Network lifetime analytical model for node-disjoint multipath routing in wireless sensor networks

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Abstract: The objective of every sensor node in a sensor network is to send the sensed data in the phenomena to a sink node. The network lifetime is maximal, when the energy consumption of each node is uniform. Many routing protocols select an optimal path to increase the network lifetime in sensor networks. The energy of the nodes along this optimal path is consumed more, causing their early death. In the multipath routing, data traffic is distributed among the multiple paths, instead of a single optimal path. This work proposes a theoretical frame work to study the node-disjoint multipath wireless sensor network reliability. With higher reliability, higher network lifetime can be achieved. Simulation results show that when the data is transmitted through multiple paths with different data rates, the network lifetime increases. The node criticality factor enhances the network lifetime analysis effectively.

Keywords: wireless sensor network; WSN; network lifetime; multipath routing; node disjoint paths; analytical model; network reliability.


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1 Introduction

Routing the sensed data from the source to sink node in a resource constrained environment in wireless sensor network (WSN) is still a challenge. There were many attempts made to route the data in the resource constrained scenarios. Optimal path between the source and destination is selected by the routing protocols to satisfy the resource constraints such as energy, bandwidth and computation power. The routing protocols take into account the metrics like minimum hop, minimum transmission cost, high residual energy etc. to route the data. Many routing protocols attempt to reduce the energy usage in the nodes in order to increase the network lifetime. This provides an optimal path between source and destination. But, this may cause early death of some nodes in that route and reduces the network lifetime.

A multipath routing approach can overcome this problem. The multipath routing protocols select the available possible paths between the source and destination. The data is distributed among the multiple paths and the energy usage for data transmission is spread among the number of nodes along these multiple paths. The transmission delay is reduced as portion of the data is being sent along different paths. The multipath routing protocols provide effective load sharing among the multiple paths to satisfy the resource constraints and to meet the required quality of service in the WSNs. The multipath routing increases the probability of reliable data delivery. The energy cost overhead for data retransmission due to link or node failure and alternate path constructions are minimised.

The rest of this paper is organised as follows. In Section 2, it discusses the existing work. The network assumptions are mentioned in Section 3. In Section 4 the WSNs analytical model for multipath routing protocol is discussed. In Section 5, network lifetime model for multipath sensor networks is proposed. Conclusions are drawn in Section 7.

2 Related work

In the recent past many proactive routing protocols like dynamic destination-sequenced distance vector routing protocol (Perkins and Bhagwat, 1994), wireless routing protocol (Murthy and Garcia-Luna-Aveces, 1996) etc. were proposed for mobile ad hoc networks (MANET). The nodes in the MANET are mobile and the network topology is dynamic in
network. These proactive routing protocols suffer from excessive routing overhead by periodic message exchanges in the network. The network resources such as node energy and bandwidth of the network are wasted in the proactive routing overhead. WSN applications take into consideration the static topology and immobile sensor node in the network. The proactive routing strategy can be used when the topology of the network is fixed and nodes in the network are immobile.

Several authors have proposed a number of multipath routing protocols for WSN. Marina and Das (2001) proposed ad hoc on demand multipath distance vector (AOMDV) routing protocol. This protocol is a link-disjoint multipath routing algorithm and it is a variant of ad hoc on demand distance vector routing (AODV) protocol. The AOMDV protocol reduces delay and routing overhead as compared to the AODV protocol. Deepak Ganesan et al. (2001) proposed a partially disjoint multipath routing protocol. This protocol increases resilience to the node failure and provides energy efficient recovery from failure in WSNs. But it is not focused on the energy efficient multiple path construction and load sharing among the multiple paths to increase the lifetime of the network.

Energy-efficient multiple paths routing algorithm (EMRA) for WSNs, proposed by Cai et al. (2008) is another multipath routing algorithm. This is a variant of directed diffusion. To find the possible alternate paths, the EMRA incurs little overhead: interest flooding, exploratory messages and reinforce messages. The EMRA reduces the message overhead as compared to directed diffusion. Bheemalingaiah et al. (2009) proposed power-aware node-disjoint multipath source routing (PNDMSR) for real-time traffic, which balances the node energy utilisation to increase the network lifetime. It takes the network congestion into account to reduce the routing delay across the network and increases the reliability of the data packets reaching the destination. It is a source initiated route discovery mechanism. The protocol maintains \( K \) optimal paths among the possible node-disjoint paths. However, the criteria for selecting these paths and how the load is shared among them are not clearly explained by the authors.

Mao et al. (2006) present an analytical framework for the optimal partitioning of real-time multimedia traffic that minimises the total end-to-end delay. Specifically, it formulates optimal traffic partitioning as a constrained optimisation problem using deterministic network calculus and derives its closed-form solution. This framework discusses how to minimise the end to end delay in the multipath routing. But minimising the energy usage and network lifetime are not taken into consideration. Vidhyapriya and Vanathi (2007) proposed an energy efficient adaptive multipath routing technique. It is a sink initiated routing protocol. The proposed protocol utilises the multiple paths between the source and the sink; the rationale behind traffic distribution is to equally share the energy consumption by all nodes. However, the traffic spreading mechanism among the multiple paths is not shown.

Guan and He (2010) proposed energy-efficient multi-path routing protocol for WSNs. It is a reactive routing protocol. In the network every node may act as a source and a sink node. The assumption of the common base station as in most of the literature is eliminated. The route discovery mechanism provides the multiple paths between source and destination using shared nodes in the query tree and search tree. The number of control messages used in the multiple route construction is high. Because, to construct query tree and search tree, query messages and search messages are broadcasted in the network. These messages are sent from the sink and source nodes respectively.
Radi et al. (2010) proposed low-interference energy-efficient multipath routing (LIEMRO) for WSNs. It is source initiated event-based, reactive routing protocol. LIEMRO is a finds the multipath between the source and destination. But, these multipaths are excluding the node disjoint property. To increase network lifetime in the sensor network, node disjoint multipath is better than the multipath routing protocols. LIEMRO proposes the load balancing algorithm. Load balancing is done based on the average interference level, average residual battery, and estimated transmission energy of each path. Xie and Gu proposed node disjoint multipath routing algorithm for wireless multimedia sensor networks (Xie and Gu, 2010). This algorithm is a proactive routing protocol, as it is picking the alternate node disjoint path from routing table. But, it fails to note how the multiple node disjoint paths are identified between source and destination. The traffic is equally distributed among the multiple paths in this algorithm. This mechanism will not be feasible for resource constrained networks like WSNs.

3 Assumptions

The following assumptions are made for this work:

- ‘n’ identical wireless sensor nodes are deployed randomly in the phenomenon, with a single sink node. All the sensor nodes send the sensed information to the destination (sink node), in multiple hops.
- The WSN is assumed to be undirected graph \( G(V, E) \) where, \( V \) is the set of nodes and \( E \) is the edge set such that where \( E \subset V \times V \). The link \((i,j) \in E \) if nodes \( i \) and \( j \) can communicate with each other. \( N_i \) is the set of all the nodes that can be reached in one hop from node \( i \).
- Each sensor node has a fixed transmission range ‘\( R \)’.
- Multiple paths are available between any source and sink node in the network. The source node selects the node-disjoint paths between the source and destination to route the sensed data to the destination.
- The lifetime of the sensor network is assumed to be:
  1. the duration until the network gets partitioned
  2. until any sensor node drains out of its energy.

4 WSNs analytical model for multipath routing protocol

The theoretical analysis of \( k \) node-disjoint path availability is discussed by Bheemalingaiah et al. (2009) and Abbas and Abbasi (2006). The probability that there exist \( k \) node-disjoint paths is estimated as follows. Let the nodes in the network be assigned unique id’s. Select \( k \) subsets of the nodes randomly. Then \( i^{th} \) subset contains \( m_i \) nodes such that \( \sum_{i=1}^{k} m_i << n \).
The probability \( \chi \) that all these \( k \) subsets are node-disjoint is

\[
\chi = \prod_{i=2}^{k} \left( n-2 - \sum_{m_i}^{i-1} m_j \right) \left( n-2 - \sum_{m_i}^{i} m_j \right)
\]

The number of nodes used in the calculation is \((n-2)\), because the source and destination are excluded. Let us assume that between a given source node ‘S’ and sink node ‘D’, there exist \( k \) node-disjoint paths. The \( i^{th} \) path has \( m_i \) nodes as shown in Figure 1. It denotes \( j^{th} \) node of \( i^{th} \) path by the subscript \( i, j \), where \( 1 \leq i \leq k, 1 \leq j \leq m_i \). It may be noted that the end points of all \( k \) paths are fixed by the source node and sink node. Consider the \( i^{th} \) path: The probability that there exists a link between nodes 1 and 2 is \( \varepsilon_{1,2} \) and subsequently probabilities that there exist in links \(<2, 3>, <3, 4>, ..., <m_i-1, m_i>\) are \( \varepsilon_{2,3}, \varepsilon_{3,4}, ..., \varepsilon_{m_i-1, m_i} \). Let \( \Omega \) be the probability that there exist links from the ‘S’ to ‘D’ in an order \(<1, 2, 3, ..., m_i>\) then

\[
\Omega = \varepsilon_{1,2}\varepsilon_{2,3}\varepsilon_{3,4}\cdots\varepsilon_{m_i-1, m_i}.
\]

We further assume that \( \varepsilon_{u,v} = \varepsilon \), \( \forall (u, v) \in E \) then

\[
\Omega = \varepsilon^{m_i+1}
\]

For \( m_i \) intermediate nodes along a path, there can be \( m_i! \) possible orderings. Suppose \( E_o |_{o=1}^{m_i!} \) denotes the event of occurrence of \( o^{th} \) such ordering. For \( O = m_i! \). Let \( P(E_i) \) be the probability that occurrence of event \( E \) with \( i^{th} \) ordering. According to the addition theorem of probability, we have

\[
P\left( \bigcup_{0=1}^{o=m_i!} E_o \right) = \sum_i P(E_i) - \sum_{i<j} P(E_iE_j) + \sum_{i<j<k} P(E_iE_jE_k) \cdots (-1)^{o+1} \sum P(E_iE_j \cdots E_o)
\]

Let \( P \) be the probability that there exists a path with \( m_i \) intermediate nodes from the source to destination, so
Let $P_k$ be the probability that there exist $k$ multiple paths between the nodes $S$ and $D$. Then,

$$P_k = \prod_{i=1}^{k} \left[1 - (1 - \epsilon^{m+1})^{m_i} \right]$$

Combining (1) and (2), the probability that there exist $K$ node-disjoint paths between a given source node ‘$S$’ and ‘$D$’ is obtained as,

$$P_{nod} = \chi P_k$$

5 Network lifetime model

The realistic lifetime model for certain types of systems is modelled using exponential distribution (Hoyland and Rausand, 1994; Le et al., 2010).

The assumptions of exponentially distributed system lifetime are:

1. A used unit is stochastically as good as new. There is no reason to replace a functioning unit.
2. For the analysis of the reliability function of a unit, if it is sufficient to collect data on the number of hours of observed time in operation and the number of failures.

These assumptions strengthen the modelling of wireless sensor node lifetime as exponential distribution. Probability density function of lifetime of a sensor node is,

$$p(x) = \lambda e^{-\lambda x}; x > 0, \lambda > 0$$

where $p(x)$ is the probability density function, $x$ is the lifetime of the node and $\lambda$ is the parameter for exponentially distributed sensor node lifetime. $\lambda$ is the reciprocal of the mathematical expectation of the sensor node’s lifetime. Lower the value of $\lambda$, higher the sensor node lifetime is expected. Here, $\lambda$ is considered as the data rate through the sensor node. Because, higher the data rate through the sensor node, lower is the sensor node lifetime expected. The cumulative distribution function that the node will work up to $x$ is,

$$P(X \leq x) = \int_0^x p(x) dx = \int_0^x \lambda e^{-\lambda x} dx = 1 - e^{-\lambda x}$$

The reliability of the node, i.e., probability that the node will not fail before $x$, is

$$P(X > x) = 1 - P(X \leq x)$$

$$= 1 - (1 - e^{-\lambda x})$$

$$= e^{-\lambda x}$$
5.1 Path failure model

Let the path $k_i$ has $m$ nodes. Then, $R_k$, the probability of failure of path $k_i$, is,

$$R_k = \text{reliability of the path}$$
$$= P\left(\min\left(X_1, X_2, X_3, \ldots, X_m\right) > x\right)$$
$$= P\left(\min\left(X_1 > x, X_2 > x, X_3 > x, \ldots, X_m > x\right)\right)$$
$$= \prod_{i=1}^{m} P\left(X_i > x\right)$$
$$= \prod_{i=1}^{m} e^{-\lambda x}$$

$$R_k = e^{-m \lambda x} \quad \text{(5)}$$

5.2 Network lifetime model for node-disjoint multipath

5.2.1 Wireless sensor networks

Consider that, the node-disjoint multipath network has ‘$k$’ number of paths and ‘$m$’ number of nodes in each path. The data rate through each path is assumed to be $\lambda$. This implies that the data is equal in each path. If it is assumed that the lifetime of the network is death of a node due to its energy depletion, then the reliability of the network is,

$$R_{ks} = \prod_{j=1}^{k} \prod_{i=1}^{m} P\left(X_i > x\right)$$
$$= e^{-k m \lambda x} \quad \text{(6)}$$

If the lifetime of the network is considered as ‘no path exist’ between the source and destination, then, the reliability of network is,

$$R_{kp} = P\left(\max\left(X_1, X_2, X_3, \ldots, X_k\right) > x\right)$$
$$= P\left(\max\left(X_1 > x, X_2 > x, X_3 > x, \ldots, X_k > x\right)\right)$$
$$= 1 - P\left(\max\left(X_1, X_2, X_3, \ldots, X_k\right) \leq x\right)$$
$$= 1 - \prod_{i=1}^{k} P\left(X_i \leq x\right)$$
$$= 1 - \prod_{i=1}^{k} \left(1 - e^{-\lambda m x}\right)$$

$$R_{kp} = 1 - \left(1 - e^{-\lambda m x}\right)^k \quad \text{(7)}$$

To increase the lifetime of the network, the source adjusts the data rate to each path according to their residual energy. Let the node-disjoint multipath network consists of $k$ number of paths and $m_1, m_2, m_3, \ldots, m_k$ be the number of nodes with $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k$ as the data rate in each path respectively, then the reliability of the network is,

$$R_{kn} = 1 - \left(1 - e^{-\lambda_1 m_1 x}\right)\left(1 - e^{-\lambda_2 m_2 x}\right)\left(1 - e^{-\lambda_3 m_3 x}\right) \ldots \left(1 - e^{-\lambda_k m_k x}\right)$$

$$R_{kn} = 1 - \left(1 - e^{-\sum_{i=1}^{k} \lambda_i m_i x}\right) \quad \text{(8)}$$
The lifetime of the sensor network follows Exponential distribution. Weibull distribution is also used to model the lifetime of the sensor networks to suit real scenarios. The Weibull distribution introduces another parameter \( \gamma \). The parameter \( \gamma \) is the sensor node criticality. The sensor node criticality may be the traffic through the node, its rate of energy consumption or node congestion level. Then the distribution function for the lifetime of the sensor network becomes:

\[
P_w(X \leq x) = 1 - e^{-(\lambda x^\gamma)}
\]  

(9)

When \( \gamma > 1.0 \), lower the network lifetime and \( \gamma < 1.0 \) higher the network lifetime can be achieved. If the \( \gamma = 1.0 \), then equation (9) behave as exponential distribution model. The reliability functions for WSNs (6), (7) and (8) are updated as (10) (11) and (12) respectively.

\[
R_{w_{ks}} = e^{-(\lambda x^\gamma)}
\]  

(10)

\[
R_{w_{kp}} = 1 - \left(1 - e^{-m(\lambda x^\gamma)}\right)^k
\]  

(11)

\[
R_{w_{kn}} = 1 - \left(1 - e^{-m_1(\lambda_1 x^\gamma)}\right)\left(1 - e^{-m_2(\lambda_2 x^\gamma)}\right)\ldots\left(1 - e^{-m_k(\lambda_k x^\gamma)}\right)
\]  

(12)

Figure 2 Single path with varied data rate (see online version for colours)

6 Results and discussion

Simulations are conducted using MATLAB R2008a tool to analyse the network reliability. Higher network reliability indicate higher network lifetime. The simulations
are conducted to study the reliability when one path is selected between the source and destination with varied data traffic through that path. The Figure 2 shows that higher the data rate, lower is the reliability and lesser the data rate, the higher the reliability. It also shows that the lifetime of the network is more when data rate is less. But in most of the single path or optimal path routing protocols, once the optimal route is selected, all the traffic is sent through the same path. The nodes on the optimal path drain its energy very fast, thus partitioning the network soon. The Figure 2 shows that when the data rate is low (lambda is 20 kbps), network reliability is higher and when data rate is high (lambda is 50 kbps) network reliability, i.e., network lifetime is less.

Figure 2 shows the network lifetime for single path and multiple paths. In the single path approach, 30 kbps of data rate is sent along a path with three nodes between the ‘S’ to ‘D’. The multipath approach has two paths with a data rate of 15 kbps in each path. It is seen that there is a significant improvement of 60% network lifetime in the multipath approach.

The global objective of the multipath routing is to increase the network lifetime by distributing the traffic among available k number of paths. The network lifetime is maximised, if the energy available on all the node is same, after a given data transmission. The data rate and number of nodes in each path are varied to analyse the network reliability. The simulations conducted for the scenarios described in Table 1. The data selected for the simulation to match the features of MOT300 (MICA) (http://stomach.v2.nl/docs/Hardware/DataSheets/Sensors/MICA_data_sheet.pdf). The maximum data rate supported by the mica mote is 50 kbps to 60 kbps. So, the data rate of 60 kbps is taken at the source. Two node-disjoint paths are available between source and destination.
Table 1 Wireless network set up

<table>
<thead>
<tr>
<th>Sl. no</th>
<th>Number of nodes in each path</th>
<th>Data rate in each path (path1, path2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3.3)</td>
<td>(30 kbps, 30 kbps)</td>
</tr>
<tr>
<td>2</td>
<td>(3.5)</td>
<td>(30 kbps, 30 kbps)</td>
</tr>
<tr>
<td>3</td>
<td>(3.3)</td>
<td>(20 kbps, 40 kbps)</td>
</tr>
<tr>
<td>4</td>
<td>(3.5)</td>
<td>(35 kbps, 25 kbps)</td>
</tr>
</tbody>
</table>

It is observed that when the data rate along the two paths is controlled in such a way that the residual energy on both the paths are equal, the network lifetime is maximised. In the scenario where both the paths have three nodes each, a data rate of 20 kbps and 40 kbps performed better as shown in the Figure 4. There is an improvement of 15% of network lifetime compare to other scenarios.

Figure 4 Multipath routing with varied data rate and number of nodes (see online version for colours)

The simulations are conducted to establish the results with additional parameter $\gamma$, i.e., node criticality. The simulation is conducted with the data given in Table 1. When ($\gamma < 1$), the network reliability is higher as compared to ($\gamma > 1$) and ($\gamma = 1$), because, as the traffic through the node is higher lifetime of the node decreases.

Figures 5 and 6 show the network reliability for the multipath network with $\gamma = 0.5$ and $\gamma = 2.0$ respectively. It is seen that when $\gamma = 2.0$, the network reliability approaches zero, when the time = 5 units. However when $\gamma = 0.5$, the network lifetime is above 40% even when the time = 5 units.
Figure 5  Multipath routing with varied data rate and number of nodes ($\gamma = 0.5$) (see online version for colours)

Figure 6  Multipath routing with varied data rate and number of nodes ($\gamma = 2$) (see online version for colours)
7 Conclusions

Increasing the network lifetime in the resource constrained sensor networks is the major concern of this work. This paper proposes a lifetime analytical model for node-disjoint multipath WSNs. The analytical model covers early node death and no path existence between the source and destination. To provide the realistic network analysis, different data rates and different number of nodes were modelled in multipath routing. The node criticality parameter in the model enhances the scope of network reliability analysis. The simulation results confirm that the network lifetime is increased when data rate along the multiple paths is varied in accordance with the available node energy. The simulation results also establish an increase of 40% in the network lifetime when $\gamma$ is changed from 2 to 0.5 and an increase of 15% when $\gamma$ is changed from 1 to 0.5.

References


