STUDENTS' COGNITIVE PROCESSES IN MATHEMATICS CLASSROOM USING LESSON STUDY AND OPEN APPROACH WITH GEOGEBRA

MR. VISA KIM

A PROPOSAL FOR THE DEGREE OF MASTER EDUCATION

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A PROPOSAL SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF MATHEMATICS EDUCATION

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CHAPTER I
INTRODUCTION

1. Rationale and Background

During the Khmer Rouge from April 1975 to January 1979, almost all of the country's infrastructures, public and private properties, economic system, and the education system were destroyed (Dy, 2009). They also destroyed the national intellectual resources, which were the country's most invaluable capital, especially killed former teachers (Howard, Quigley, & Robinson, 2012). The eradication of teachers as well as the education system has been a cause of the huge setback to the development of Cambodia even long after the civil war had ended. Current mathematics Education in Cambodia, as to face many problems like lack of well qualified teachers, and lack of expertise's on method of teaching, so on (Chan, 2012). Students are passive recipients of information who listen to the teacher and/or read a textbook (Moore & Hansen, 2011) and are facing challenges in developing one of the most important skills needed – critical thinking. Students' opportunities to understand mathematical concepts and procedures are not maximized when students are passively listening to teachers (Takahashi, 2006). However, students need to be actively involved in mathematics and to do mathematical activities rather than just listening to teachers talk (Brown, 1994 cited from Takahashi, 2006). Therefore, Cambodian teachers should be changed mathematics classroom from passive learning to active learning, where the students can engage in with more meaningful mathematics because according to Davis and Maher (1997) cautioned that effective mathematical leaning requires active student participation in meaningful experiences which deal in an accurate and relevant way with important part of real mathematics.

While analyzing international experience and tendencies of developmental, I have considered different approaches to organizing research activity of learners. As a result of the preliminary research, special interest was concentrated on Problem-Solving. A number of studies have reported that the problem-solving is to be a key characteristic of teaching mathematics (National Council of Teachers of Mathematics [NCTM], 1980). Furthermore, in Principles and Standards of School Mathematics stated that educators are urged to teach mathematics through problem-solving (NCTM, 2000). In problem-solving, when students are approaching mathematical situations, they need to conceptualize the construction of mathematical knowledge as an activity in which they have to actively participate in order to identify and communicate ideas that emerge (Moreno-Armella & Sriraman, 2005). Learning, constructing, or developing mathematical knowledge via problem solving activities continues to be an important goal in curriculum proposals and a central theme in research programs around the world (Santos-Trigo & Gooya, 2015). From these ideas, the teaching approach that teach students by focus on problem-solving from that which was mentioned in the early part of this paper to be an Open Approach incorporating in Lesson Study (Nimtrakul, Sangaroon, & Inprasitha, 2014).

Lesson Study is the process of learning a professional that Japanese teacher engage in the career of its ongoing worldwide to examine methods to teach a systematic their teaching content and curriculum, as well as the process in which their students of learning and understanding, in order to achieve their goals of education (Yoshida, 2008), and is a process in which the teacher constantly tries to improve the teaching techniques of his/her work with other teachers to examine different teaching techniques and critics (Baba, 2007). Lesson study, teachers share responses (including misunderstandings) from students in the past lessons, have a lot of expectations about students' ideas and prepare their questions to extract students' ideas and their reaction against students' ideas (Isoda, 2006). Lesson study is to recognize the central importance and difficulty of teaching – of absolutely leading towards a life standards, frameworks, and best practices in the classroom (Lewis & Hurd, 2011). One adaptation teaching approach with lesson study is open approach (Moonsri & Pattanajak, 2013). Open Approach is a teaching approach that has been used for integration of mathematical activity and students' thinking (Nohda, 1998), and is a teaching approach with the emphasis on students' mathematical thinking (Isoda, 2010). The instruction in the classroom, using open approach to the teaching method innovative, which has cooperated with lesson study is effective in the development of mathematical activity using open-ended problems for the promotion and development of the concept of students (Suthisung & Sangaroon, 2010). Each student has opportunities to gain his/her own unique solutions since open-ended problems allows different appropriated solutions (Sawada, 1997). According to the above mentioned, the integration of the Lesson Study and Open Approach in mathematics classroom provide many educational benefits for students and help them reach their mathematical knowledge. In addition, these two teaching approaches are important for students to address the problem in a number of important ways, and provides the students the opportunity to make use of their mathematical knowledge and skills are widely.

In fact, a substantial amount of research studies have shown that Lesson Study improves teachers' learning and supports teachers to grow professionally (see Stigler & Hiebert, 1997; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999; Lewis, 2000; Fernan-dez, & Yoshida, 2004 cited from Meng, & Sam, 2011; Inprasitha, 2006; Inprasitha, 2011; Inprasitha, 2015). Along with this, many research studies have shown that open approach is an approach of teaching professional development for enhancing the mathematical learning process (Loipha & Inprasitha, 2004) and providing students the opportunity to discuss and share their ideas as well as thoughts in various (Thinwiangthong, 2012). However, so far there is no relevant research not yet been found to encourage the use of innovative ICT tool, GeoGebra, in the teaching and learning of mathematics among secondary school teachers through open approach incorporating lesson study. Therefore, this makes our motivation and interest to adopt open approach incorporating lesson study as a way to encourage the use of innovative GeoGebra among secondary school mathematics teachers.

In teaching and learning mathematics, according to Nissim, Barak, and Ben-Zvi, (2011), Information and Communication Technology (ICT) is plays an important role to facilitate the process of complex information and the construct of knowledge through visualization, inquiry, problem solving, and reflection. Recently, ICT has been widely used in classrooms for teaching and learning purposes (Delen & Bulut, 2011). Use of technology enhances exploration of the problem (Healy & Hoyles, 2001), students are given with a sense of mighty over their environment; they are thinking about their thinking, examing their work and reflecting (Chrysanthou, 2008). ICT promotes discussion and helps students to develop their thinking and understanding, particularly their mathematics thinking and their individual reasoning (Thompson, 2003). The use of ICT, providing a rich learning environment to promote social interaction, critical thinking skills and a holistic understanding of their learning experiences has brought about the urgency to incorporate technology in the classroom (Shadaan & Eu, 2013). ICT makes the process of teaching more efficient and strengthens knowledge wherever it is possible in the classroom; there are affirms that ICT has the potential to develop cognitive learning, enhance problem-solving and higher-level thinking skills and extend physical and mental abilities (Loveless, 1995). Thus, it is widespread belief that ICT can and will empower teachers and learners, transforming teaching and learning processes from being highly teacher-dominated to student-centered, and that this transformation will result in increased learning gains for students, creating and allowing for opportunities for learners to develop their creativity, problem-solving abilities, informational reasoning skills, communication skills, and other higher-order thinking skills (Trucano, 2005). GeoGebra is one of the ICT tools, which has the potential to clearly demonstrate to students the close connection between geometry and algebra and is becoming a recognized part of mathematical knowledge (Jones & Edwards, 2006; Hohenwarter & Jones, 2007).

GeoGebra is a relative new software system, which acts both as a physical medium and as a cognitive tool, because, on one hand, it accommodates mathematical objects, and on the other hand, it encodes data to provide support for cognitive activities (Karadag & McDougall, 2011). GeoGebra includes possibilities of both dynamic geometry and computer algebra in one tool (Chrysanthou, 2008) as well as allows for a closer connection between, manipulation of symbols and visualization capabilities, and dynamic changeability (Hohenwarter & Fuchs, 2004). Edwards and Jones (2006) believe a significant feature of GeoGebra is its activities which require high-level thinking and enable pupils to engage with the potential that technology brings, such as learning through feedback, seeing patterns, making connections, working with dynamic images, etc. Introduction of GeoGebra in mathematics classroom can be a way of providing opportunities for the mathematical investigation, encouraging discussion and group work and generally it is possible to make mathematics a more open and practical subject that can access and control to more students (Hohenwarter & Fuchs, 2004). Through the above interpretation, it shows us that GeoGebra supports and makes students can visualize virtual environment of problem and allow for opportunities for learners to develop their reasoning skills as well as provide them effective involve the problems and gain a deeper understanding of mathematical concepts.

Through adopt of Open Approach incorporating Lesson Study as a means to encourage the innovative use of GeoGebra, as illustrated above, students' visualization processes, construction processes, and reasoning processes may be occurred. These three types of process are the cognitive processes in geometrical activity (Duval, 1998). In terms of *visualization*, Arcavi (2003) defined that visualization is a skill that helps students to recognize shapes, to create new shapes or objects, and to reveal relationships between them. Mathematical visualization aims at an abstract framework for fundamental objects appearing in visualization and at the application of the manifold visualization techniques to problems in geometry, topology and numerical mathematics (Hege & Polthier, 2003). Visualization is generally considered helpful in supporting intuition and concept formation in mathematics learning (Drefus, 1991), mathematical comprehension (Borba & Villarreal, 2005). To reach that comprehension, it is necessary to take into account that visualization, which is usually associated with graphical representations, occurs not as an isolated topic, but inside a mathematical context that also includes numerical and symbolic representations (Zimmermann & Cunningham, 1991). Whereas *Construction processes*, construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented (Duval, 1998). About *reasoning*, in general, reasoning involves inferences that are drawn from principles and from evidence, whereby the individual either infers new conclusions or evaluates proposed conclusions from what is already known (Johnson-Laird & Byrne, 1993). Along with this, English (2009) also averred that mathematical reasoning is imaginative in the sense that it draws upon a number of powerful, illuminating devices that structure these concrete or base level experiences and transform them into models of abstract thought. There are many ways in which children use such reasoning, including as a source of hypotheses about an unfamiliar situation, as a source of problem-solving operators and techniques, and as an aid to learning and transfer (Halford, 1993). The results of which, demonstrate that by implementing Lesson Study and Open Approach with GeoGebra to teaching, improvements in the depth of both student visualization and reasoning can be achieved.

2. Objective of Research

The objectives of this study is to analyze students' cognitive processes in mathematics classroom using Lesson Study and Open Approach with GeoGebra.

3. Scope and Limitation of Research

3.1 Research Scope

The scope of this research is to analyze students' cognitive processes in mathematics classroom using Lesson Study and Open Approach with GeoGebra. The research findings do not generalize to the other classroom and schools in Cambodia.

3.2 Limitation of Research

Instruments of the study will develop by the researcher, so the interpretation of the results depends on the validity and reliability of these instruments. Though the researcher verified these psychometric characteristics.

4. Anticipated Outcomes

4.1 Students can solve the problem and develop their cognitive processes as well as gain a deeper understanding of mathematical concepts.

4.2 Open Approach incorporating Lesson Study as a means to encourage the innovative use of GeoGebra could be a teaching approach for mathematics teachers.

5. Definition of Terms

Cognitive processes: Duval (1998) defined cognitive processes in three types of processes as follows:

- *Visualization process* with regard to visual representation for the geometrical statement, for the heuristic exploration of a complex situation, for a subjective verification.
- *Construction process by tools:* construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented.

- *Reasoning* in relationship to discursive processes for extension of knowledge, for proof, for explanation.

Lesson Study: Lesson study concentrates on cooperation among teachers, researchers and experts in the development of lesson plans, observation of classroom activities and reflection on the outcome of learning activities (Inprasitha and Loipha, 2007). The process of lesson study consists of three phases: collaboratively design a research lesson; collaboratively observe the research lesson; and collaboratively discuss and reflect on the research lesson (Inprasitha, 2011).

Open Approach: The Open Approach is a teaching approach used in cooperated with lesson study to design learning units and lesson plans and comprises the following four steps: posing open-ended problem, students' self-learning, whole class discussion and comparison of concepts, and summarization through connecting students' mathematical ideas emerged in the classroom (Inprasitha, 2010).

GeoGebra: GeoGebra as a methodological or didactical resource that supports the teaching and learning of mathematics by helping teachers and their students visualize formal mathematical knowledge and promote their sense of ownership through dynamic constructions (Faggiano & Ronchi, 2011).

Mathematics Classroom Using Lesson Study and Open Approach with GeoGebra: Mathematics classroom using Lesson Study and Open Approach with GeoGebra is the classroom which addresses the problem in a number of important ways, and provides the students the opportunity to make use of their mathematical knowledge and skills are widely. It is the classroom which supports and makes students can visualize virtual environment of problem and allow for opportunities for learners to develop their reasoning skills as well as provide them effective involve the problems and gain a deeper understanding of mathematical concepts.

CHAPTER II
LITTERATURE REVIEW

The following section begins by introducing the theoretical concepts, followed by a presentation of the theoretical frameworks for Teaching practice based on Lesson Study incorporating Open Approach (Inprasitha, 2010 & 2011), and cognitive processes (Duval, 1998). The latter will be used for structuring and analyzing the data.

1. Traditional teaching

Traditional teaching methods are ineffective, discourage students to question what they have learned or to associate with the knowledge obtained previously, and seriously stunted growth of students' mathematical reasoning and problem solving skills because teaching is not allow for much questioning, investigating or individual development of understanding. Teacher works alone and does not provide students any opportunities to express their ideas. For instance, Mahira and Azamat (2013) stated that traditional method of teaching is teacher-dominated interaction. Teaching is deeply teacher-centered and teachers are the source of the knowledge, while learners are passive receivers that must memorize things. This approach puts emphasis on learning through acquiring information, and in this approach, the teacher's main responsibility is teaching students concepts that correspond to specific learning outcomes (Prosser & Trigwell, 1999; Howard, McGee, Schwartz, & Purcell, 2000; Doruk 2014). Students who taught by traditional approaches at the school level and in higher education settings have been shown that there are problems in applying knowledge, finding relevant in topics and the transfer of skills within and between the disciplines (Astin 1993; Hanson 2006; Plush & Kehrwald, 2014). And Traditional teachers are responsible for disseminating information to students. They look for the correct answer to know whether students learn what is taught (Yilmaz & Sahin, 2011).

Some limitations which may prevail in traditional teaching method are: 1) teaching in classroom using chalk and talk is "one way flow" of information, 2) teachers often continuously talk for an hour without knowing students response and feedback,

3) the material presented is only based on lecturer notes and textbooks, 4) teaching and learning are concentrated on "plug and play" method rather than practical aspects, 5) the handwriting of the lecturer decides the fate of the subject, 6) there is insufficient interaction with students in classroom, 7) more emphasis has been given on theory without any practical and real life time situations, 8) learning from memorization but not understanding, and 9) marks rather than result oriented.

2. Mathematical Problem Solving in classroom using Lesson Study and Open Approach

2.1 Mathematical Problem Solving

Problem solving is a principal component of mathematics education from the time of its emergency as a self – sufficient science until today (Voskoglou M.G., 2008). According to Schoenfeld (1983) a problem is only a problem (as mathematicians use the word) if you don't know how to go about solving it. A problem that has no "surprises" in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!) it is an exercise. In solving problem, Polya (1957) divided it in four steps: understanding the problem, divising a plan, carrying out the plan, and looking back. Therefore, Problem solving is a process that require solver visualize and understand virtual environment of the problem, find the connection between the data and the unknown, carry out plan of the solution, and examine the solution obtained.

The characteristics of problem solving are applied to this guide as follows:

- First, students can learn mathematical problem solving; it is neither an innate talent nor happenstance that creates skilled problem solvers.
- Second, mathematical problem solving is relative to the individual. What is challenging or non-routine for one student may be comparatively straightforward for a more advanced student.
- Third, mathematical problem solving need not be treated like just another topic in the pacing guide; instead, it can serve to support and enrich the learning of mathematics concepts and notation.
- Fourth, often more than one strategy can be used to solve a problem. Learning multiple strategies may help students see different ideas and

> approaches for solving problems and may enable students to think more flexibly when presented with a problem that does not have an obvious solution. (National Center for Education Evaluation and Regional Assistance [NCEERA], 2012)

In mathematics education, problem solving is refers to the process wherein students encounter a problem – a question for which they have no immediately apparent resolution, nor an algorithm that they can directly apply to get an answer (Schoenfeld, 1992) and is the basis of reform- and inquiry-based instruction in mathematics (Clark, 1997). The National Council of Supervisors of Mathematics (1989) defines problem solving as the process of applying previously acquired knowledge to new and unfamiliar situations. Additionally, problem solving is one of the five National Council of Teachers of Mathematics (2000) process standards. In recent years, therefore, researchers have studied problem solving and decision making with experienced subjects in knowledge-rich domains.

Problem solving plays a vital role in the study of mathematics making it not only a goal of learning mathematics but also the means in doing so. Problem solving is an integral part of all mathematics learning. Good problem will integrate multiple topics and will involve significant mathematics (NCTM, 2000). Problem-solving is cognitive processing aimed at accomplishing certain goals when the solution is unknown (Mayer & Wittrock, 1996). In cognitive informatics, problem solving is identified as a cognitive process of the brain at the higher cognitive layer that searches a solution for a given problem or finds a path to reach a given goal (Wang, 2007a). Problem solving is one of the fundamental cognitive processes modeled in the layered reference model of the brain model (Wang et al., 2006). It is recognized that there is a need to seek an axiomatic and rigorous model of the cognitive process of human problem solving in order to develop a solid and coherent theoretical foundation for integrating various theories, models, and practices of problem solving (Wang, 2007b).

Problem solving is one of the fundamental human cognitive processes; as a higher-layer cognitive process, problem solving interacts with many other cognitive processes such as abstraction, searching, learning, decision making, inference, analysis, and synthesis on the basis of internal knowledge representation by the object–attributerelation model (Wang & Chiew 2008). Problem solving is a cognitive process of the brain that searches a solution for a given problem or finds a path to reach a given goal. When a problem object is identified, problem solving can be perceived as a search process in the memory space for finding a relationship between a set of solution goals and a set of alternative paths (Wang Y. & Chiew V., 2010). Problem solving is an approach that can drives students access to their required.

2.2 Lesson Study

Lesson study, a form of collaborative practice, is a school-based professional development initiative that aims to enhance teaching and learning through the methodology of professional sharing of practice (Burghes & Robinson, 2009). Lesson Study is a format to build and analyze classroom teaching where teachers and researchers combine to design lessons, predict how the lessons might be expected to develop, then carry out the lessons with a group of observers bringing multiple perspectives on what actually happened during the lesson (Tall, 2015). Additionally, Lesson Study is a teacher-led instructional improvement cycle in which teachers work collaboratively to: formulate goals for student learning, plan a lesson, teach and/or observe the lesson, reflect on the gathered evidence, revise the lesson for improvement, reteach the revised lesson (Lewis, 2002). The process of lesson study consists of three phases: collaboratively design a research lesson; collaboratively observe the research lesson; and collaboratively discuss and reflect on the research lesson (Inprasitha, 2011).

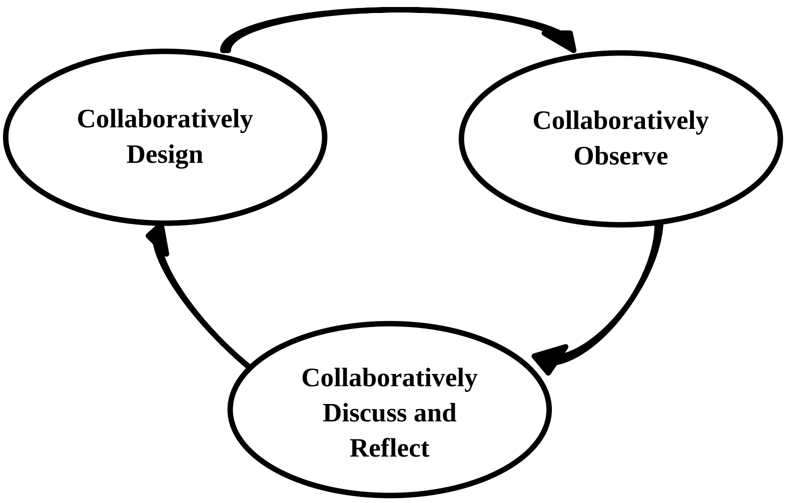

Figure 1: The process of lesson study (Inprasitha, 2011)

By using Lesson Study, teachers can view teaching and learning as they occur in mathematics classroom and it has the potential to build learning communities within schools and ultimately result in instructional improvement and increase in teachers' knowledge with focus on the student and the content (Meyer & Wilkerson, 2011). Lesson Study places teachers at the center of the professional activity with their interests and a desire to better understand student learning based on their own teaching experiences (Murata, 2011).

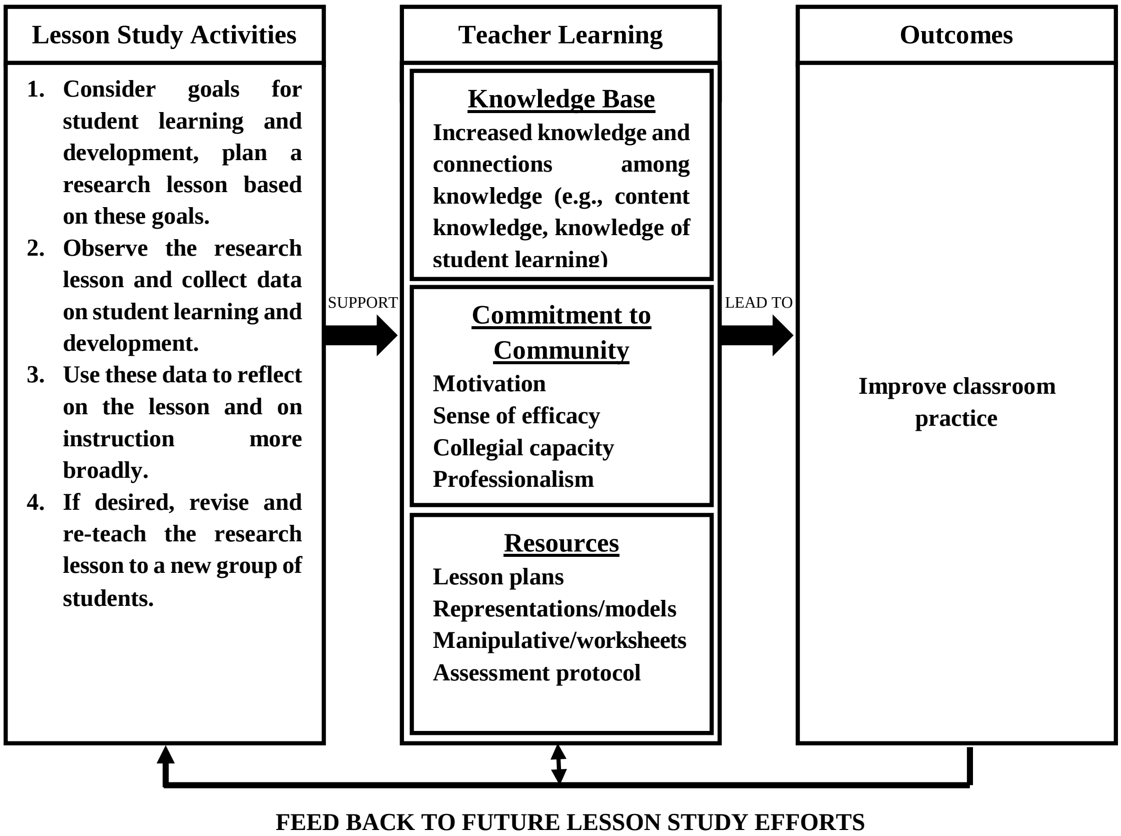

Figure 2: Lesson study activities, teachers learning, and outcomes (Murata et al. 2004)

Lesson Study is conducted by developing an annual pedagogical theme and forming teams for each subject and grade, and is employed as a strategy for cultivating teaching skills (Baba, 2007). Researchers have noted that lesson study embodies many features effective in changing teacher practice, such as using concrete practical materials to focus on meaningful problems, taking explicit account of the contexts of teaching and the experiences of teachers, and providing on-site teacher support within a collegial network (Takahashi, 2006). The Lesson Study was prominent including continuous and regular development focusing on classroom changes (Loipha & Inprasitha, 2004). According to this characteristic, some kinds of innovations to make changes were needed. These were carried-out by integrating into the topic to be developed continuously which was the approach of mathematics teaching model focusing on Open Approach (Thinwiangthong et al., 2012).

2.3 Open Approach

A teaching approach using Open Approach is aimed to provide the students an opportunity to learn mathematics in their own ways and respond to their abilities, and allow the students to negotiate a mathematical meaning with other students (Nohna, 2000). The Open Approach is a teaching approach used in cooperated with lesson study to design learning units and lesson plans and comprises the following four steps: posing open-ended problem, students' self-learning, whole class discussion and comparison of concepts, and summarization through connecting students' mathematical ideas emerged in the classroom (Inprasitha, 2010). This approach as problem solving approach used in Japan is one shared theory for developing children who learn mathematics by/for themselves (Isoda, 2010). According to the above mentioned, there is one feature for adaptive lesson study incorporating open approach as a teaching approach into lesson study process in order to create problem-solving classroom in Cambodia.

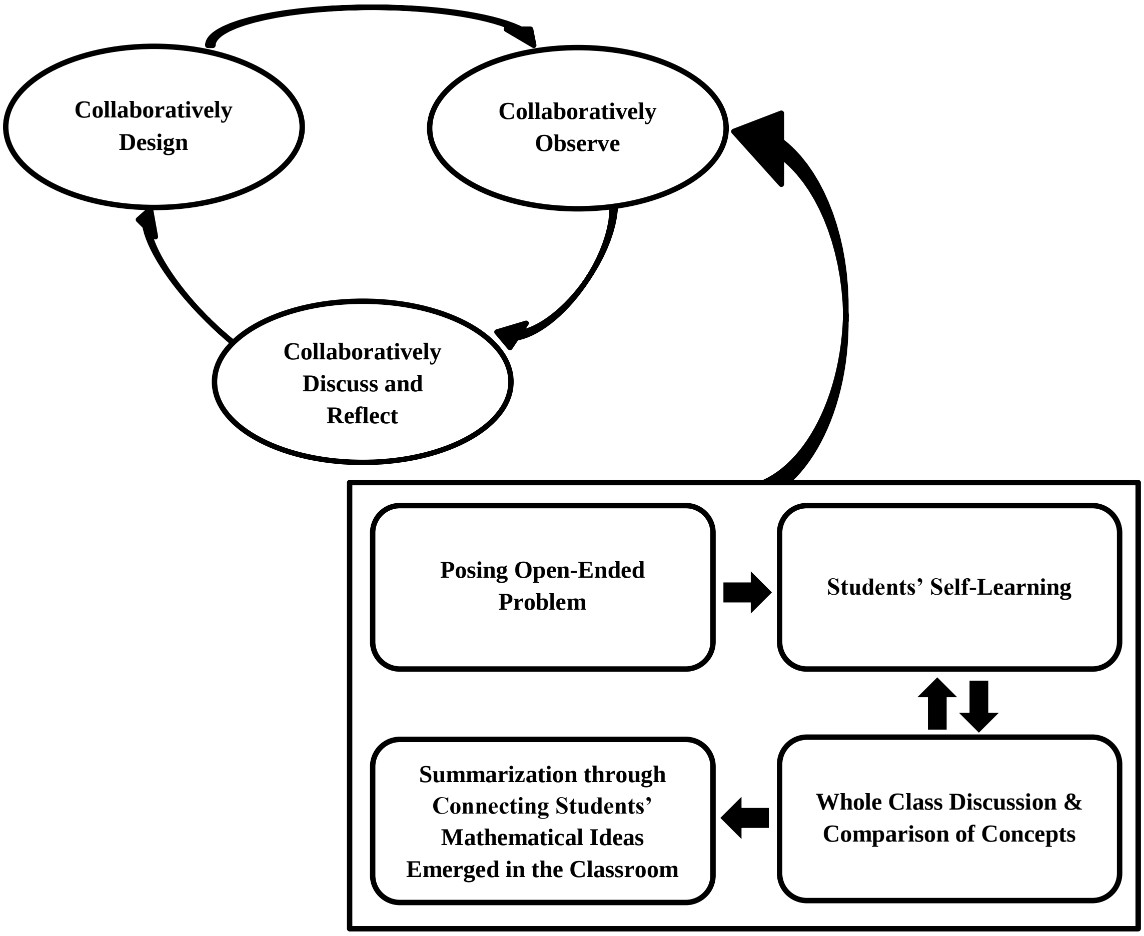

Figure 3: Teaching practice based on Lesson Study incorporating Open Approach (Inprasitha, 2010 & 2011)

3. Potentials and Benefits of using ICT, GeoGebra

Education and technology are accepted two key elements which are effective for the human life (Fidan, 2008). When considered from this point of view, education helps a person to discover his/her hidden, potential talent(Nisbet & LaGoy), to grow stronger, more mature, more creative and constructive as a person (Cavus, N. 2008). Technology helps mankind to be more efficient to use the knowledge and talent, which comes via education, and to use them more systematic consciously (Alkan, 2005). Thus Technology causes the changes on every part of life and these changes effect the structures and functions of education (Akpinar, 2003). The knowledge technology and their tools have important role to develop the process of education. They also provide full-time learning, knowledge management, simulation and visuality (Morrison, 2002). Knowledge technology and their tools are a base for learning environment (Inelmen, E.

2009). They provide society with opportunities of lifelong education, improving knowledge and opening up horizon, becoming skilful at their career (Cavas et.al, 2003).

Additionally, Oldknow and Taylor (2002) indicate that ICTs offer young children the ability to explore and solve problems involving large numbers at an early age, investigate characteristics of shapes using dynamic geometry software and organize large sets of data. They further indicate that ICTs offer educators options for adapting instructions to special needs of learners that is unlikely in a traditional mathematics class. The educator can always come with activities for different levels of learners that are in class, and learning occurs at the place of learners. ICT can impact on the quality of pupils' work, and on internal cognitive processes as well as upon internalization and externalization processes. In the US the National Council of Teachers of Mathematics (2000) considers technology as one of their six principles for school mathematics: 'Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. In England the Teacher Training Agency (TTA, 1999) offers a rationale for making use of ICT to support children's learning of mathematics.

In mathematics education, Mistretta (2005) has also found the benefits of using ICT to include mathematics education as follows:

- Promoting learners' higher order thinking skills.
- Developing and maintaining learners' computation and communication skills.
- Facilitating learners' algebraic and geometric thinking.
- Showing the learners the role of mathematics in an interdisciplinary.

Technology is the future of Education. We cannot live in denial and believe that we don't have to change. Mathematics requires thorough understanding to assist the critical thinking, reasoning, and logic. Technology can do that, allowing the students to see, graphically and visually, the complex ideas and topic. It also reduces repetitive and mindless computations. In in the new era of the 21st century, we have to allow students to learn and be proficient in technology. It's what they need in order to compete in the growing their mathematical knowledge and problem solving skills.

The ability to solve problems is a function of the nature of the problem, the way that the problem is represented to the solver, and a host of individual differences that mediate the process (Jonassen, 2000). Problem representation is key to successful problem-solving (Liu et al., 2004). In studying how computers can assist problem-solving, Tennyson and Breuer (2002) suggested using computer-based complex-dynamic simulations to improve problem solving. Jonassen (2003) suggested, "What makes experienced problem-solvers more effective is their richer, more coherent, and interconnected representations of problems" (p. 365). He suggested using cognitive tools for externalizing problems. According to Zhang and Norman's theory of distributed representations (1994), performing a cognitive task distributed representations across the internal mind and the external environment. External representations are not simply inputs and stimuli to the internal mind, but indispensable to the representational system of any distributed cognitive task (Zhang, 1997). In examining students' tool use patterns in a hypermedia environment in which cognitive tools were provided as a form of external representations, Liu and Bera (2004) found some empirical evidence to support this notion that external representations can guide, constrain, and determine certain cognitive behaviors. The growth of these communication and computer systems, their ease of use, the power and diversity of information transfer allow teachers and students to have access to a world beyond the classroom. It has the potential to transform the nature and process of the learning environment and envision a new learning culture. Interactivity, flexibility and convenience have become the order of the day in the ICT supported environment. ICT opens up opportunities for learning because it enables learners to access, extend, transform and share ideas and information in multi-modal communication styles and format. It helps the learner to share learning resources and spaces, promote learner centered and collaborative learning principles and enhance critical thinking, creative thinking and problem solving skills. Congruently, GeoGebra is an appropriate cognitive tools can assist the students in performing complex open-ended problem-solving tasks more like experts.

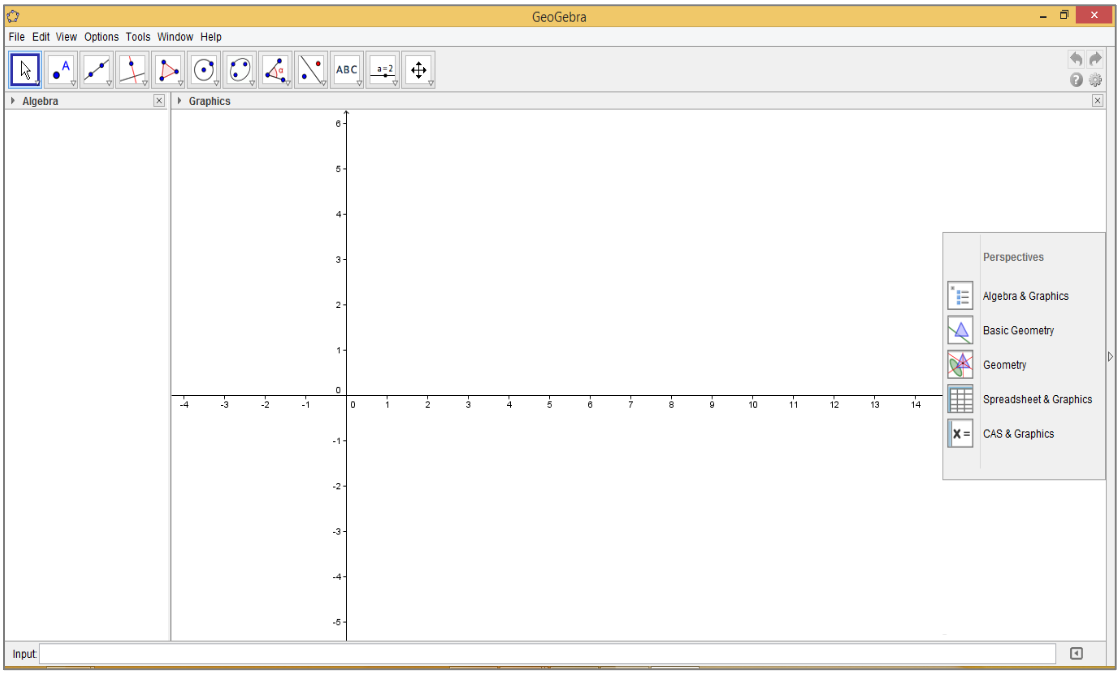

Figure 4: GeoGebra

The software GeoGebra is a technology tool which is originated in the Master's thesis project of Markus Hohenwarter at the University of Salzburg in 2002. During the past years, GeoGebra has developed into an open-source project with a group of 20 developers and over 100 translators across the world. The latest version of GeoGebra offers dynamically linked multiple representations for mathematical objects (Hohenwarter & Jones 2007) through its graphical, algebraic, and spreadsheet views. GeoGebra, which is currently available in 50 languages, has received several educational software awards in Europe and the USA (e.g. EASA 2002, digita 2004, Comenius 2004, eTwinning 2006, AECT 2008, BETT 2009 finalist, Tech Award 2009, NTLC Award 2010).

GeoGebra is a Dynamic Mathematics Software for teaching and learning mathematics that combines many aspects of different mathematical packages (Hohenwarter and Lavicza, 2007). GeoGebra is a cognitive tool to explore and track students' mathematical thinking process. For instance, International GeoGebra Institute (2015) states that GeoGebra is a multi-platform mathematics software that gives everyone the chance to experience the extraordinary insights that math makes possible:

- *It makes math tangible.* GeoGebra makes a link between geometry and algebra in an entirely new, visual way – students can finally see, touch, and experience math.
- *It makes math dynamic, interactive and fun.* GeoGebra teaches students math in a new and exciting way that goes beyond the blackboard and leverages new media.
- *It makes math easier to learn.* GeoGebra creates the interactions that students need in order to “absorb” mathematical concepts.

GeoGebra as a Dynamic Geometry Software can be used as an effective tool in learning by way of visualization to promote learning and enhance understanding.

4 Cognitive processes

A process in the brain which does information processing and/or information retrieval and/or information storage in the brain is a cognitive process (Renumol et al. 2010). Key cognitive processes underlying the use of the elimination strategy are (1) the interpretation of the available information in the context of other information and the problem scenario, (2) the identification of relevant information, and (3) the use of data to draw appropriate causal inferences and conclusions (Chung et al. 2002). Students are aware of their cognitive processes and are able to report them with sufficient accuracy to provide valuable information concerning how they learn from teaching; by discovering more about the manner in which cognitive processes facilitate achievement, it is possible to provide teachers with information concerning how they can best approach the task of designing a learning environment conducive to effective use of those processes (Solmon & Lee, 1996). Duval (1998) defined cognitive processes in three types of processes as follows:

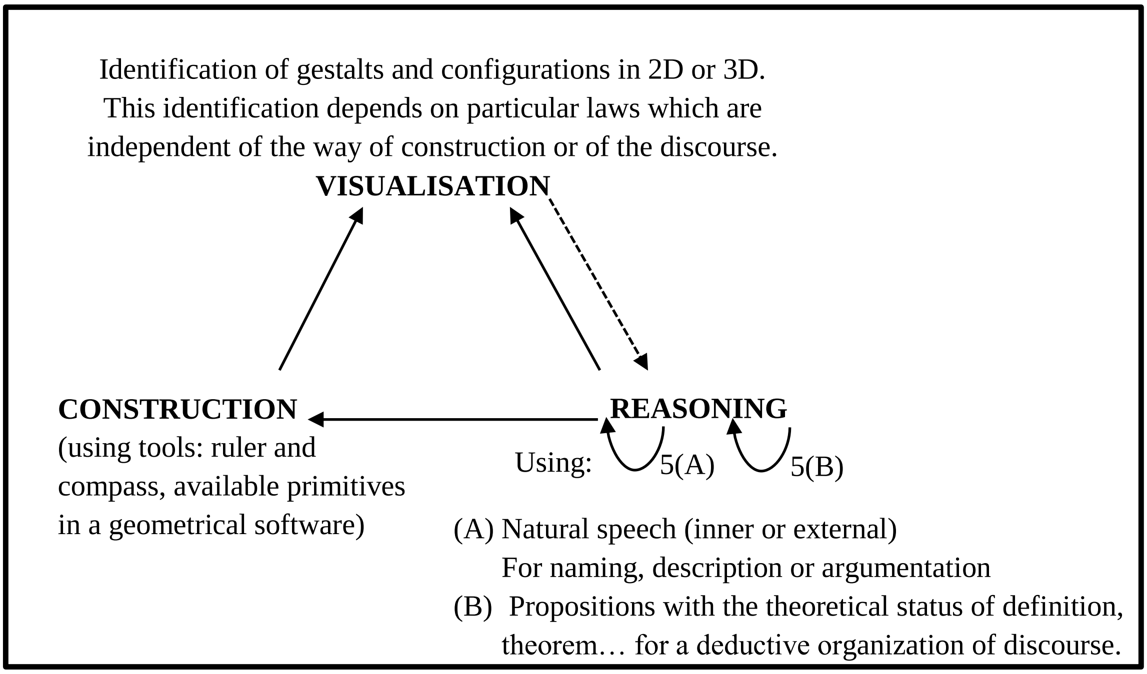

Figure 5: The underlying cognitive interactions involved in geometrical activity

4.1 Visualization processes

Visualization process with regard to visual representation for the geometrical statement, for the heuristic exploration of a complex situation, for a subjective verification (Duval, 1998). Visualization is a skill that helps students to recognize shapes, to create new shapes or objects, and to reveal relationships between them (Arcavi, 2003). Visualization provides students with activities that are aided by visual representations, such as models, animation, and graphs, in order to make the abstract more concrete and to clarify scientific principles and concepts (Barak & Dori, 2005). Visualization can help teachers and students present thoughts, discuss different ideas, and share knowledge. Students can learn from building, critiquing, and manipulating physical and virtual models (Barak, 2011). In this way, it is possible to encourage learners to develop their understanding of scientific ideas through the creation of mental models (Tversky, 2005). Many studies have shown about relationships between visualization and mathematical problem solving (Moses, 1977; Van Garderen & Montague, 2003), as well as between visualization and mathematics achievement (Fennema & Sherman, 1977; Ünal et al., 2009). Visualization is generally considered helpful in supporting intuition and concept formation in mathematics learning (Drefus, 1991), mathematical comprehension (Borba & Villarreal, 2005). To reach that comprehension, it is necessary to take into account that visualization, which is usually associated with graphical representations, occurs not as an isolated topic, but inside a mathematical context that also includes numerical and symbolic representations (Zimmermann and Cunningham, 1991).

4.2 Reasoning processes

Reasoning in relationship to discursive processes for extension of knowledge, for proof, for explanation (Duval, 1998). Reasoning is a skill that is demonstrated during the advanced stages of thought (Umay, 2003), in other words, during problem-solving processes (Yıldım, 2000), and which represents high-order mathematical thinking (Kenney & Lindquist, 2000). Webster (1982) also defines reasoning as “the ability to think coherently and logically and draw inferences or conclusions from facts known or assumed” (cited in Mansi, 2003:9). Reasoning skills are an important component of education, and reasoning skills are necessary for understanding mathematics in particular, and they present an important means of developing ideas (NCTM, 2000). Mathematical reasoning refers to the ability to formulate and represent a given mathematics problem, and to explain and justify the solution or argument (Kilpatrick, Swafford & Findell, 2001). Individuals may use more than one reasoning approach in problem-solving situations. Moreover, their ability to use these approaches and reasoning skills is closely related to the depth of their conceptual knowledge and to their corresponding associative skills (Briscoe & Stout, 2001; Gerald, 2002; Lithner, 2000). In addition, reasoning is a process that provides depth to the existing body of knowledge (Duval, 1998). Toole (2001) and Kramarski; Mevarech and Lieberman (2001) have emphasized that a direct relation exists between reasoning skills and success in mathematics, where individuals who demonstrate better reasoning skills display good problem-solving profiles with the interrelations they are able to identify, while also having better communication skills. Through reasoning, students could develop an appreciation of mathematical justification in the study of the mathematical content (Kaur and Lam, 2012). Reasoning, in general, involves inferences that are drawn from principles and from evidence, whereby the individual either infers new conclusions or evaluates proposed conclusions from what is already known (Johnson-

Laird & Byrne, 1993). Findings from these studies indicate that reasoning is a cognitive process that provides students depth to the existing body of knowledge and make students closely related to the depth of their conceptual knowledge and to their corresponding associative skills.

This research project to a large extent was inspired by Alex and Mammen (2012) interpretation of the van Hiele theory, and study on van Hiele model of thinking and conception in geometry and the researchers' experiences and concerns regarding the poor geometry performance of learners in mathematics classroom in Cambodia. According to NCTM (2000) document suggests that instructional programs in mathematics should pay attention to geometry and spatial sense so that, amongst other things, learners use visualization and spatial reasoning to solve problems both within and outside of mathematics, therefore, this study seek to find the level of geometric thinking of the learners by using the Duval framework together with the results of subsequent research as a framework in determining the Duval cognitive processes of learners in a selected classroom secondary schools.

4.3 Construction processes

Construction processes by tools: construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented (Duval, 1998).

In this research study, researchers will use the conceptual framework of Duval (1998) to analyze students' cognitive processes: visualization processes, construction processes, and reasoning processes. These three types of cognitive processes can be performed separately.

CHAPTER III
RESEARCH METHODOLOGY

The present study adopts of Open Approach incorporating Lesson Study as a means to encourage the innovative use of GeoGebra that considers competence and knowledge to be skills that students develop through interaction within a social setting. This study emerges from the idea that visualization, construction, and reasoning are beneficial to students' learning. The framework of this study merges these perspectives and explores the benefits of collaborative problem-solving activities as well as students' individual cognitive processes. Because the aim of the study is to analyze students' cognitive processes in mathematics classroom using Lesson Study and Open Approach with GeoGebra. Furthermore, the data collection needs to capture students' language and actions to examine their interactions with GeoGebra, the visualization, and the construction and, finally, their reasoning as articulated through spoken arguments and field note(s). The analysis will be conducted using theoretical concepts involving teaching practice based on Lesson Study incorporating Open Approach (Inprasitha, 2010 & 2011), and students' cognitive processes (Duval, 1998).

1. Target Group

Target group will be one mathematics classroom of secondary school in Phnom Penh, Cambodia. The teachers will use Lesson Study incorporating Open Approach to be teaching approach. There is instruction for students to reveal thinking concept during doing mathematical activity and to create and do teaching material together.

3. Research Instruments

In this study, the required instruments to collect data are field note, video- and audio recorders. The video- and audio-tape recorder for recoding all activities in accordance to 3 steps of Lesson Study: collaboratively plan, do, and see. Instrument will use to analyze cognitive processes of students is the conceptual framework of Duval (1998).

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4. Data Collection

The following procedure will adopt to conduct the analyze:

The permission of school principle will do obtain, who directed the class teacher to provide the necessary assistance. The researcher will meet the subject teacher with the request to provide information about what subject matter the students have already studied in mathematics and what content they had to study in the next.

In the light of the above, it is decide that the topic will could be easily taught during the period of the study span. The achievement lesson plans for teaching purposes will design accordingly. The lesson will teach to the students in the light of lesson plans prepared by the analyst and teacher, using the format and the procedure of problem solving.

Students will divide into small groups. The students would sit in their assigned groups on each class day, and after a brief introduction to the problem by using GeoGebra, start working on the problem. The small group interactions and the class discussions are means to help students to build their cognitive processes. Researcher listen to and note students' discussion.

Data will collect and analyze qualitatively. It will collect via notes from students' work and discussion during class, and through reflective notes that researcher write down at the end of the class. This study centered on the engagement in the learning activities as target task(s). To collect the data on the students' cognitive processes, the occurrences in classroom in the learning step will be videotaped, and the analysis also will be concerned field notes, the teachers and the observers' interviews and students' performance analysis for explaining as well as interpreting the students' speaking and action.

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