

## FUNDAMENTAL PROBLEMS IN METROLOGY

### AN ALTERNATIVE SET OF DEFINING CONSTANTS FOR USE IN REDEFINING THE FOUR UNITS OF THE INTERNATIONAL SYSTEM OF UNITS

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*Different sets of constants the fixed values of which may be selected for new definitions of the four units (kilogram, mole, ampere, and kelvin) of the International System of Units are discussed. The concept of the "order of a constant" in a given system of units is proposed. Criteria for arriving at an optimal selection of defining constants as well as a set of constants consisting of Planck's constant  $h$ , Avogadro's constant  $N_A$ , Boltzmann's constant  $k$ , and the magnetic permeability of a vacuum (magnetic constant)  $\mu_0$  are considered. The proposed set is an alternative to the base set consisting of  $h$ ,  $e$ ,  $k$ , and  $N_A$ .*

**Keywords:** *redefinition of the units of the International System of Units, dimension of a physical quantity, defining constant, Planck's constant, Avogadro's constant, Boltzmann's constant, magnetic permeability of a vacuum, magnetic constant.*

Existing systems of the units of the physical quantities are a constituent part of the necessary set of measuring instruments used to conduct scientific studies. Modern technologies, the development of industry, and international trade presuppose a corresponding scientific level of the units of the physical quantities. The International Prototype of the Kilogram produced from a platinum-iridium compound continues to be used today, despite the fact that it was found to be temporarily unstable at a level of  $5 \cdot 10^{-10}$  kg/yr [1, 2]. This is not acceptable when performing precision measurements and the subsequent use of the results of these measurements over a long period of time. A redefinition of the four base units of the International System of Units (SI) (review of SI), which is now at the stage of preparation, will solve this problem [3, 4]. The planned review of the SI is based on a proposal that involves redefining the four units of measurement by establishing exact values of selected physical constants, following the method used to redefine the ampere in 1946 and that used to redefine the meter in 1983 [5–7]. A new term, *defining constant* (DC), is used in the ninth edition of the SI Brochure [4] in place of physical constants with fixed values. The main obstacle to making a transition to the new definitions is the insufficient level of precision of the experimental values of the proposed DCs.

A proposal to redefine a number of the units of the SI appeared in 1999 [8] and a more complete set of proposals in 2006 [6]. Several different sets of DCs were considered. In particular, it was suggested that the values of Planck's constant  $h$ , the elementary charge  $e$ , Boltzmann's constant  $k$ , and Avogadro's constant  $N_A$  should be established precisely without any experimental uncertainty in order to redefine the kilogram, ampere, kelvin, and mole, respectively [4, 6, 7]. This set of DCs is now considered the most preferable. The International System of Units with new definitions of the four base units was presented in [4], i.e., a Modified System of Units (MSI) or New System of Units is realized on the basis of this set.

The existing situation with a transition to the MSI is reflected in Resolution 1 of the 24th CGPM in 2011, that is, to “continue work on improving the statements of the definitions of the base units of the SI based on the fundamental constants, which possess sufficient scientific rigor and clarity, in order to enable the simplest possible understanding of the constants by users.” Nevertheless, in 2014 the 25th CGPM, noting the progress that had been achieved following the 24th CGPM, recommended that Resolution 1 of the 24th CGPM continue to be implemented in view of the insufficient precision of the measured values of the DCs. It was proposed that the MSI should be adopted at the 26th CGPM in 2018 [3].

The objective of the present study is to present a critical discussion of the proposed new definitions of the four base units of the SI in light of recent theoretical and experimental results [9–13]. A new concept, that of the *order of a constant* in a given system of units, is introduced and a set of criteria that may be used for selecting an appropriate set of DCs is proposed. An alternative set of DCs, comprising  $h$ ,  $k$ ,  $N_A$ , and the magnetic permeability of a vacuum (magnetic constant)  $\mu_0$ , is proposed. The advantages of this set by comparison with the base set of  $h$ ,  $e$ ,  $k$ , and  $N_A$ , which is the one that is currently the selected set, are noted. It should be expected that a review of these questions will be useful for selecting an optimal set of DCs and the subsequent use of this set.

The article proposes a new classification of the DCs in accordance with their dimensions relative to the base units based on the newly introduced concept of the order of a constant and discusses criteria for arriving at a preferential selection of DCs. Different sets of DCs that have been proposed for a redefinition of the four units of the SI are then analyzed. New definitions of the kilogram, mole, ampere, and kelvin based on fixed values of the  $h$ ,  $\mu_0$ ,  $k$ , and  $N_A$  (system  $D$ ) are considered next. The use of the concept of the order of a DC is considered separately and criteria for selecting DCs are described. The advantages and drawbacks of system  $D$  relative to other sets are noted.

**Classification of the defining constants and criteria for their preferential selection.** In [4], the Modified System of Units is presented on the basis of fixed values of  $h$ ,  $e$ ,  $k$ , and  $N_A$ . The general concept of a DC is used in place of more specialized concepts, such as physical constant, natural constant, International Prototype of the Kilogram, physical parameter, etc. The set of constants is divided into four classes: the fundamental physical constants, special atomic constants, conversion factors, and technical constants. Such a classification has a number of drawbacks, however. For example, the speed of light in a vacuum  $c$  is a fundamental physical constant, though once its value is established, it becomes a conversion factor in the sense used within the framework of the MSI [4]. Once  $N_A$  is established, it also becomes a conversion factor between the unit of the amount of a substance and the unit used to count the number of objects [4, 14], though it is also a natural constant (Avogadro’s law). It is known that the number of constants varies together with the number of base units [15]. Moreover, within the framework of a given system of units, the number of constants depends on the physical theory or model employed. For example, within the framework of nonrelativistic classical mechanics there are no constants  $c$  or  $h$ , whereas they are present in relativistic quantum mechanics. In [16], a vector space is associated with the indicators of the degrees of units for the dimensions of quantities, and a criterion for determining a complete set of constants is also proposed. Different classifications of the physical constants based on their role in the description of physical phenomena are considered in [17].

Below, a new characteristic for constants defined in some system of units is proposed. A constant that does not depend on the base units of a given system is said to be a *zero-order constant*. A constant that does depend on a single base unit is called a *first-order constant* and so on. Such a classification of the constants in accordance with their orders is used below for selecting a set of DCs.

At first glance, it would appear that the value of a physical constant cannot be established in view of the average value and uncertainty of the result of measurements which are assigned to the value of a physical constant. However, in view of the relationship between a physical constant and a unit of measurement, the value of a physical constant may be established precisely if, at the same time, there is no substantial increase in uncertainty in the realization of a redefined unit and if, moreover, this leads to additional advantages in the measurement of other quantities. Such a physical quantity with fixed value will be a DC for the given unit of measurement. It would be desirable for a set of DCs to be consistent and minimal and not lead to any contradictions when several DCs are used simultaneously. Of course, the status of any selected DC is not absolute and may vary in a generalization of the particular theory in use or with an increase in the precision of measurements of certain physical constants.

In any case, the requirement imposed on the MSI, that it not worsen the situation that now exists for users of the SI, is critical. This means that there may exist a definite succession or continuity relative to the existing SI. Succession

presupposes the use of current revisions of the base units and their values in effect at the present time in order to preserve the vast set of accumulated measurement data without having to perform any corrections.

It is also obvious that a review of the SI must not reduce the stability of the realization of the units. For example, the new prototypes of the unit of mass approved by the International Bureau of Weights and Measures must possess a temporary instability that is less than  $5 \cdot 10^{-10}$  a year at any time and in any place. Recall that when this condition was not observed for the International Prototype of the Kilogram, this led to a scheduled transition to the MSI [1, 2]. In order that no contradictions arise between the system or set of DCs and existing physical theories, the number of DCs must be minimal and the relationships between the DCs and the units of measurement must be as simple as possible. It is desirable that these requirements be observed in order to successfully teach the foundations of metrology in universities and colleges, and this may be achieved if the base unit of measurement and the corresponding DC are of the same dimension or if the relation between their dimensions is the simplest possible. For example, the dimension of the speed of light and that of the meter and, similarly, the dimension of the charge of an electron and that of the ampere each differ only in terms of the defining degree of time. The dimension of the kilogram coincides with the dimension of the atomic unit of mass  $m_u = m_{12C}/12$ , where  $m_{12C}$  is the mass of  $^{12}\text{C}$ . In terms of the concept of the orders of constants introduced above, a criterion necessary for this purpose may be stated as follows: *the order of a defining constant must be the least possible.*

Thus, we wish to propose the following criterion for selecting an optimal set of DCs for new definitions of the units [18–20]: (a) a succession between the old and new definitions; (b) stability in the transition of the units of measurement; (c) minimal number of DCs; (d) least possible order of a DC.

**Analysis of sets of defining constants proposed for a redefinition of the four base units of the International System.** Discussions on the advantages and drawbacks of the proposed new definitions of the four base units of the SI are continuing today [9, 19, 20–22]. The set of DCs consisting of  $h$ ,  $e$ ,  $k$ , and  $N_A$  is the preferred variant. Other variants have also been proposed [8, 9, 19, 20]. For example, in the opinion of the authors of [9], the attained level of the data of CODATA-2014 [13] determines a new context for selecting an optimal set of DCs by comparison with what had been selected in 2007 at the 23rd CGPM [6, 23]. Five possible variants of the SI with different sets of DCs, including the existing set, are considered in [9]. In the existing SI, the definitions of the kilogram, ampere, kelvin, and mole are based on fixed values of the mass of the International Prototype of the Kilogram  $m(\text{K})$ ,  $\mu_0$ , the triple point of water  $T_{tpw}$ , and the molar mass of  $^{12}\text{C}$   $M(^{12}\text{C})$ . In [9], the effective SI is denoted System A. Other variants of the MSI are based on fixed values of sets of constants containing  $h$ ,  $e$ ,  $k$ ,  $N_A$ ,  $m_0$ , and  $\mu_0$ . Recalling that in all these variants the values of  $k$  and  $N_A$  are fixed, these variants may be distinguished by selecting two DCs from among  $h$ ,  $e$ ,  $m_u$ , and  $\mu_0$ . Thus, system B contains  $h$  and  $e$ ; system C,  $e$  and  $m_u$ ; system D,  $h$  and  $\mu_0$ ; and system E,  $m_u$  and  $\mu_0$ . Note that the pair  $h$  and  $m_u$  is excluded in [9], since it leads to a redefinition of the second, while the pair  $e$  and  $\mu_0$  is excluded because of the lack of any advantage by comparison with variants B, C, D, and E. Let us consider in more detail the three systems B, C, and E (system D will be described subsequently). System B is a variant of the MSI proposed in [6] and noted in Resolution 1 of the 24th CGPM as a base system. System B assures null uncertainty of the Josephson constant  $K_J$  and the von Klitzing constant  $R_K$ , which are used as practical electromagnetic standards, i.e., a current problem of electrical metrology, namely, the existence of fixed values of  $K_J$  and  $R_K$  adopted in 1990 and consistent with experimental data within the limits of their uncertainty, is solved in this variant. However, this does not correspond to the actual situation today, i.e., the values of 1990 must be replaced by new values on the basis of recent CODATA data for  $h$  and  $e$  published at the time of a revision based on their known relation to  $K_J$  and  $R_K$ .

Note that for system B the two constants  $\mu_0$  and  $M_u$  (molal mass constant) must vary as functions of their current experimental data. For example, a dependence for  $\mu_0$  may be represented as  $4\pi(1 + \delta) \cdot 10^{-7} \text{ N/A}^2$ , where  $\delta = \alpha/\alpha_{2018} - 1$  with  $\alpha_{2018}$  the best attained experimental value of  $\alpha$  at the moment of redefinition [9]. Values of  $u_r$  for  $\delta$  and  $\alpha$  are equal, provided that the value of  $\alpha_{2018}$  is exact. Thus, if the deviations of  $\mu_0$  and  $M_u$  from the values of  $4\pi \cdot 10^{-7} \text{ N/A}^2$  and  $1 \text{ g} \cdot \text{mol}^{-1}$  are sufficiently small, there are basically no problems. But the factor  $(1 + \delta)$  is hard to explain when introducing  $\mu_0$  into textbooks and in lectures on electromagnetism, i.e., the presence of this factor does not agree with Resolution 1 of the 24th CGPM that demands that the MSI must be easily understood by users in general and must be consistent with scientific rigor and clarity [3]. This situation is similar to the problem of a varying molal mass constant  $M_u$  within the framework of systems B and D.

The new definitions of the kilogram and mole in systems E and C may be implemented either by a watt balance or by silicon spheres. The equivalence of these methods follows from the well-known relationship

$$N_A h = A_r(e) M_u c \alpha^2 / (2R_\infty),$$

where  $A_r(e) = m_e/m_u$  is the relative atomic mass of an electron;  $M_u = M(^{12}\text{C})/12$ , where  $M_u = N_A m_u$  coincides with  $M_{u0} = 1 \text{ g}\cdot\text{mol}^{-1}$  in the current SI, and  $R_\infty$  is the Rydberg constant. The value of Planck's molal constant is known with  $u_r = 4.5 \cdot 10^{-10}$  [13], so that there is no loss in precision when determining the value of  $N_A$  by means of  $h$  and conversely. There are advantages to the use of systems  $E$  and  $C$  in defining the kilogram and the mole [24] that had not been previously noted due to the high level of uncertainty for the values of  $R_K$  and  $K_J$ , leading to a decrease in the precision of electromagnetic measurements. System  $C$  is considered in detail in [20, 22, 25–27]. It is an intermediate system between system  $B$  [6] and system  $E$  [9] and is optimal relative to the proposed criteria.

**New definitions of the kilogram, ampere, kelvin, and mole in system  $D$ .** Each variant set of DCs has advantages and drawbacks which were best considered and discussed in the preparation of the resolution of the 26th CGPM. In the present study, we will consider in detail an alternative set of DCs used for the new definitions of the four units of the SI. This set (system  $D$ ) consists of  $h$ ,  $\mu_0$ ,  $k$ , and  $N_A$  and was first proposed by the Working Group on the Base Units of the SI and Fundamental Constants of the Academy of Sciences in Paris (it was suggested that Planck's charge be assigned the fixed value  $q_p = (2\varepsilon_0 h c)^{1/2}$ ) [30].

In system  $D$ , the new definitions of the mole and kelvin are based on fixed values of  $N_A$  and  $k$  as in systems  $B$ ,  $C$ , and  $E$ , the new definition of the kilogram, on a fixed value of  $h$  as in system  $B$ , and the new definition of the ampere on a fixed value of  $\mu_0$  as in the existing SI. Because of the lack of a fixed value of the charge of an electron  $e$  in system  $D$ , there are experimental uncertainties for  $R_K$  and  $K_J$ , though they are very small ( $\sim 10^{-10}$ ) and may be ignored. For exact electromagnetic measurements, it is only necessary that the values of  $R_K$  and  $K_J$  be stable and known with uncertainties less than some limiting value. The fine structure constant  $\alpha$  plays a central role in the determination of the uncertainty of the values of  $R_K$  and  $K_J$ . The value of  $\alpha$  was recently confirmed by the result of an exact experiment based on the yield of an atom (measurement of  $h/m_u$  ( $^{87}\text{Rb}$ ) [28]) that was independent of quantum electrodynamics. Improvements in methods of determining  $\alpha$  will continue and these will lead to a high degree of precision in the determination of the two constants  $R_K$  and  $K_J$ .

Let us now turn to a discussion of the advantage of the fixed value of  $\mu_0$  in system  $D$ ,  $\mu_0 = 4\pi \cdot 10^{-7} \text{ N/A}^2$ . First, setting a fixed value for  $\mu_0$  is compatible with setting a fixed value of  $c$ , a step that had been taken in in 1983. Since the dielectric permittivity  $\varepsilon_0$  and the magnetic permeability  $\mu_0$  of a vacuum obey the relationship  $\varepsilon_0 \mu_0 c^2 = 1$ ,  $\varepsilon_0$  is also fixed and equal to  $8.854187817\dots \text{ F}\cdot\text{m}^{-1}$ . Recalling that  $\alpha = e^2/(2\varepsilon_0 h c)$ , we obtain  $u_r(e^2) = u_r(\alpha)$ . Note that for the purpose of estimating the uncertainties of different constants in system  $D$ , the results of spectroscopic measurements depend only on the units of length and time. For example, it is sufficient to express  $m_u$  and  $e$  in terms of the fixed constants  $h$  and  $\mu_0$  and the measurable constants  $\alpha$ ,  $A_r(e)$ , and  $R_\infty$ . A fixed value of  $\mu_0$  presupposes that the vacuum impedance  $Z_0$  is also fixed. Thus, not only may  $R_K$  be determined by a direct comparison to  $Z_0$  by means of an experiment with a computable capacitor, but  $K_J$  may also be determined using a watt scale, without having to verify whether the formulas relating  $e$ ,  $h$ , and  $\alpha$  are valid. Thus, system  $D$  is more appropriate for assuring scientific and technological progress.

**Orders of defining constants and use of criteria for selecting systems  $B$ ,  $C$ ,  $D$ , and  $E$ .** Let us find the orders of the DCs  $h$ ,  $e$ ,  $k$ ,  $N_A$ ,  $m_u$ , and  $\mu_0$  which have been proposed for a redefinition of the four units of the SI. This is needed in order to use the criteria for selecting the DCs. The dimensions of these constants relative to the base physical quantities of the SI are as follows:

$$[h] = ML^2T^{-1}, \quad [e] = TI, \quad [N_A] = N^{-1}, \quad [k] = ML^2T^{-2} \theta^{-1}, \quad [m_u] = M, \quad [\mu_0] = LMT^{-2}I^{-2}.$$

We may now calculate the orders of these constants in accordance with the definition of the order of a constant:

$$O(h) = 3, \quad O(e) = 2, \quad O(N_A) = 1, \quad O(k) = 4, \quad O(m_u) = 1, \quad O(\mu_0) = 4,$$

where  $O(\cdot)$  is the order of a DC.

Moreover, the order of a system of constants

$$S = \{c_1, c_2, c_3, \dots\}$$

may be defined in the following way:

$$O(S) = \Sigma O(c_i).$$

Then,  $O(B) = 10$ ,  $O(C) = 8$ ,  $O(D) = 12$ , and  $O(E) = 10$ .

In systems  $B$ ,  $C$ ,  $D$ , and  $E$ ,  $k_u$  and  $N_A$  are used for new definitions of the units of temperature and quantity of matter, respectively. It remains for us to select two constants for the new definitions of the units of mass and current intensity. If criterion  $d$  is used directly, we obtain the constants  $m_u$  and  $e$ . In this case, system  $C$  proposed in [8] and reviewed in detail for the redefinition of the kilogram, mole, ampere, and kelvin in [20, 22, 25, 26] must be selected. Implementation of the new definitions must be performed on the basis of criterion  $b$ .

Above, the most optimal system  $C$  for redefinition of the four base units of the SI by means of the criteria considered earlier was selected unambiguously. However, there are other criteria that could be used to select the DCs. For example, the authors of the ninth edition of the Brochure of the SI [4] selected system  $B$ , using the fact that system  $B$  possesses great advantages for electromagnetic measurements, an important reason for selecting the constants  $h$ ,  $e$ ,  $k$ , and  $N_A$ . In turn, other sets possess their own advantages, for example, system  $D$ , which is based on the set  $h$ ,  $\mu_0$ ,  $k$ , and  $N_A$ . The advantage of this set lies in the stability of the values of  $h$  and  $\mu_0$ , which reflects the stability of the properties of a vacuum relative to quantum phenomena. At the same time, establishing fixed values of  $e$  and  $m_u$  reflects the stability of the properties of the atomic particles related to mass and electromagnetic fields. There exist two systems,  $B$  and  $C$ , which each contain  $e$ , and the two systems  $D$  and  $E$  which each contain  $\mu_0$ . It is known that the value of  $e$  depends on the interaction constant or energy of a process, for example,  $\alpha^{-1} \sim 137$  at low energies, but  $\alpha^{-1} \sim 128$  at the scale of the mass  $Z$  of the boson [28]. From this point of view, it is preferable to use system  $D$  or system  $E$  for the new definitions of the four base units of the SI.

**Conclusion.** The proposed new classification of the DCs based on the new concept of the *order of a constant* in a given system of units, together with the criteria that have been presented here, make it possible to select DCs for the redefinition of the units of the SI. However, the criteria that have been considered here are not the only ones possible. There are other arguments regarding the properties of the new set of DCs. In the present study, we have considered an alternative system of constants for use in the redefinition of the four base units of the SI (system  $D$ ), consisting of  $h$ ,  $\mu_0$ ,  $k$ , and  $N_A$ . This system reflects the stability of the properties of a vacuum in which quantum phenomena occur. This is compatible with the fixed value of  $c$  established in 1983 and leads to a fixed value of  $\epsilon_0$  and of the impedance of a vacuum  $Z_0$ .

However, system  $B$ , which is now considered the most preferable system, has advantages when electromagnetic measurements have to be performed. In this case, the practical units  $\Omega_{90}$  and  $V_{90}$  (or, henceforth,  $\Omega_{2018}$  and  $V_{2018}$ ) turn into units of the MSI. Nevertheless, note that from the theoretical point of view, establishing a fixed value of  $e$  has drawbacks because of the variation in  $\alpha$  as the energy of a process varies as well as because of possible space-time variations [29]. To this, we might add the fact that the practice of establishing fixed values of  $K_J$  and  $R_K$  for conducting practical electromagnetic measurements with the existing degree of precision in the measurement of  $e$  and  $\alpha$  as well as with an increase in precision may be preserved in systems  $D$ ,  $C$ , and  $E$ . A further increase in the precision with which  $\alpha$  is measured makes it possible, moreover, to verify existing theories and to find new variants of the fundamental interactions. Therefore, system  $D$  which has been considered here is the best variant of the MSI, despite the fact that it was not selected by the CCU [30].

The goal of obtaining exact values of Planck's and Avogadro's constants with relative uncertainty  $u_r = 2 \cdot 10^{-8}$  was achieved in 2015 thanks to the results of experiments carried out by NMIJ, BIPM, PTB, INRIM, NIST, and NRC [10–12]. The goal had been formulated in the resolutions of the 24th and 25th CGPM [3]. The adoption of new definitions of the four base units of the SI can now be expected in 2018 [3, 4]. Nevertheless, there is still time to consider the achievements and drawbacks of the proposed definitions of the four base units in light of new theoretical and experimental results.

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