Inverse QR 2D-RLS Adaptive Channel Estimation for OFDM Systems

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SUMMARY This letter proposes Inverse QR two-dimensional Recursive Least Square (IQR-2D-RLS) adaptive channel estimation for Orthogonal Frequency Division Multiplexing (OFDM) systems (using Givens Rotations and Householder Transformations). It is more stable numerically than 2D-RLS algorithm. MATLAB simulations show that BER performance of IQR-2D-RLS algorithm is similar to that of 2D-RLS algorithm.

key words: OFDM, 2D-RLS, IQR-2D-RLS

1. Introduction

OFDM is seen as one of the most promising solutions to broadband wireless communications. Its performance depends on the channel state information (CSI) which is estimated using pilot symbols (preambles). In OFDM systems, two-dimensional Minimum Mean-Squared Error (2D-MMSE) channel estimation in frequency and time domain is optimum, if the noise is additive [1]. However, 2D-MMSE channel estimation requires more pilot symbols if weak channel correlation exists between the data and pilot symbol transmission which increases computational complexity (O(N^2) where N is the order of the filter) as well as signal overhead [2].

In [1], two-dimensional Recursive Least Square (2D-RLS) adaptive channel estimation for OFDM systems is proposed. Unlike 2D-MMSE, it does not require accurate channel statistics. With properly chosen parameters, it converges in several OFDM symbols’ time only and requires pilot symbols only during training period. However, this algorithm diverges and becomes unstable when the inverse of input auto-correlation matrix loses the property of positive definiteness or Hermitian symmetry.

In this letter, Inverse QR two-dimensional Recursive Least Square (IQR-2D-RLS) adaptive channel estimation for OFDM systems has been proposed. Instead of propagating the inverse of input auto-correlation matrix (\(\phi^{-1}(n)\), explained in Sect. 3), it propagates the inverse square-root of input auto-correlation matrix (\(\phi^{-1/2}(n)\), described in Sect. 4). The unitary matrix \(\Theta(n)\) required in QR decomposition is calculated using Givens Rotations and Householder Transformations. IQR-2D-RLS algorithm guarantees the property of positive definiteness and is numerically more stable than 2D-RLS algorithm. However, the computational complexity for both algorithms is similar [3].

The rest of the letter is organized as follows: In Sect. 2 OFDM channel model is described. Section 3 explains 2D-RLS algorithm. Section 4 describes IQR-2D-RLS adaptive channel estimation. In Sect. 5, the stability analysis for IQR-2D-RLS algorithm has been done. Section 6 shows computer simulations for 2D-RLS and IQR-2D-RLS algorithms. Conclusions can be found in Sect. 7.

2. OFDM Channel Estimation

Consider an OFDM system working in a time-varying multi-path Rayleigh fading environment. Initially, \(N_c\) input samples are taken and BPSK modulated. If \(K\) subcarriers are considered for OFDM then the BPSK samples are grouped in symbols of length \(K\). Since the channel is assumed to be slow time-varying, it is considered constant for \(M\) OFDM symbols. The first ‘L’ symbols (preambles) are used for the initial ‘L’ Least Square (LS) estimates of the channel, using which the input vector \(P(n)\) is constructed [4].

OFDM symbol length is less than the coherence time of slow fading frequency selective channel therefore, intercarrier-interference (ICI) can be neglected. Now, the OFDM system can be described as set of parallel Gaussian channels. Accordingly, at transmission time \(n\), the received signal on the \(k\)th sub-carrier can be expressed as

\[
Y(n, k) = H(n, k)X(n, k) + N(n, k)
\]

where \(X(n, k)\) represents transmitted signal on \(k\)th subcarrier at time \(n\) and \(N(n, k)\) represents FFT of additive complex Gaussian noise with zero mean and variance \(\sigma^2\), which is uncorrelated for different \(n\) or \(k\).

For the initial \(M\) input OFDM symbols, first ‘L’ symbols are preambles and the rest are data symbols. The output \(Y(n-1, k)\) for the first preamble \(X(n-1, k)\), after removal of CP and taking \(K\)-point FFT, is used for first LS estimate 
\(\tilde{H}(n-1, k)\) of the channel as

\[
\tilde{H}(n-1, k) = \frac{Y(n-1, k)}{X(n-1, k)}
\]

Using similar approach, other LS channel estimates \(\tilde{H}(n-1, k)\) are calculated.

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2, k), \( \hat{H}(n-3, k) \) ... \( \hat{H}(n-L, k) \) are obtained. The ‘L’ LS estimates are stored in \( LK \times 1 \) input vector as
\[
P(n) = (\hat{H}(n-1, 1) \ldots \hat{H}(n-1, K) \ldots \hat{H}(n-L, 1) \ldots \hat{H}(n-L, K))^T
\]  
and a \( K \times 1 \) reference vector \( \hat{H}_{ref}(n) \) is constructed as
\[
\hat{H}_{ref}(n) = (\hat{H}(n-1, 1) \hat{H}(n-1, 2) \ldots \hat{H}(n-1, K))^T
\]  
where \([.]^T\) represents transpose.
Using the vectors \( P(n) \) and \( \hat{H}_{ref}(n) \), IQR-2D-RLS adaptive channel estimation algorithm is used for estimating the channel \( \hat{H}(n) \). Input estimation for these \( M \) symbols is done using new channel estimate \( \hat{H}(n) \). For every next \( M \) OFDM symbols, input is estimated using the previous channel estimate and Decision Quantizer [5] maps the input estimates to the corresponding BPSK symbols. Using these input estimates, new ‘L’ LS estimates of channel are obtained, \( \hat{H}_{ref}(n) \) and \( P(n) \) are updated and passed to the adaptive channel estimation algorithm for constructing new channel estimate \( \hat{H}(n) \).

3. 2D-RLS Algorithm

Given vectors \( \hat{H}_{ref}(n) \) and \( P(n) \), the channel estimation in general form is defined as [1]
\[
\hat{H}(n) = \frac{G(n)^H P(n)}{e(n) = \hat{H}_{ref}(n) - \hat{H}(n)}
\]  
where \( \hat{H}(n) \) is estimation of \( H(n) \), \( G(n) \) is \( LK \times K \) weight coefficient matrix and \((.)^H\) represents Hermitian transpose. So the channel estimation error is
\[
e(n) = \| G(n) \|^2_f + \delta \lambda^n \| G(n) \|^2_f
\]  
where \( \lambda \) is the exponential weighting factor with a positive constant value less than unity, \( \delta \) is a positive real number called regularization parameter and \( \| . \|^2_f \) represents the squared Frobenius norm of a matrix.

Deriving gradient vector of cost function (7) with respect to \( G(n) \) and manipulating it to zero, gives the result
\[
\Phi(n) G(n) = \Psi(n)
\]  
where
\[
\Phi(n) = \sum_{i=1}^{n} \lambda^{n-i} P(i) P^H(i) \delta \lambda^n I
\]
\[
\Psi(n) = \sum_{i=1}^{n} \lambda^{n-i} P(i) \hat{H}_{ref}(i)
\]  
having \( I \) as \( LK \times LK \) identity matrix. Equations (9) and (10) can be rewritten as
\[
\Phi(n) = \lambda \Phi(n-1) + P(n) P^H(n)
\]
\[
\Psi(n) = \lambda \Psi(n-1) + P(n) \hat{H}_{ref}(n)
\]
Assuming \( \Phi(n) \) to be non-singular,
\[
G(n) = \Phi^{-1}(n) \Psi(n)
\]  
For further simplicity of description, a \( LK \times LK \) matrix \( Q(n) \) is defined as
\[
Q(n) = \Phi^{-1}(n)
\]  
According to matrix inversion lemma [6], it can be derived
\[
Q(n) = \lambda^{-1} Q(n-1) - \lambda^{-1} k(n) P^H(n) Q(n-1)
\]  
where \( k(n) \) is called the gain vector
\[
k(n) = \frac{\lambda^{-1} Q(n-1) P(n)}{1 + \lambda^{-1} P^H(n) Q(n-1) P(n)}
\]  
Also, \( k(n) \) can be defined as
\[
k(n) = Q(n) P(n)
\]  
Substituting (14), (15) and (17) into (13), defines \( G(n) \) as
\[
G(n) = G(n-1) + k(n) \xi^H(n)
\]  
where \( \xi(n) \) is the priori estimation error given as
\[
\xi(n) = \hat{H}_{ref}(n) - G^H(n-1) P(n)
\]  
4. Inverse QR-2D-RLS Algorithm

In this section, IQR-2D-RLS adaptive channel estimation algorithm has been established. Considering the recursive equations for \( Q(n) \) and \( k(n) \), update equation (15) can be rewritten as
\[
Q(n) = \lambda^{-1} Q(n-1) - \lambda^{-1} Q(n-1) P(n) \lambda^{-1} P^H(n) Q(n-1)
\]  
where \( r(n) = 1 + \lambda^{-1} P^H(n) Q(n-1) P(n) \).
Arranging terms in RHS of the equation (20) in matrix form \( A(n) \) gives
\[
A(n) = 
\begin{bmatrix}
1 + \lambda^{-1} P^H(n) Q(n-1) P(n) & \lambda^{-1} P^H(n) Q(n-1) \\
\lambda^{-1} Q(n-1) P(n) & \lambda^{-1} Q(n-1)
\end{bmatrix}
\]  
Now, \( A(n) \) can be partitioned and written as product of two square-root cholesky factors [6], on which IQR-2D-RLS algorithm operates, as
\[ A(n) = \begin{bmatrix}
1 & \lambda^{-1/2} P^H(n) Q^{1/2}(n-1) \\
0 & \lambda^{-1/2} Q^{1/2}(n-1)
\end{bmatrix}
\times \begin{bmatrix}
1 \\
\lambda^{-1/2} Q^{H/2}(n-1) P(n) \\
\lambda^{-1/2} Q^{H/2}(n-1)
\end{bmatrix}
\] (21)

Using matrix factorization lemma [6] on first product term in (21) yields
\[ \Theta(n) = \begin{bmatrix}
1 & \lambda^{-1/2} P^H(n) Q^{1/2}(n-1) \\
0 & \lambda^{-1/2} Q^{1/2}(n-1)
\end{bmatrix}
\]
\[ r^{1/2}(n) \\
k(n) Q^{1/2}(n)
\]
(22)
The unitary matrix \( \Theta(n) \) is determined by using either Givens Rotations or Householder Transformations [3]. From RHS of equation (22), dividing the \( LK \times 1 \) column vector \( k(n) Q^{1/2}(n) \) with scalar \( \lambda^{1/2}(n) \) gives the updated gain vector \( k(n) \). Also, the updated \( LK \times LK \) matrix \( Q^{1/2}(n) \) is obtained. The updated weight coefficient matrix \( G(n) \) is determined by using the equation (18). Using updated \( Q^{1/2}(n) \), new updated LHS of equation (22) is computed.

5. IQR-2D-RLS Stability Analysis

IQR-2D-RLS algorithm is more stable numerically than 2D-RLS algorithm as it propagates a square-root factor of \( Q(n) \), rather than \( Q(n) \) itself as done in 2D-RLS implementation. This reduces the danger of \( Q(n) \) loosing its positive-definiteness due to numerical inaccuracies. The recursion in IQR-2D-RLS propagates matrix \( Q^{1/2}(n) \), defined as square root of \( Q(n) \). The relation between \( Q(n) \) and \( Q^{1/2}(n) \) is defined by
\[ Q(n) = Q^{1/2}(n) Q^{H/2}(n) \] (23)
where the matrix \( Q^{H/2}(n) \) is Hermitian transpose of \( Q^{1/2}(n) \). The nonnegative definite character of \( Q(n) \) as a correlation matrix is preserved by virtue of the fact that the product of any square matrix and its Hermitian transpose is always a nonnegative definite matrix [3, 7].

In particular, IQR-2D-RLS algorithm propagates the square root of inverse correlation matrix \( Q(n) \). Hence, the condition number of \( Q^{1/2}(n) \) equals the square root of the condition number of \( Q(n) \). This results in a significant reduction in dynamic range of input handled by algorithm and in turn a more accurate computation than 2D-RLS algorithm.

6. Computer Simulations

Both 2D-RLS and IQR-2D-RLS adaptive channel estimation algorithms are simulated in Matlab for SISO-OFDM system for IEEE 802.11b standard. The total bandwidth of 20 MHz is divided into 64 sub-carriers and the effective OFDM symbol length \( T_s \) is 3.2 µs. BPSK modulation scheme is used. The fixed and time-varying Rayleigh fading channel is modeled using smith model [8] as equal spaced (0.05 µs) tap delay-line (TDL) structure with exponential power delay profile. Maximum multipath delay \( T_d \) of the channel is taken as 0.5 µs and the maximum Doppler shift of 100 Hz is considered. The number of symbols, \( M \), is taken as 5 and the number of previous LS channel estimates \( L \) is taken as 2. Regularization parameter \( \delta \) is a small constant between \( 0 < \delta < 1 \) and exponential weighting factor \( \lambda \) is taken such that \( 0 < \lambda < 1 \) [6]. At time instant \( n = 0 \), \( G \) and \( Q \) are defined as \( G(0) = 0 \) and \( Q^{1/2}(0) = \delta^{-1/2} I \).

Table 1 shows the number of operations required for 2D-RLS algorithm and IQR-2D-RLS algorithm using Givens Rotations and Householder Transformations. It is observed that operation count for 2D-RLS algorithm and IQR-2D-RLS algorithm using Givens Rotations are similar. But fewer operations per iteration are required for Householder Transformations than Givens Rotations.

In Fig. 1, BER performance of 2D-RLS algorithm and IQR-2D-RLS algorithm for different values of Signal to Noise Ratio (SNR) is compared. CP (CP \( \geq T_d \)) of length 0.8 µs is chosen. It is evident that performance of both the algorithms is almost similar but IQR-2D-RLS removes the basic instability problem of 2D-RLS algorithm by propagating \( \phi^{-1/2}(n) \) instead of propagating \( \phi^{-1}(n) \).

Figure 2 depicts comparison between NMSE performance of 2D-RLS and IQR-2D-RLS algorithms at SNR 10 dB where CP \( \geq T_d \). It is observed that the NMSE performance for these algorithms is almost similar. But there is a slight increase in iterations required for convergence in the
7. Conclusion

In this paper, IQR-2D-RLS adaptive channel estimation has been proposed for OFDM systems. Due to its smaller condition number, the matrix $Q^{1/2}$ in IQR-2D-RLS algorithm is close to non-singularity and hence proposed algorithm is numerically more stable than 2D-RLS algorithm as seen in Fig. 3. Also, both algorithms have computational complexity of $O(N^2)$. MATLAB simulations show that IQR-2D-RLS and 2D-RLS algorithms have similar BER performance. NMSE performance shows that convergence rate of IQR-2D-RLS algorithm is slightly less than the 2D-RLS algorithm.

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