A Model Based Channel Shortening Technique for IEEE 802.11a OFDM System

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Abstract—A new computationally efficient channel shortening technique, to make the OFDM systems robust for delay spreads exceeding the cyclic prefix is presented. The proposed technique gives same BER performance with shorter equalizer. The method is based on pole-zero modeling of the channel impulse response. This gives low computational complexity, both during initialization and data transmission. Moreover no delay search is required as compared with other methods. IEEE 802.11a is considered as underlying OFDM system.

I. INTRODUCTION
High speed data applications over wireless channels is restricted by the frequency-selective fading. Orthogonal Frequency Division Multiplexing (OFDM) is a technique, which inherently converts frequency-selective fading channels to flat fading channels. Thus OFDM allows high speed data transmission, with no or minimal equalization. Moreover, OFDM systems can be implemented digitally using DFT/IDFT. These advantages make OFDM suitable for various wireless systems such as WLAN (IEEE 802.11a and HiperLAN/2), Digital Audio and Video Broadcasting (DAB/DVB).

OFDM employs a time domain guard interval between successive symbols, duration of which must be equal to channel memory. This guard interval minimizes the equalization requirements, down to a single tap equalizer per subcarrier. A guard interval of length $L_g$ is sufficient for the channel of length $L_g + 1$. The guard interval is cyclic prefix (CP) of OFDM symbol, consisting of last $L_g$ samples. This equalization structure of OFDM fails, for the channel length more than CP. Moreover, the CP carries no useful information, hence it also reduces system efficiency. For the FFT size of $N_s$, using CP of length $L_g$ reduces system efficiency to $\frac{N_s}{N_s+L_g}$. IEEE 802.11a OFDM system, allows a CP length equal to one fourth of symbol duration. So, for a specified Bit Error Rate (BER) criterion, the CP can be sufficient but reducing the system efficiency, or CP is not sufficient hence increasing BER.

To avoid both the above cases, shortening of channel impulse response is required. There are various approaches proposed in literature for channel shortening (CS). For multicarrier systems, a Minimum Mean Square error (MMSE) based CS technique was proposed by Cioffi and Dhahir in [1]. This method finds the optimum Target Impulse Response (TIR) of desired length, and from it calculates the optimum CS equalizer coefficients. Moreover, an optimum synchronization delay search is also required. Melsa et al [2] proposed a technique based on, maximization of a performance measure namely, Shortening Signal to Noise Ratio (SSNR). In this method need for finding TIR was eliminated. The desired equalizer is found by, maximizing the ratio of energy in desired CP window to the energy outside the window i.e. SSNR. Both of these methods require to solve the eigenvalue problem.

A detailed and unified analysis of MMSE and MSSNR techniques is given by Martin et al in [3]. They made observation that performance of all the techniques varies only 10%, while their computational complexity varies by large amounts. Moreover, the complexity of training based methods, depend on computations required both during training and data transmission phases. These computations depends mainly on length of equalizer. This paper proposes a new Model Based Shortening (MBS) technique, which gives same performance (BER) as other techniques, but with much shorter equalizer. This results in low computational complexity during training & data transmission phase.

Remaining of the paper is organized as follows: section II gives the OFDM system model for IEEE 802.11a, section III presents the proposed model based channel shortening technique, section IV gives simulation results and the paper is concluded in section V.

II. OFDM SYSTEM MODEL
This paper considers IEEE802.11a as underlying OFDM model. It utilizes 20 MHz bandwidth with 64 subcarriers. Figure (1) shows the block diagram of the baseband system model. It mainly consists of data scrambler, convolutional encoder with varying coding gain, two stage data interleaver, different PSK and QAM modulation schemes, and 64-point IFFT/FFT block. After IFFT a cyclic prefix of desired length is added. At the receiver cyclic prefix is removed before FFT. A single tap equalizer follows FFT block to equalize the channel. Other operation are complementary to transmitter side operations. When using a CS equalizer it precedes the CP removal block. The implementation detail can be obtained from [4].

III. MODEL BASED CHANNEL SHORTENING
This technique is based on modeling of the channel impulse response. The key idea is to obtain the pole-zero model of the channel impulse response, and take denominator coefficients as the equalizer coefficients, while keeping number of zeros equal to desired CP length. Consider the original channel impulse response $h(n)$ or $(H(z)$ in $z$ domain), is modeled as the impulse response of a pole-zero system ($\frac{B(z)}{A_p(z)}$), with $p$ poles and $q$
The error in (3) can be expressed as,
\[ e(n) = h(n) + \sum_{l=1}^{P} a_p(l)h(n-l) - b_q(n) : 0 \leq n \leq q \]
\[ e(n) = h(n) + \sum_{l=1}^{P} a_p(l)h(n-l) : n > q \quad (4) \]
Since we have to find only poles of the model, and error in (4) depends only on the poles, we choose to find \( a_p(k) \) by minimizing square of this error, using least squares criterion. Consider length of the equalizer as \( (L_e = p + 1) \), length of the channel as \( L_c \), and length of the CP as \( (L_q = q) \). The shortened impulse response has the length \( (N = L_c + L_c - 1) \). With this the squared error \( \xi \) is given as:
\[ \xi = \sum_{n=L_q+1}^{N-1} e(n)e^*(n) \]
With equalizer coefficients \( (w(n)) \) being same as poles \( a_p(n) \) of the model, optimum equalizer coefficients can be found by setting the partial derivatives of \( \xi \) with respect to \( w^*(k) \) equal to zero i.e.
\[ \frac{\partial \xi}{\partial w^*(k)} = \sum_{n=L_q+1}^{N-1} \frac{\partial e(n)e^*(n)}{\partial w^*(k)} = \sum_{n=L_q+1}^{N-1} e(n)\frac{\partial e^*(n)}{\partial w^*(k)} = 0 \quad : k = 1, 2, \cdots, L_e - 1 \quad (5) \]
From Eq. (4) the derivative of \( e(n) \) with respect to \( w^*(k) \) is \( h^*(n-k) \). Hence Eq. (5) becomes:
\[ \sum_{n=L_q+1}^{N-1} e(n)h^*(n-k) = 0 \quad ; \quad k = 1, 2, \cdots, L_e - 1 \quad (6) \]
This is known as Orthogonality Principle. Substituting \( e(n) \) from Eq. (4) into Eq. (6), gives us:
\[ \sum_{n=L_q+1}^{N-1} \left[ h(n) + \sum_{l=1}^{L_c-1} w(l)h(n-l) \right] h^*(n-k) = 0 \quad ; \quad k = 1, 2, \cdots, L_e - 1 \]
Above equation can be modified as:
\[ \sum_{l=1}^{L_e-1} w(l) \sum_{n=L_q+1}^{N-1} h(n-l)h^*(n-k) = - \sum_{n=L_q+1}^{N-1} h(n)h^*(n-k) \quad : k = 1, 2, \cdots, L_e - 1 \quad (7) \]
By defining autocorrelation of the channel as
\[ \sum_{n=L_q+1}^{N-1} h(n-l)h^*(n-k) = r_h(k,l) \quad (8) \]
Eq. (7) can be written as:

$$\sum_{l=1}^{L_e-1} w(l)r_h(k, l) = -r_h(k, 0) : \quad k = 1, 2, \cdots, L_e - 1 \quad (9)$$

Autocorrelation function is conjugate symmetric i.e.

$$r_h(k, l) = r_h^*(l, k)$$

Using this Eq. (9) can be written in matrix form as:

$$\begin{bmatrix}
    r_h(1, 1) & r_h^*(2, 1) & \cdots & r_h^*(L_e - 1, 1) \\
    r_h(2, 1) & r_h(1, 1) & \cdots & r_h^*(L_e - 2, 1) \\
    \vdots & \ddots & \ddots & \vdots \\
    r_h(L_e - 1, 1) & r_h(L_e - 2, 1) & \cdots & r_h(1, 1)
\end{bmatrix}
\begin{bmatrix}
    w(1) \\
    w(2) \\
    \vdots \\
    w(L_e - 1)
\end{bmatrix}
= -
\begin{bmatrix}
    r_h(1, 0) \\
    r_h(2, 0) \\
    \vdots \\
    r_h(L_e - 1, 0)
\end{bmatrix}$$

or,

$$Rw = -r$$

Eq. (10) is a set of \((L_e - 1)\) linear equations in \((L_e - 1)\) unknowns, with matrix \(R\) being Hermitian. The optimum equalizer coefficients can be found as:

$$w = -R^{-1}r \quad (11)$$

Solving this equation results in \((w(k) : k = 1, 2, \cdots, L_e - 1)\), and considering that coefficients are normalized \((w(0) = 1)\) we get the length \(L_e\) equalizer.

With this formulation the MBS method can be summarized as follows:

1) For a given length of the equalizer(\(L_e\)) determine the autocorrelation matrix\((R)\) and vector \((r)\) as in (10) using (8).

2) Find the inverse of \(R\) and solve for \((w(k) : k = 1, 2, \cdots, L_e - 1)\) using (11).

3) With \((w(0) = 1)\) get the length \(L_e\) equalizer.

In contrast with MMSE [1] and MSSNR [2] methods, this algorithm does not requires any delay search. In MMSE to synchronize the TIR and equalizer outputs optimum delay value is required to be found. The optimum delay search is required in MSSNR method also. But in proposed method, the shortened impulse response starts with first sample. So, no delay search is required. An example of Modeling Based Shortening(MBS) method is shown in Figure (3).

\section*{A. Computational complexity}

A long impulse response can be modeled as an IIR with very few coefficients(poles and zeros). In modeling based shortening, channel impulse response is modeled as an IIR system. Hence, the number of poles(number of zeros are fixed) required are very few, consequently the equalizer length is also short. To find the computational complexity of this algorithm, we count the number of MAC(Multiply and Accumulate) operations.

Given length of equalizer \(L_e\), channel length \(L_c\) and guard length \(L_g\), there are two main steps in model based shortening algorithm. First is to determine the Hermitian autocorrelation matrix \(R\), which requires \(L_c^2(L_c - L_g)\) operations. Second is to solve the equation (10). Considering various methods given in [6] to solve such equations, they require an order of \(L_c^3\) operations. So total number of operations required are:

$$N_{op} = L_c^3 + \frac{L_c^2(L_c - L_g)}{2} \quad (12)$$

\section*{IV. SIMULATION RESULTS}

Simulations in MATLAB are done to examine the performance of channel shortening techniques discussed. Multipath channel models proposed by Joint Technical Committee(JTC) [7] and European Telecommunication Standards Institute(ETSI) [8] are considered. The noise is considered to be Additive White Gaussian Noise(AWGN). The data rate is 54 Mbps. For sampling rate of 20 MHz, CP length is 16 samples of each OFDM symbol.

\textbf{Case I:} For the case of \(CP\) insufficient, an ETSI defined channel E of length 36 is considered. The channel is shortened to 16 length. This decreases the BER as seen in Figure (4). From the figure, it can be observed that shortening methods performs equally well. The MMSE and MSSNR requires 32 length equalizer, while MBS requires 12 length equalizer.

\textbf{Case II:} For the case of \(CP\) sufficient, \(CP\) length requirement can be relaxed by shortening the channel. Performance of OFDM for ETSI channel B of length 17, with \(CP\) length of 16, is shown in Figure (5). To increase the system efficiency \(CP\) of length 8 is used. At the receiver CS equalizer to shorten the channel to length 8 is used. This gives same BER performance as full CP, but now 60 Mbps data rate can be achieved. For this MMSE and MSSNR requires 20 length equalizer, but MBS requires 4 length equalizer. Similarly, simulation is done for JTC defined channel B of length 15. For this CP of length 4 is used. From Figure (6), it can be observed that this gives
same BER as full CP. But, now we can send data at 64 Mbps. For this MMSE and MSSNR require 32 length equalizer while MBS require 15 length equalizer. The number of MAC operations required for the above cases is shown in Table I and corresponding length of equalizers are in Table II. For MMSE and MSSNR complexity equations given in [3] are used.

V. CONCLUSIONS

A channel shortening equalizer can be used, to make the OFDM systems robust, against excessive channel delay spread. As compared to MMSE and MSSNR shortening methods MBS requires much shorter length equalizer. As a consequence of which complexity of equalization, during data transmission, can be reduced by 50-75%, as shown in Table II. The computational complexity during initialization also reduced accordingly, as shown in Table I.

REFERENCES


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<th>Case</th>
<th>CP length</th>
<th>MMSE</th>
<th>MSSNR</th>
<th>MBS</th>
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<td>Case II</td>
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<td>3.3 × 10^5</td>
<td>6.8 × 10^5</td>
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