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Joint Source-Channel Coding Using Multiple Label Mapping

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Abstract—This paper proposes a technique to compress the data with equal length code words. A novel source coding technique, multiple label mapping (MLM), is introduced. With MLM it is possible to produce a source code which uses equal length code words. Moreover, it is shown that with the MLM technique, it is possible to achieve near limit compression without using variable length coding (VLC). However this requires that the source probability grouping is performed so that after MLM each code word has almost equal appearance probability, and that full a priori feedback is available. Numerical results demonstrate proper operability of the proposed system.

I. INTRODUCTION

Lossless source codes, e.g., Huffman codes or arithmetic codes can compress data very efficiently and reach very close to the theoretical limit of the compression rate, i.e., the entropy rate of the source. Both Huffman codes and arithmetic codes are forms of variable length entropy encoding. However, the main drawback of variable length coding (VLC) is that because of boundary problems, it is very sensitive to errors, which result in many cases in long burst errors.

Shannon’s source-channel separation theorem states that as long as the entropy of the source is less than the channel capacity, there exists a separable source and channel coding (SCC) scheme which allows transmission over the channel with an arbitrarily low probability of error. However, this theorem assumes that the source can be treated as a stationary stochastic process which satisfies the asymptotic equipartition property (AEP). Moreover, Shannon’s source compression limit requires infinitely long sequences.

The mobile access of multimedia data is one of the key applications in the current and future generation wireless services. In such applications, source compression is usually performed using standardized techniques, which, in order to achieve high compression gains, often employ VLC. In Delay-and/or complexity-constrained transmission scenarios, joint optimization of SCC techniques are often more advantageous than the classical separation of source and channel encoding. Hence, joint SCC schemes has been in focus during this decade [1], [2].

In many approaches to joint SCC scheme, the implicit residual source redundancy after source encoding is additionally used for error protection in the decoder. This is useful in order to reduce the allocated bandwidth or latency for the overall transmission system because excessively powerful forward error correction coding (FEC) is not necessary [3]. Hagenauer [1] proposes joint SCC by combining the trellis diagrams of the source model and the channel code for Viterbi decoding. In [4], [5], residual redundancy of the source is used in joint SCC by exploiting the memory structure of hidden Markov models (HMMs). Correspondingly, source memory is also exploited in [6] by using the Burrows Wheeler transform. Fingscheidt et al. [7] use softbit speech decoding as an approach to error concealment. Moreover, softbit source decoders are also investigated in [8], [9]. Recently, a technique called over-complete source-mapping is proposed for joint optimization of iterative source and channel decoding (SCD) in [10]. VLC is employed on joint SCC in [3], [11]. Joint SCC and SCD for multiple correlated sources is investigated in [12]–[14].

In this paper, we propose a method to remove the error propagation problem in source coding without imposing information loss. A new source compression technique, MLM, is presented. Usually, entropy achieving source codes employ VLC, but in contrast to previous work, MLM uses equal length code words. The MLM and its inverse operation, referred to as DeMLM algorithm, are based on the extended mapping presented in [15]. The convergence analysis of the system is made by using EXIT charts. It is shown that that the proposed technique can achieve optimal performance, if the source probability grouping is performed so that after compression each code word has almost equal appearance probability, and if full a priori feedback is available.

II. SYSTEM MODEL

Fig. 1 presents a block diagram of the transmitter side of the system model. At the very beginning of the model in the transmitter side is the Markov source. In this paper, the source is the transition emitting model (TEM) source shown in Fig.
After the interleaver \( \Pi_1 \), source bits \( u_{2,1}, u_{2,2}, \ldots, u_{2,n_i} \) is compressed by using MLM, which is described in Section III, resulting in compressed sequence \( c_{2,1}, c_{2,2}, \ldots, c_{2,n_c} \). The compressed sequence is permuted with interleaver \( \Pi_2 \) and coded by using a systematic repeat accumulate (RA) code [16], resulting in the sequence \( c_{3,1}, c_{3,2}, \ldots, c_{3,R,n_c} \). Finally, bits are sent through an Additive White Gaussian Noise (AWGN) channel. The receiver receives the noisy bit sequence \( y_j = x_j + n_j, j = 1, 2, \ldots, R\cdot n_c \), where \( x_j \in \{-1, 1\} \) is the transmitted sequence and \( n_j \in \mathbb{R} \) is zero mean white Gaussian noise with variance \( \sigma_n^2 \).

### III. MLM and DeMLM Techniques

#### A. Design of Mapping Parameters

The MLM follows the idea of extended mapping [15], for bit interleaved coded modulation with iterative detection (BICM-ID). The basic idea of extended mapping is to increase the length of labeling of the bit patterns for a given constellation diagram. For example, in the standard QPSK case, the labeling length is \( l_{\text{map}} = \log_2(4) = 2 \), but in extended mapping, the labeling length is \( l_{\text{map}} = \log_2(4) + r \), where \( r \) is the number of extended bits. The idea of MLM is to compress the data using equal length code words. For example assume 3-bit segmentation where the MLM compressor allocates each possible 3-bit pattern to 2-bit code words. This allocation is shown in Fig. 3, where code symbol \( A \) corresponds to the source words 000 and 101, code symbol \( B \) corresponds to the source words 010 and 111, etc. In the extended mapping, the optimal mapping rule with full \( a \text{ priori} \) information available is obtained so that the Hamming distance of source words is as large as possible when the Euclidean distance between the code symbols is small. In the MLM the Euclidean distance between the code symbols can be considered as the Hamming distance between the code symbols. For example, in Fig. 3 the Hamming distances between the source words that are allocated to the code symbols \( A \) and \( C \) are all 2. After the allocation, code symbols are sent to the channel encoder so that each code symbol corresponds to two bits: \( \{ A, B, C, D \} = \{00, 01, 10, 11\} \).

The ultimate data compression rate is the entropy rate of the source. The compression rate can be expressed as

\[
R_b = \frac{n_c}{n_i}, \quad n_i, n_c \in \mathbb{Z}_+,
\]

where \( n_i \) is the number of information bits and \( n_c \) is the number of compressed bits. The compression rate has to be chosen so that the entropy of the source is smaller than or equal to the compression rate, i.e., \( n_i H(C_1) \leq n_c, C_1 \in \{0, 1\} \). In the MLM case, the entropy after the source compression is maximized if the probabilities of the symbols \( A, B, C \) and \( D \) are equal. The expected length \( L(C) \) of a source code \( C(c_2) \) for a random variable \( C_2 \) with probability mass function \( p(c_2) \) is given by [2]

\[
L(C) = \sum_{c_2 \in C_2} p(c_2) l(c_2),
\]

where \( l(c_2) \) is the length of the code word for a source symbol \( c_2 \). With MLM, this property leads to the requirement

\[
L(C) = \frac{1}{n} \sum_{c_2 \in C_2} l(c_2) = \frac{1}{n} n_c = n_c,
\]

which can be chosen so that \( n_c \approx n_i H(C_1), C_1 \in \{0, 1\} \). It is possible to make \( p(c_2) \) uniform if the symbol allocation is performed so that the sum of the probabilities of the words that are allocated to each symbol is approximately \( 1/l_{\text{map}} \). The mapping scheme must be reconfigured to make it optimal, for which, for example, the binary switching algorithm (BSA) [15] can be used. Other algorithms to determine an optimal mapping rule are reactive tabu search [17] and quadratic integer programming [18]. In this paper, optimal mapping schemes that are optimal for equiprobable source words are considered. In order to achieve near limit compression, the appearance probabilities of the source code words after compression should be equal. Note that to satisfy this requirement, the number of the segment patterns allocated to one code word should not necessarily be equal. The MLM algorithm is shown in Algorithm 1. Assume that the length of the binary input is long enough to

\[\text{f} \text{unction symbol is different. Only difference is the function } f \text{ which is the mapping rule.}\]
track the correlation model of the source. The entropy rate of
the source model in Fig. 2 can be calculated as

\[
H(C_1) = - \sum_{ij} p(S_i) P_{ij} \log P_{ij},
\]

where \( p(S_i) \) is the steady state probability of state \( S_i \) and \( P_{ij} \)
is the probability of transition from state \( i \) to state \( j \). By using
simple mathematics, it can be seen that when the algorithm
has been performed, the compression rate is

\[
R_s = \frac{n_c}{n_i} = \frac{[k]}{n_i} = \frac{[k] - k + k}{n_i} = \frac{\epsilon + k}{n_i} = \frac{n_i H(C_1)}{n_i},
\]

(4)

where \([k]\) means the nearest integer from \( k \) towards infinity,
and \( \epsilon = [k] - k \). It can be concluded that the compression
rate approaches to the entropy of the source when the \( \epsilon \) value
approaches to zero and, consequently, the number of loops in
step 5) in Algorithm 1 approaches infinity. [2], [15], [19]

**Algorithm 1** The Multiple Label Mapping Algorithm

1) **Input** 0’s and 1’s.
2) Calculate the entropy of the source \( H(C_1) \)
3) Choose \( \epsilon \in [0, 1] \)
4) Set \( q = 1 \)
5) 1) **repeat**
   2) \( k := qH(C_1) \)
   3) \( q := q + 1 \)
   4) **until** \([k] - k \leq \epsilon \)
6) \( n_c = [k], n_i = q - 1 \)
7) **Probability grouping (BSA)**

B. Soft Decompression

Next we will derive how the decoding of MLM compressed
data is performed. Consider the function node depicted in
Fig. 4. For notational simplicity, in the following formulation,
the DeMLX box is marked as \( f \) (function node). The edges for a **priori** information \( A(c_2) \) and extrinsic information \( E(c_2) \) is marked as \( C_{2,1}, C_{2,2}, \ldots, C_{2,n_c} \). The edges for extrinsic information \( E(u_2) \) and a **priori** information \( A(u_2) \) is marked as \( U_{2,1}, U_{2,2}, \ldots, U_{2,n_i} \). Another incoming messages \( L_{C_{2,k}\rightarrow f(c_2,k)} \) are the log-domain **priori** messages from the previous decoder. Incoming messages \( L_{U_{2,k}\rightarrow f(u_2,k)} \) are the log-domain **priori** messages from the next decoder. The outgoing log-domain messages \( L_{f\rightarrow U_{2,k}}(u_2,k) \) and \( L_{f\rightarrow C_{2,k}}(c_2,k) \) can be calculated as follows: in the probability
domain, the sum-product (SP) algorithm [20] is now

\[
\mu_{f\rightarrow U_{2,k}}(u_2,k) = \sum_{u_2,k} f(u_2,1, \ldots, u_2,n_i, c_2,1, \ldots, c_2,n_c) \\
\prod_{l \neq k} \mu_{U_{2,l}\rightarrow f(u_2,l)} \prod_{w} \mu_{C_{2,w}\rightarrow f(c_2,w)},
\]

(5)

where \( w = \{1, 2, \ldots, n_c\} \) and \( f(u_2,1, \ldots, u_2,n_i, c_2,1, \ldots, c_2,n_c) \) equals one, when \( c_2,1, \ldots, c_2,n_c \) is the code word for \( u_2,1, \ldots, u_2,n_i \) and

case the Jacobian logarithm is a function
\[
\text{Definition 1: (Jacobian logarithm [20]). For any } M \in \mathbb{Z}_{+},
\text{ the Jacobian logarithm is a function } \Upsilon : \mathbb{R}^M \rightarrow \mathbb{R}
defined according to the following recursive rule:
\[
\Upsilon(L_1, \ldots, L_M) = \Upsilon(L_1, \Upsilon(L_2, \ldots, L_M),)
\]

(6)

where

\[
\Upsilon(L_1, L_2) = \max(L_1, L_2) + \log \left( 1 + e^{-|L_1 - L_2|} \right)
\]

(7)

and

\[
\Upsilon(L_1, \ldots, L_M) = \log \left( \sum_{m=1}^{M} e^{L_m} \right).
\]

(9)

Similarly to the summation operator \( \sum \), abbreviation
\( \Upsilon(L_1, \ldots, L_M) = \Upsilon_{|l=1}^{M}(L_l) \) is used.

The log-domain messages from function node \( f \) to an edge
\( U_{2,k} \)

\[
L_{f\rightarrow U_{2,k}}(u_2,k) = \ln \left( \sum_{u_2,k} f(u_2,1, \ldots, u_2,n_i, c_2,1, \ldots, c_2,n_c) \\
\prod_{l \neq k} \mu_{U_{2,l}\rightarrow f(u_2,l)} \prod_{w} \mu_{C_{2,w}\rightarrow f(c_2,w)} \right).
\]

(10)

With the help of (9) it is possible to rewrite previous equation as

\[
L_{f\rightarrow U_{2,k}}(u_2,k) = \Upsilon_{|u_2,k} \left( \ln f(u_2,1, \ldots, u_2,n_i, c_2,1, \ldots, c_2,n_c) \\
+ \sum_{l \neq k} L_{U_{2,l}\rightarrow f(u_2,l)} + \sum_{w} L_{C_{2,w}\rightarrow f(c_2,w)} \right).
\]

(11)

By utilizing the SP algorithm, outgoing messages from function
node \( f \) to an edge \( C_{2,k} \)

\[
L_{f\rightarrow C_{2,k}}(c_2,k) = \Upsilon_{|c_2,k} \left( \ln f(u_2,1, \ldots, u_2,n_i, c_2,1, \ldots, c_2,n_c) \\
+ \sum_{l} L_{U_{2,l}\rightarrow f(u_2,l)} + \sum_{w \neq k} L_{C_{2,w}\rightarrow f(c_2,w)} \right).
\]

(12)
IV. NUMERICAL RESULTS

In this section, results of the EXIT chart analysis performed for the three-stage system of Fig. 5 is presented. The BCJR decoder [21] in Fig. 5 is used for decoding the TEM Markov source, where the value $p = 0.17$ is used in the simulations. Similarly to the two-stage system [22], the signal to noise ratio (SNR) threshold value of the three-stage system can be determined. In EXIT simulations the length of the bit sequence transmitted through the channel is 240000 and it is repeated 42 times. In BER simulations the length of the bit sequence transmitted through the channel is 6000 and the process is repeated 20 times due to the long simulation time. In BER simulations the length of the bit sequence transmitted through the channel is 240000 and it is repeated 42 times. The code rate of the systematic RA code is $R_c = 1/3$ where the variable node degree $d_v = 2$ and the check node degree $d_c = 1$ [16]. 6 iterations are performed inside the RA decoder. 

In a system with 3 serially concatenated codes, there are four extrinsic values, $E(u_3), E(c_2), E(u_2)$ and $E(c_1)$, connecting the three decoders and therefore extrinsic mutual information (MI) at the four points have to be evaluated. Hence, for a fixed SNR, the EXIT chart for three serially concatenated code is four-dimensional, and hence visualization of the convergence property using the EXIT chart is not so straightforward.

As shown in Fig. 5, DeMLM exploits two a priori inputs, namely, $A(c_2)$ and $A(u_2)$. At the same time, it generates two extrinsic outputs, i.e., $E(c_2)$ and $E(u_2)$. Hence, in order to describe the EXIT characteristics of DeMLM, following two 3D EXIT functions are needed [23]–[25]:

$$I_E(u_2) = T_{u_2}(I_{A(u_2)}, I_{A(c_2)})$$

$$I_E(c_2) = T_{c_2}(I_{A(u_2)}, I_{A(c_2)})$$

For the BCJR decoder and RA decoder only one a priori input is available; let their corresponding EXIT functions be denoted as

$$I_E(u_3) = T_{u_3}(I_{A(u_3)}, E_b/N_0)$$

(15)

for the RA decoder and

$$I_E(c_1) = T_{c_1}(I_{A(c_1)})$$

(16)

for the BCJR decoder. In (15) the second parameter, $E_b/N_0$, indicates that the extrinsic information also depends on the channel SNR. Equations (14) and (15) are plotted in Fig. 6(a), and equations (13) and (16) are plotted in Fig. 6(b). The entropy rate of the source is $H(C_1) \approx 0.6577$. The compression is performed in a way described in Section III. First, the entropy of the one-bit segments is calculated and that is, as mentioned above, 0.6577. The entropy of two-bit segments is $2 \cdot 0.6577 = 1.3154$. The entropy of three-bit segments is $3 \cdot 0.6577 = 1.9731$, which is close enough to an integer, so the code word length $n_c = 2$ is chosen and the compression rate is $R_c = 2/3$.

First consider the extrinsic information exchange between the RA decoder and the DeMLM. Because the mutual information does not change after interleaving or deinterleaving, the following equivalence holds

$$I_A^{(l)}(c_2) = I_A^{(l-1)}(u_3),$$

$$I_A^{(l)}(u_3) = I_A^{(l-1)}(c_2),$$

(17)

(18)

where $l$ is the time index. For a given a priori information $I_A(u_2)$, the a priori information for DeMLM writes as

$$I_A^{(l)}(c_2) = T_{u_3}(T_{c_2}(I_A^{(l-2)}(u_2), I_A^{(l-2)}(c_2)), E_b/N_0),$$

(19)
with $I_A^{(0)}(c_2) = 0$ and
\[ I_E^{(0)}(u_2) = T_{u_2}(I_A^{(0)}(u_2), I_A^{(0)}(c_2)). \]  
(20)

$I_A^{(0)}(c_2)$ and $I_E^{(0)}(u_2)$ converge to a value between 0 and 1 after iterations. That value depends on the channel SNR and on the \textit{a priori} input $I_A(u_2)$. Mathematically this is
\[ I_A(c_2) = \lim_{l \to -\infty} I_A^{(l)}(c_2), \]  
(21)

and
\[ I_E(u_2) = \lim_{l \to -\infty} T_{u_2}(I_A^{(l)}(u_2), I_A^{(l)}(c_2)). \]  
(22)

Hence, the overall EXIT function of the combined module of the RA decoder and the DeMLM is a function of $I_A(u_2)$ and $E_b/N_0$. This can be expressed as
\[ I_E(u_2) = T_{u_2}(I_A(u_2), E_b/N_0), \]  
(23)

which can be plotted in a 2-dimensional chart when $E_b/N_0$ is constant.

The extreme values of $I_A(c_2)$ in (21), which corresponds to different $I_A(u_2)$ values, can be visualized as the intersection of the two EXIT surfaces seen in Fig. 6(a). In Fig. 6(b) it can be seen that in order to reach the convergence point, i.e., $(1,1,1)$ point, in DeMLM output $I_E(u_2)$, both of the inputs $I_A(u_2)$ and $I_A(c_2)$ have to reach the convergence point. That is also an essential condition to reach the convergence point in BCJR decoder output. Hence, in Fig. 6(a) there has to be an open tunnel to the convergence point between the two EXIT surfaces. Let the axis in Fig. 6(a) be denoted as $I_A(u_2)$, $I_E(c_1) := x$, $I_E(c_2)$, $I_A(u_3) := y$ and $I_A(c_2)$, $I_E(u_3) := z$. The inner code inputs are on the $x$ and $y$ axes, and the output is on the $z$ axis. The outer code inputs are on the $x$ and $z$ axes, and the output is on the $y$ axis. When the 3D EXIT chart is plotted in this way, it can be concluded that the convergence gets stuck when the outer code surface is completely above the inner code surface. Finally, the EXIT function of (23) plotted for the combined module of RA and DeMLM is shown in a 2D EXIT chart in Fig. 7(a). The $E_b/N_0$ value is $-4.6$ dB and the tunnel is open and the trajectory reaches the convergence point. In order to maximize the bit rate, the convergence with lower SNR can be checked, which has been demonstrated in Fig. 7(b). It can be seen that the convergence gets stuck in a $E_b/N_0$ value $-4.7$ dB. This observation suggests that the convergence threshold is somewhere between $-4.6$ dB and $-4.7$ dB.

The BER performance is shown in Fig. 8. It can be seen that there exist a BER floor below BER=$10^{-5}$, even in the SNR value range where the tunnel in the EXIT chart is open. This is due to the fact that the gap between the EXIT curves is extremely narrow near to the convergence point. In many cases, the BER floor can be reduced by increasing the length of the interleaver. However, BER=$10^{-5}$ can already be thought as arbitrarily small. The theoretical $E_b/N_0$ limit for reliable communication in Fig. 8 is $-5.9$ dB, which is $1.5$ dB away from the BER=$10^{-5}$ of the proposed system. This loss is caused by RA code which is not optimized.

By investigating the densities of the LLRs yields that the consistency and symmetry condition does not hold for $I_A(u_2)$. That is why $I_A(u_2)$ is produced as shown in Fig. 9. At first, the source bits $c_1$ are sent through the side channel, which is also called an extrinsic channel. After the BCJR decoder and the interleaver $\Pi_1$ these noisy symbols become the \textit{a priori} value $I_A(u_2)$, which depends on the variance $\sigma^2$. The value $I_A(u_2)$ is saved and the two decoders, RA and BCJR, are activated as long as $I_E(u_2)$ converges with a certain accuracy, which in this paper is set to 0.01. This method to produce the asymmetric inconsistent \textit{a priori} information can be applied in other decoders too. The \textit{a priori} information can be produced by sending a consistent Gaussian distributed random variable through the decoder, which yields the asymmetry and inconsistency. However, there is no proof that it can track the distribution perfectly in any situation.
Markov source and the combined module of the RA and the DeMLM.

Fig. 8. BER performance for BCJR decoder for the transition emitting of each code word after MLM is almost equal, and if full EXIT chart analysis.

The MLM can achieve the entropy of the source, if probability grouping of the probabilities of the source alphabet given the entropy of the source. This is to find the best probability grouping of the probabilities of each code word after MLM is almost equal, and if full a priori feedback is available. The most crucial process of MLM is to find the best probability grouping of the probabilities of the source alphabet given the entropy of the source. This is left as future research work.

V. CONCLUSIONS

In this paper we have examined the possibility that source and channel coding schemes are jointly designed and jointly decoded by using the turbo concept. A new compression technique, the MLM technique, has been proposed which produces equal length code words. This prevents efficiently the error propagation which may occur due to code word synchronization error. The convergence analysis of the receiver, the chain RA-DeMLM-BCJR, has been performed by using EXIT chart analysis.

The MLM can achieve the entropy of the source, if probability grouping is performed so that the appearance probability of each code word after MLM is almost equal, and if full a priori feedback is available. The most crucial process of MLM is to find the best probability grouping of the probabilities of the source alphabet given the entropy of the source. This is left as future research work.

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