Processing and Analysis of Digital Images on the Basis of the Gibbs Distribution

V. N. Vasyukov
Novosibirsk State Technical University, pr. Karla Marksa 20, Novosibirsk, 630092 Russia
e-mail: vasyukov@ktor.ref.nstu.ru, vasyukov_vn@mail.ru

Abstract—An approach to digital image processing and analysis based on the Gibbs distribution is considered. Gibbs hierarchical models are used in segmentation of texture images, reconstruction of gray-scale and color images corrupted by nonlinear and linear inhomogeneous distortions, and estimation of the parameters of Gibbs models from observed images.

INTRODUCTION

The Gibbs distribution has long been extensively used in describing various classes of images as realizations of Markov random fields on finite lattices [7]. The universality of the mathematical apparatus of the Gibbs distribution, which makes it possible to uniformly describe digital random fields of arbitrary dimension, and the possibility of using the Bayesian approach to the statistical synthesis of decision making algorithms have maintained researchers’ interest in Gibbs models. The iterative character of the algorithms based on stochastic relaxation impedes applying them to real-time problems, but this does not matter due to potentially high parallelization. The Gibbs distribution has already been applied to high-level problems such as image interpretation [5]. In this paper, we present some results on the application of the Gibbs distribution to image segmentation and reconstruction and suggest possible directions of further research.

SEGMENTATION AND RESTORATION OF IMAGES DESCRIBED BY THE GIBBS DISTRIBUTION

The local nature of the relations between the random values constituting a digital image is the characteristic feature of the Gibbs distribution. Due to this feature, it is applied extensively to the synthesis of information systems of image processing and analysis. These relations are specified by defining a system of cliques, i.e., sets of pixels considered neighboring. In accordance with this model, images are treated as realizations of Markov random fields. The probability (or probability density) of a realization \( x \) of a field \( X \) is determined by [7]

\[
P(X = x) = Z^{-1} \exp \left\{ -\sum_{c \in C} V_c(x) \right\},
\]

where \( C \) is a clique set, \( V_c(\cdot) \) is the potential of the clique \( c \), \( Z \) is the normalizing constant defined by

\[
Z = \int \exp \left\{ -\sum_{c \in C} V_c(x) \right\} d\mu(x),
\]

and \( \mu(x) \) is a measure on the set \( X \) of realizations of the random process or field. Obviously, \( Z \) depends on the potentials determining the properties of the field, but the explicit form of this dependence can be determined only in exceptional cases. Images described by Gibbs models are usually processed by algorithms employing only conditional probabilities, which do not depend on \( Z \). Taking various cliques and potentials, we can construct models of arbitrary dimension and complexity. To make decisions on the basis of observed realizations, iterative methods of stochastic relaxation are usually applied, which implement the Bayesian approach. The simplicity of the Gibbs description based on cliques and potentials makes it possible to take into account various factors of image formation, including linear and nonlinear distortions and noise; the noise may be additive or multiplicative, or it may interact

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with the image in a more complex way (the only requirement is that the interaction must be invertible). In [1–3], the Gibbs distribution apparatus was developed and successfully applied to solve various problems, such as segmentation of texture images (Fig. 1), reconstruction of grayscale images corrupted by nonlinear (Fig. 2) and linear inhomogeneous (Fig. 3) distortions and additive noise, and estimation of the parameters of Gibbs random fields.

One way to improve Gibbs models in image processing and analysis is to introduce additional unobserved (hidden) levels, which are statistically related to the observed images and contain information useful for solving the problem [6]. The hidden levels can be added as the problem becomes more complicated, depending on the character of the decision to be made as a result of image processing or analysis. We must take into account only the statistical relations between the elements of different levels by including the corresponding local characteristics of the Gibbs distribution in the model. It is important that the complication of the model is purely quantitative and results only in increasing analysis time. Figure 4 shows a blurred image, and Fig. 5 shows the same image reconstructed on the basis of a two-level model, which includes, in addition to the observed blurred (defocused) image and the ideal undistorted image, a field of contour lines separating regions of approximately constant intensities [6]. Taking into account these edges makes it possible to

Fig. 2. Compensation of nonlinear distortions.

Fig. 3. Compensation of linear inhomogeneous distortions.

Fig. 4. Blurred image.

Fig. 5. Reconstructed image.

Fig. 6. Contour preparation obtained in image reconstruction.
weaken the mutual influence of the regions and improves the quality of reconstruction. Moreover, during reconstruction, the contour field is estimated, and the obtained estimate can be used to segment and, then, analyze and recognize the image. The contour preparation obtained as a byproduct of reconstruction is shown in Fig. 6.

DIRECTIONS OF FURTHER RESEARCH

The further development of the approach based on the Gibbs distribution is oriented to the inclusion, in the model, of hidden levels containing various elements of image description (such as edges, regions, lines of a certain shape, graphical primitives, etc.). In particular, we intend to use the Gibbs distribution for describing the statistical properties of contour images (outlines) represented by chains of beamlets, i.e., straight line segments connecting sides of squares in a recursive-dyadic partition of an image [4]. Connected beamlets forming a polygonal line can be used to define neighborhoods. By specifying cliques and the corresponding potentials on sets of beamlets, we can extend the Gibbs approach and stochastic relaxation procedures to detection and analysis of the edges and regions of a certain shape. Another promising research direction is related to constructing hierarchical Gibbs models, which make it possible to accomplish low-level tasks (detection) and high-level tasks (recognition) in the framework of one model, which might improve the efficiency of analysis due to the simultaneous use of statistical properties at different levels.

REFERENCES