For high-speed transmission and network flexibility, orthogonal frequency-division multiplexing (OFDM) in optical communications has been considered as a promising technology for next-generation optical access networks, because it has advantages such as high spectral efficiency and inherent dynamic bandwidth capabilities [1]. In the optical OFDM system, spectral bandwidth for transmission is divided to the number of subcarriers and spectra of individual subcarriers are partially overlapped [1], [2]. As long as the optical channel is linear, the subcarriers at a receiver can be demodulated without the interference between subcarriers and the need for analog filtering [3]. However, high power signals at a transmitter of the optical OFDM system for a single-carrier transmission cause fiber nonlinear effects such as self-phase modulation (SPM), which causes inter-symbol interference (ISI) between subcarriers in long haul lightwave systems [4]. Main components which are sensitive to nonlinearity in the OFDM-based optical transmission are the fiber and the optical modulator [4]. The high peak-to-average power ratio (PAPR), sensitivities to phase noise, and frequency offset are the main drawbacks of OFDM-based communication systems [2], [3].

A partial transmit sequence (PTS) technique is one of PAPR reduction methods [5]. Because the input data block is split into disjoint sub-blocks and those sub-blocks are combined after being phase-shifted by constant phase factors, the PTS is a distortionless technique [6]. However, it requires side information for phase factors to recover data and search complexity for the optimal phase-factor combination exponentially increases with the number of sub-blocks. Therefore, it is not practical for a large number of sub-blocks to more reduce PAPR. In order to reduce the computational complexity, some approaches have been proposed recently. The iterative flipping algorithm for the PTS in [7] had the computational complexity which was proportionally linear to the number of sub-blocks. But the flipping sign of the phase factors occurred in the case of the allowed phase factor of 2. A neighborhood search approach, called as gradient descent search, was proposed to achieve PAPR reduction which was close to that by the ordinary PTS technique with reduced search complexity and little performance degradation [8]. An approach in [9] modified the computation complexity problem into an equivalent problem of minimizing the sum of phase-rotated vectors. A simulated annealing method was proposed in [10]. An intelligent genetic algorithm for PAPR reduction was developed in [11]. A PTS method based on the modified artificial bee colony algorithm (ABC-PTS) [12], which used the artificial bee colony (ABC) algorithm to solve combinatorial and numerical optimization problems, was proposed to reduce the PAPR of OFDM signals with reduced complexity.

To more reduce computation complexity, a PTS method is
proposed by adapting the modified ABC algorithm. Since the new phase factors for the next search cycle in the proposed method are randomly generated near to currently optimal phase factors, the phase factors for PAPR reduction are found with reduced search complexity. The effectiveness of the proposed method is confirmed by simulation results of complementary cumulative distribution function (CCDF) and BER in a coherent optical OFDM (CO-OFDM) system.

II. PTS WITH LOW COMPLEXITY COMPUTATION

A. PTS Scheme for PAPR Reduction in OFDM Systems

In the OFDM-based systems with N sub-carriers, a high-rate data stream is divided to N low-rate data streams. Let a vector \( \mathbf{X} \) be an input OFDM block. The \( \mathbf{X} \) consists of \( N \) symbols (i.e., \( \mathbf{X} = [X_0, X_1, ..., X_{N-1}]^T \)) and each symbol is modulated to one subcarrier of \( \{f_n : n=0,1, ..., N-1\} \). And, the \( N \) subcarriers in the OFDM systems are orthogonally chosen, that is, \( f_n = f_0 + n\Delta f \), where \( \Delta f = 1/NT \) and \( T \) is the symbol period. Here, \( NT \) is the duration of the OFDM data block \( \mathbf{X} \). After the inverse fast Fourier transform (IFFT), the complex envelope of the transmitted OFDM signal is given by:

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_nt}, \quad \text{for } 0 \leq t < NT.
\]

(1)

The PAPR of the transmitted signal \( x(t) \) in (1) is defined as the ratio of the maximum instantaneous power to the average power, and it is written as:

\[
PAPR = \max_{0 \leq t < NT} \frac{|x(t)|^2}{E[|x(t)|^2]},
\]

(2)

where

\[
E[|x(t)|^2] = \frac{1}{NT} \int_0^{NT} |x(t)|^2 dt.
\]

(3)

However, most systems use discrete-time OFDM signals. Since symbol-spaced sampling of (1) sometimes misses some of the single peaks, oversampling for signal samples is required to accurately estimate the PAPR of the OFDM signals. The OFDM signal with the oversampling factor \( L \) \((L>1)\) is given by:

\[
x(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi nk/L}, \quad \text{for } k = 0, 1, ..., LN - 1.
\]

(4)

According to [13], the oversampled factor \( L = 4 \) provides a sufficiently accurate estimation for the PAPR of the OFDM signals. The PAPR for the \( L \)-times oversampled discrete-time signals is rewritten from (2) as:

\[
PAPR = \max_{0 \leq k < LN-1} \frac{|x(k)|^2}{E[|x(k)|^2]}.
\]

(5)

To reduce the PAPR, the OFDM signals can be transmitted by using the PTS method, as shown in Fig. 1. In the ordinary PTS (OPTS) approach, the input data \( \mathbf{X} = [X_0, X_1, ..., X_{N-1}]^T \) is partitioned into \( M \) sub-blocks \( \mathbf{X}_m = [X_{m,0}, X_{m,1}, ..., X_{m,N-1}]^T \), \( m = 1, 2, ..., M \), such that \( \sum_{m=1}^{M} \mathbf{X}_m = \mathbf{X} \). Then, the \( L \)-times oversampled time-domain signal of \( \mathbf{X}_m \) is obtained by taking the IFFT of length \( LN \) of \( \mathbf{X}_m \) padded with \( (L-1)N \) zeros as \( \mathbf{x}_m = [x_{m,0}, x_{m,1}, ..., x_{m,LN-1}]^T \), \( m = 1, 2, ..., M \). The partial transmit sequences \( \mathbf{x}_m \) is combined with complex phase factor \( b_m = e^{j\phi_m} \), where \( \phi_m \in [0, 2\pi] \) for \( m = 1, 2, ..., M \), to minimize the PAPR value in the time-domain. After the combination, the transmitted signal in the time domain is given by:

\[
x'(b) = \sum_{m=1}^{M} b_m x_m,
\]

(6)

where \( x'(b) = [x'_0(b), x'_1(b), ..., x'_{LN-1}(b)]^T \). The phase factors should be limited to a set with finite number of elements in order to keep the number of required side information bits and the search complexity within a reasonable limit. The set of the phase factors is generally used with:

\[
B = \left\{ e^{j2\pi l/W} | l = 0, 1, ..., W - 1 \right\},
\]

(7)

where \( W \) is the number of allowed phase factors. In a case that one phase factor is fixed, the optimum set of phase factors is found in \( W^{M-1} \) sets of phase factors. The search complexity for the optimum phase factor set exponentially increases with the number of sub-blocks, \( M \).

B. PTS Method based on Modified ABC Algorithm

The artificial bee colony (ABC) algorithm was introduced by D. Karaboga [14] to estimate a nearly optimum set of solutions with reducing search cycles, which is only suitable for continuous numerical optimization problems. In the ABC algorithm, employed bees, onlooker bees, and scout bees are tasked to find optimum food source with the best nectar amount. In PAPR reduction problem, some modifications are needed for the basic ABC algorithm to apply for PTS. In [12], the authors have modified ABC algorithm to search the better combination of phase factors for PTS due to that fact that the phase factor is the discrete coordinate.

In this paper, we introduced a different modification of ABC algorithm based on angular concept and distance between angles, named as the modified ABC-based PTS (mABC-PTS) method to improve the convergence capability of ABC algorithm due to the fact that the phase factor is the phase weighting rotation. We chose the phase factor \( \mathbf{b} = \)}
In each cycle of our proposed method, at the initial step, phase factor optimizations memorize a global optimum phase vector with the best fitness from the randomly produced phase vectors. Next, phase factor test sets produce the neighbor candidate phase vector around the optimum phase vector and its fitness value is evaluated. If the fitness value of the new phase vector is better than that of the previous one, they memorize the new one, otherwise they keep the previous one. After phase factor test sets complete the search process, they will share the fitness value of the phase vector and the phase vector information with the phase factor optimizations. The phase factor optimizations determine a phase vector with a high probability based on its fitness value and produce the neighbor candidate phase vector around the optimum phase vector. The phase factor optimizations use the greedy selection process in that if the fitness of the candidate phase vector is better than that of the present one, they forget the present one and memorize the candidate phase vector. Otherwise, they keep the present one in the memory. After all phase factor optimizations are distributed, they compare the fitness of the global optimum phase vector with that of the final best candidate phase vector. If the fitness of the final best candidate phase vector is better than that of the global optimum one, they will memorize the final best candidate phase vector as the new global optimum phase vector for the next cycle. Otherwise, they keep the global optimum phase vector in memory for the next cycle. After the phase factor test sets and the phase factor optimizations complete their searches, if the fitness value of the phase vectors cannot be improved by a predetermined number that is called limit, a renew phase factor set is produced to replace the exhausted phase vector associated with that fitness value. This prevents the algorithm from being trapped in the same phase vector for the whole cycles. All these steps will be repeated until the maximum number of cycles $K$ is reached.

The fitness value of the phase vector $b_i$ is determined as:

$$\text{fitness}(b_i) = \frac{1}{1 + f(b_i)},$$

where

$$f(b_i) = \max \left\{ |x'(b_i)|^2 \right\} = \max \left\{ \sum_{j=1}^{M} b_{i,j}^2 \right\}. \quad (9)$$

For each phase factor test set, to produce a neighbor candidate phase vector from the previous one, we modify the formula of the basic ABC algorithm based on angular concept as:

$$u_{i,j} = \arg(b_{i,j}) + \text{rand}(-1,1) \times \| \arg(b_{i,j}) - \arg(b_{k,j}) \|, \quad (10)$$

where $\arg(.)$ denotes the angle, $\|\|$ denotes the distance between two angles, and $j \in \{1, 2, ..., M\}$. In basic ABC algorithm, $k$ is the random chosen index, $k \in \{1, 2, ..., S\}$, and $i \neq k$. In our proposed method, $k$ is the index of the best previous phase vector. The relationship between the angle $u_{i,j}$ represented in range of 0 to 2$\pi$ and its discrete coordinate $b_{i,j}$ is expressed as:

For $W = 2$,

$$b_{i,j} = \begin{cases} -1, & \text{for } \pi/2 \leq u_{i,j} \leq 3\pi/2, \\ 1, & \text{otherwise}, \end{cases} \quad (11)$$

and, for $W = 4$,

$$b_{i,j} = \begin{cases} j & \text{for } \pi/4 \leq u_{i,j} \leq 3\pi/4, \\ -1 & \text{for } 3\pi/4 \leq u_{i,j} \leq 5\pi/4, \\ -j & \text{for } 5\pi/4 \leq u_{i,j} \leq 7\pi/4, \\ 1 & \text{otherwise}. \end{cases} \quad (12)$$

For each phase factor optimization, a phase vector is chosen depending on the probability $p_i$ as:

$$p_i = \frac{\text{fitness}(b_i)}{\sum_{i=1}^{S} \text{fitness}(b_i)}. \quad (13)$$

The phase factor test set associated with the exhausted phase vector becomes a renewed phase factor set by the following formula:

$$u_s = \min(u_s) + \text{rand}(0,1) \times \| \max(u_s) - \min(u_s) \|. \quad (14)$$

Due to angular concept, $\max(u_s)$ and $\min(u_s)$ are $2\pi$ and 0, respectively. The flowchart of the mABC-PTS is illustrated in Fig. 3.
III. COMPLEXITY COMPUTATION COMPARISON

In [12], Y. Wang et al. has compared computational complexity among the ABC-PTS and the existing PAPR reduction methods such as particle swarm optimization algorithm-based PTS (PSO-PTS), the iterative flipping algorithm for PTS (IPTS), gradient descent search (GD), and minimum distance guided genetic algorithm (MDGA). It is shown that compared to the exiting PAPR reduction methods, the ABC-PTS algorithm has the lowest search complexity for larger PTS sub-blocks at the same time and can get better PAPR reduction performance. In the mABC-PTS algorithm, the randomly initial phase vectors with the size $S$ are produced, and then the phase factor test sets and the phase factor optimizations carry out searching according to the above algorithm. When the maximum cycles $K$ is reached, the optimum phase vector with the minimum PAPR is outputted. Therefore, the computational complexity in our proposed method is proportional to $SK$, which is the same to the ABC-PTS in [12]. The complexity of the PTS technique with an exhaustive search (OPTS) is $W^{M-1}$ by fixing a phase vector without any performance loss.

IV. RESULTS AND DISCUSSION

To evaluate the proposed PTS method, we used two performance parameters such as the complementary cumulative distribution function (CCDF) and bit-error-rate (BER). We introduce the CCDF representing the probability that the PAPR...
of an OFDM symbol exceeds the given threshold \( PAPR_0 \),
\[
CCDF = \Pr\{PAPR > PAPR_0\},
\]  
which is commonly used to quantify the efficacy of a PAPR reduction scheme. In order to get CCDF, \( 10^5 \) random OFDM symbols are generated with 16-QAM modulation. In our simulation, 256 subcarriers is chosen while the allowed phase factor \( W = 2 \) and 4.

In Fig. 4, we compared CCDF performance among our proposed method (the mABC-PTS), the ABC-PTS in [12] and OPTS with \( M = 16 \) sub-blocks using adjacent partition method. We chose \( S = K = 30 \), \( \text{limit} = 5 \) with 16-QAM modulation. When \( CCDF = 10^{-3} \), PAPR of the original OFDM signal is around 10.89 dB. PAPR by the mABC-PTS with \( W = 2 \) and 4 are 7.24 dB and 7.03 dB, respectively while that of the ABC-PTS in [12] with \( W = 2 \) and 4 are 7.26 dB and 7.19 dB, respectively for the same search complexity 900. It can be seen that there is a reduction of 0.16 dB of PAPR by the mABC-PTS compared with the ABC-PTS in [12] when \( W = 4 \) while this reduction is about 0.02 dB when \( W = 2 \). It means that our proposed method for PAPR reduction outperforms the ABC-PTS in [12]. The reason is that the neighbor candidate phase factor which is randomly generated is close to the previous optimum one in our proposed algorithm. PAPR by OPTS is about 6.63 dB, but there is an exhausted search complexity \( 2^{15} \cdot 32768 = 32768 \). The gap between PAPR by the mABC-PTS with \( W = 4 \) and by OPTS is only 0.41 dB. However, the search complexity of the mABC-PTS is only 900, which is about 900/32768 = 2.75\% of that by OPTS. When compared with the PAPR of original OFDM signal, that of the mABC-PTS with \( W = 4 \) reduces approximately 3.85 dB. The decreased PAPR is so attractive for distortionless transmission because there is no requirement for a high dynamic range of power amplifiers, AD/DA-converters, and optical modulator while the search complexity is reduced which is extremely attractive for high-speed circuit with less processing time. Hence, our proposed method is feasible for the practical implementation.

In order to determine the gain from using PAPR reduction in a system we have carried out further simulation. We have considered a CO-OFDM system, as shown in Fig. 5, which represents the more spectral efficiency, receiver sensitivity, and robustness against polarization dispersion than direct-detection optical OFDM [1]. In simulation setup, we use IEEE802.16-2004 standard [15] as the PHY protocol; there are 192 data carriers, 8 pilot tones for channel estimation and equalization, 56 unused tones for guard band, 64 tones for cyclic prefix (CP). The data rate is 10 Gbits/s with 4-QAM modulation and 4 partitioning sub-blocks which is adopted in simulation. The baseband OFDM signal with PAPR reduction scheme is converted to optical domain by using the RF-to-optical (RTO) up-converter [1] with the wavelength of laser diode (LD) of 1550 nm. For a fiber transmission, a 120-Km of single mode fiber (SMF) is used with the dispersion parameter \( D = 16 \text{ ps}/(\text{Km}\cdot\text{nm}) \), the dispersion slope \( S = 0.057 \text{ ps}/(\text{Km}\cdot\text{nm}^2) \), the attenuation coefficient \( \alpha = 0.2 \text{ dB/Km} \), the nonlinear coefficient \( \gamma = 1.3 \text{ W}^{-1}/\text{Km} \), and no polarization mode dispersion (PMD). The optical signal is coherently detected by the optical-to-RF (OTR) down-converter [1] and transferred to baseband OFDM signal. Finally, it is demodulated with channel estimation and equalization to recover data.

Fig. 6 shows the comparison of the bit-error-rate (BER) performance of the mABC-PTS, the ABC-PTS, OPTS and original signal with \( W = 2 \) and 4. At the low laser launch power (e.g. \(-9 \text{ dBm}\) ), there are transparent fiber nonlinear effects, thus the BERs of mABC-PTS, ABC-PTS, OPTS, and original signals are close. Then, they decrease due to the improved optical signal-to-noise ratio (OSNR) when the laser launch power increases. The lowest BER occurs at around \(-4 \text{ dBm}\) laser power in all cases with or without the PTS methods. When the laser launch power is larger than the power for the lowest BER, BERs will increase because of the increased nonlinear distortion caused by high laser power. At the high laser launch power (e.g. \( 2 \text{ dBm}\) ), the BER performance of original signal without the PTS is the worst due to the presence of high fiber nonlinear effects. The BER performance by the mABC-PTS is always better than that by the ABC-PTS with
the same W, because the reduced PAPR in the mABC-PTS decreases the launch power before propagating in optical link and it results in less fiber nonlinear effects. It can be seen that the BER by the mABC-PTS $W=4$ is close to that by OPTS. Fig. 7 shows the CO-OFDM systems maximum possible transmission distance at different laser powers to guarantee a $10^{-3}$ BER in simulation. The maximum fiber transmission distance, which occurs at around $-4 \text{ dBm}$ laser power, is approximate 220 Km in all cases. It can be seen that, at 2dBm laser power, the original signal is only transmitted over 134 km while the signal by OPTS, the ABC-PTS $W=2$, the mABC-PTS $W=2$, the ABC-PTS $W=4$, and the mABC-PTS $W=4$ can be transmitted over 179 Km, 154 Km, 160 Km, 171 Km, and 178 Km, respectively. A longer fiber transmission can be obtained with the PTS method. At the different laser power, the maximum transmission distance by the mABC-PTS is always higher than that by the ABC-PTS for the same W. It means that the CO-OFDM system with the proposed method can handle high power and reach the lower BER compared to the system with the ABC-PTS [12] for the same fiber transmission distance. This result is consistent with the results shown in Fig. 4 and Fig. 6.

V. CONCLUSION

The proposed PTS method provides better performance in terms of PAPR reduction and computation complexity. From the results, reduced PAPR at the transmitter decreases nonlinearity in the OFDM-based transmission and it results in better BER performance in the coherent optical OFDM system, which extends the maximum transmission distance. Also, the proposed method is practical due to reduced search complexity even in the large number of sub-blocks for the PTS method.

REFERENCES


