Abstract—Spectrum sensing optimisation techniques maximise the efficiency of spectrum sensing while satisfying a number of constraints. Many optimisation models consider the possibility of the primary user changing activity state during the secondary user’s transmission period. However, most ignore the possibility of activity change during the sensing period and assume the activity state is constant without validating or enforcing this assumption. The observed primary user signal during sensing can exhibit a duty cycle which has been shown to severely degrade detection performance. This paper shows that (a) the probability of state change during sensing cannot be neglected and (b) the true detection performance obtained when incorporating the duty cycle of the primary user signal can deviate significantly from the results expected with the assumption of no such duty cycle.

I. INTRODUCTION

Cognitive radio (CR) is a possible solution to the problem of spectrum scarcity by promoting an opportunistic spectrum access model. Under such model, licensed primary users (PU) grant permission for non-licensed secondary users (SU) to utilise PU’s spectrum as long as the interference to primary user activity is minimal [1], [2]. The key function that ensures PU protection is spectrum sensing, whereby the SU detects the presence or absence of PU signal and decides whether or not to transmit.

SU cannot sense the spectrum and transmit data simultaneously; it must periodically halt data transmission for spectrum sensing. A trade-off results as the SU must balance the quality of sensing against the speed of sensing. A sensing cycle typically consists of a sensing period followed by a transmission period. Thus a longer sensing duration provides better detection performance but shorter transmission period; a shorter sensing duration provides higher SU throughput as the transmission period is longer. Depending on PU traffic pattern, choosing particular sensing parameters can achieve optimal throughput while satisfying different constraints [3]–[6].

Spectrum sensing optimisation is performed by finding a set of sensing parameters such that the objective function (typically SU throughput) is maximised while satisfying imposed constraints. Typical sensing parameters considered include single channel sensing time [4], sequential channel sensing time [3] and transmission time [5], [6]. Different constraints include interference to PU and average sensing time. Optimisation methods can be loosely categorised into fixed and variable duration of sensing. The former fixes a sensing duration and optimise the transmission duration with respect to the objectives and constraints [5], [6]. In the latter method, sensing and transmission durations are jointly optimised to achieve the desired objective [3], [4].

Early studies on CR model PU activity as constant throughout the sensing period and transmission period [1]. Newer studies in the area of spectrum sensing optimisation model PU activity as a random process. This model does not guarantee that the activity of the PU remains constant throughout the sensing period and transmission period [5], [6]. Therefore optimisation techniques often factor in the possibility of PU state changes within the transmission period into interference constraints. However, there is a common assumption among optimisation techniques that the PU activity is either fully present or completely absent during the sensing period. Only little attention is directed to the possibility of PU changing activity states within the sensing period such as studies in [7]. Fixed sensing period optimisations often assume that the sensing duration is sufficiently short without justifying or validating that the possibilities of state changes during sensing is negligible. For example [5] and [6] chose sensing time of 5ms and 1ms respectively. If the sensing period varies and increases to become comparable with PU traffic, the probabilities of observing PU state change also increases. Methods in [4] allowed sensing duration to range between 20ms and 70ms without enforcing the assumption of no PU state change.

The PU will only occupy a portion of the sensing period if it changes activity state within the sensing period. Authors in [7] defined the duty cycle of the PU as the portion of the sensing period occupied by a PU signal. The results demonstrated that reducing the duty cycle of the PU significantly degrades the probability of detection. Since the effect of PU state change during sensing period is rarely considered, optimisation techniques rarely justify whether the chosen sensing parameters satisfy the assumption of no PU state change. Failure to consider this behaviour during sensing optimisation may result in a sensing parameters that are optimised in design but in reality the constraints may be violated.

This paper aims to derive the statistics of detection performance when the PU is only partially present during the sensing
period based on parameters calculated by existing spectrum sensing optimisation studies. The remainder of this paper is organised as follows: Section II calculates the probability of PU state changes within the sensing period. Section III provides a statistical description on the distribution of PU duty cycle. Section IV demonstrates the resulting change in detection performance based on sensing parameters calculated in existing spectrum sensing optimisation algorithms. Finally Section V concludes this paper.

II. PROBABILITY OF STATE CHANGE

We begin our analysis by calculating the probability of the PU undergoing state changes within the sensing period. The results are used to validate the claim that change in PU activity during the sensing period cannot be neglected. It can also be used to test whether the assumption that the probability of PU state change is true.

Following convention, we model the traffic activity of the PU as an i.i.d. ON/OFF random process [3]. ON and OFF states represent the busy and idle periods respectively and are exponentially distributed with death rate $\alpha$ and birth rate $\beta$. The mean holding time of the two states are $\mu_{on} = 1/\alpha$ and $\mu_{off} = 1/\beta$. The steady state probability of ON and OFF states are $P_{on} = \frac{\beta}{\alpha+\beta}$ and $P_{off} = \frac{\alpha}{\alpha+\beta}$ respectively [3].

This study considers spectrum sensing of a single PU channel by a single SU. SU senses the PU’s channel for a duration $\tau$. Without loss of generality, we denote the initial state of the PU when sensing begins as state $A$, and the PU alternates between state $A$ and state $B$ during the sensing period. The mean holding time for state $A$ and $B$ are $\mu_A$ and $\mu_B$ respectively. State $A$ can refer to either state ON or OFF; the model is similar as long as the value of $\mu_{on}$ and $\mu_{off}$ correspond to the correct values of $\mu_A$ and $\mu_B$.

Let $M$ be a positive integer representing the number of PU states within $\tau$ with $M = 1$ indicating no state change. As illustrated in Fig. 1 with an example of five observed states, $T_{A_j}$ is the duration of the $j^{th}$ state $A$, $T_{B_j}$ is the duration of the $j^{th}$ state $B$ where $j = 1, 2, \ldots, K$ and $K$ is the index of the last state. For $M$ is odd, $K = (M + 1)/2$ with last state $T_{A_K}$. If $M$ is even, then $K = M/2$ with last state $T_{B_K}$. Sensing is unlikely to finish at the same exact moment as the last state ends, thus the total duration of all states, $T$, is greater than $\tau$. $T_p$ is the total duration of states prior to the last state.

Sensing begins at a random time instant during the current PU state. Therefore the duration of the first state observed during sensing, $T_{A_1}$, is a random truncation of the full duration of PU’s initial state, denoted as $T_0 \sim \exp(\mu_A)$. We assume the starting time for sensing is uniformly distributed between 0 and $T_0$. Therefore the duration of the first observed state is given as $T_{A_1} \sim U(0, \exp(\mu_A))$. The probability density function of $T_{A_1}$, $P(T_{A_1})$ is given as,

$$P(T_{A_1}) = \int_{T_{A_1}}^{\infty} \frac{1}{T_0} P(T_0) \, dT_0. \quad (1)$$

For $M$ being odd, the duration of last state is $T_{A_K}$ and the total time of $M - 1$ states prior to the last state is $T_{AP} = T - T_{A_K}$. Under the special case $M = 1$, $T_{A_K} = T_{A_1}$, hence $P(M = 1) = P(T_{A_1} \geq \tau)$. Similarly, when $M$ is even, the duration of last state is $T_{B_K}$ and $T_{BP} = T - T_{B_K}$. $T_{AK}$ and $T_{BK}$ are exponentially distributed with mean $\mu_A$ and $\mu_B$ respectively. Thus the probability of observing $M$ states is,

$$P(M|M_{odd}) = P(T_{AP} < \tau \leq T_{AP} + T_{AK}) = \int_0^\tau P(T_{AP} \geq \tau - T_{AP}) \, dT_{AP}, \quad (2)$$

$$P(M|M_{even}) = P(T_{BP} < \tau \leq T_{BP} + T_{BK}) = \int_0^\tau P(T_{BP} \geq \tau - T_{BP}) \, dT_{BP}. \quad (3)$$

$T_{AP}$ and $T_{BP}$ are dependent on the number of observed states and are given as,

$$T_{AP} = T_{A_1} + \sum_{j=2}^{K-1} T_{A_j} + \sum_{j=1}^{K-1} T_{B_j}, \quad (4)$$

$$T_{BP} = T_{A_1} + \sum_{j=2}^{K} T_{A_j} + \sum_{j=1}^{K-1} T_{B_j}. \quad (5)$$

The probabilities calculated above are conditional to the probability that the initial state $A$ is either ON or OFF. The probability that considers both initial states requires scaling (2) and (3) by the probability $P_{on}$ and $P_{off}$ to get

$$P(M = m|m_{odd}) = P_{on} P(M|M_{odd} \cap P_{on}) + P_{off} P(M|M_{odd} \cap P_{off}), \quad (6)$$

$$P(M = m|m_{even}) = P_{on} P(M|M_{even} \cap P_{on}) + P_{off} P(M|M_{even} \cap P_{off}). \quad (7)$$

The probability of number of observed states is obtained by evaluating $P(M = m)$ for all values of $m$. Theoretically there can be infinite number of state changes within $\tau$. However the probability of observing large number of state changes within a finite duration will converge to zero. Thus it is only necessary to calculate values of $m$ that give significant probabilities. As $T_{A_j}$ and $T_{B_j}$ are exponentially distributed, $\sum T_{A_j}$ and $\sum T_{B_j}$ follow the Gamma distribution. However, a closed form solution for sum of two non i.i.d. Gamma random variables and $T_{A_1}$ is difficult to obtain, hence $T_{AP}$ and $T_{BP}$ are evaluated numerically.

The probability of no state changes, $P(M = 1)$ is investigated for sensing periods $0.01 \leq \tau \leq 0.05$. The PU
traffic parameters from the studies of [4]–[6] are outlined in Table I. Fig. 2 shows that assumption of no state changes only applies for specific PU traffic parameters and sensing durations (Li2009 at \( \tau = 0.01 \)), where \( P(M = 1) = 0.98 \). However, parameters in Lee2008 and Pei2007 causes a drop to \( P(M = 1) < 0.95 \). This effect is compounded when the sensing duration is lengthened, resulting in \( P(M = 1) < 0.85 \) at \( \tau = 0.05 \). Conventional optimisation techniques design their interference constraints based on assumption of no state change, thus \( P(M = 1) \approx 0.8 \) indicates that the interference constraint is only satisfied 80% of the time. This leads to a dangerous situation as there is no guarantee that the SU has fulfilled PU’s protection requirements.

### III. DISTRIBUTION OF DUTY CYCLE

In this section we look at the distribution of PU duty cycle observed during spectrum sensing. PU duty cycle \( D \) is defined as the fraction of the total duration of ON states, \( T_{on} \), over the total sensing period \( \tau \), i.e. \( D = T_{on}/\tau \) [7]. For example, \( D = 0.5 \) is equivalent to PU signal present for half of the sensing duration.

Conventionally, the null and alternate hypotheses for signal detection assumes only noise is captured under \( H_0 \) (\( D = 0 \)) and full PU signal captured under \( H_1 \) (\( D = 1 \)). However when there are multiple ON and OFF states, \( 0 < D < 1 \) and the existing model of \( H_0 \) and \( H_1 \) no longer applies. Therefore we redefine \( H_0 \) and \( H_1 \) depending on the ending state of the PU as outlined in Table II by assuming that SU sensing is directly followed by transmission. \( H_0 \) is defined as scenarios where PU is in OFF state when sensing finishes. This implies the PU will also be in OFF state when transmission period starts, allowing SU to utilise the spectrum. Similarly, \( H_1 \) is defined as PU finishing in ON state during sensing, since PU will begin in ON state during transmission period, prohibiting the SU to transmit.

As shown in Table III, the probability of observing up to three states is sufficient for calculations as \( P(M > 3) < 0.001 \) and can be deemed not significant. The activity of the PU for different initial state and number of observed states are illustrated in Table IV. The cases that contribute to \( H_0 \) are: Initial OFF, \( M = 1 \) and \( M = 3 \) and Initial ON, \( M = 2 \), as the PU ends in OFF state. Cases for \( H_1 \) are: Initial OFF, \( M = 2 \) and Initial ON, \( M = 1 \) and \( M = 3 \), as the PU ends in ON state. The distribution of \( T_{on} \) under \( H_0 \) and \( T_{on} \) under \( H_1 \) can then be described as the sum of all cases of \( T_{on} \) that correspond to each hypotheses.

\[
P(T_{on} \leq x | H_0) = p_{off} \left( P(T_{on} \leq x \cap M = 1) + P(T_{on} \leq x \cap M = 3) \right) + P_{on} P(T_{on} \leq x \cap M = 2),
\]

(8)

\[
P(T_{on} \leq x | H_1) = p_{off} P(T_{on} \leq x \cap M = 2) + P_{on} \left( P(T_{on} \leq x \cap M = 1) + P(T_{on} \leq x \cap M = 3) \right).
\]

(9)

The distribution of \( T_{on} \) for each combination of initial state and number of states are presented. \( T_{A_j} \) and \( T_{B_j} \) are exponentially distributed with mean parameter \( \mu_A \) and \( \mu_B \) respectively. The value of \( \mu_A \) and \( \mu_B \) depends on whether the initial state is ON or OFF: \( T_{A_1} \) is defined in (1).

1) Initial OFF, \( M = 1 \) (\( T_{on} = 0 \))

\[
P(T_{on} \leq x \cap M = 1) = P(M = 1 \cap p_{off}).
\]

(10)

2) Initial OFF, \( M = 2 \)

\[
P(T_{on} \leq x \cap M = 2) = P(\tau - T_{A_1} \leq x \cap M = 2)
= \int_{\tau-x}^\tau P(T_{B_1} \geq T_{A_1}) P(T_{A_1}) dT_{A_1}.
\]

(11)
TABLE IV
PRIMARY USER ACTIVITY EXAMPLES FOR DIFFERENT SCENARIOS.

<table>
<thead>
<tr>
<th>Initial State of PU</th>
<th>M = 1</th>
<th>Number of States</th>
<th>M = 2</th>
<th>M = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFF</td>
<td>( T_{A1} )</td>
<td>( T_{A1} T_{B1} )</td>
<td>( T_{A1} T_{B1} T_{A3} )</td>
<td></td>
</tr>
<tr>
<td>ON</td>
<td>( T_{A1} )</td>
<td>( T_{A1} T_{B1} )</td>
<td>( T_{A1} T_{B1} T_{A3} T_{B3} )</td>
<td></td>
</tr>
</tbody>
</table>

3) Initial OFF, M = 3

\[
P(T_{on} \leq x \cap M = 3) = \int_0^x \int_0^{\tau - T_{B1}} P(T_{A2} \geq \tau - T_{A1} - T_{B1}) P(T_{A1}) \, dT_{A1} \times P(T_{B1}) \, dT_{B1}.
\]

4) Initial ON, M = 1 \((T_{on} = \tau)\)

\[
P(T_{on} \leq x \cap M = 1) = \begin{cases} 0 & \text{for } 0 \leq x < \tau \\ P(M = 1 \cap P_{on}) & \text{for } x = \tau.
\end{cases}
\]

5) Initial ON, M = 2

\[
P(T_{on} \leq x \cap M = 2) = \int_0^x P(T_{B1} \geq \tau - T_{A1}) P(T_{A1}) \, dT_{A1}.
\]

6) Initial ON, M = 3

\[
P(T_{on} \leq x \cap M = 3) = \int_0^{\tau} \int_{\tau - T_{B1}}^{\tau} P(T_{A2} \geq \tau - T_{A1} - T_{B1}) P(T_{A1}) \, dT_{A1} \times P(T_{B1}) \, dT_{B1}.
\]

IV. DETECTION PERFORMANCE

Existing spectrum sensing optimisation techniques do not consider the possibility of PU state changes within the sensing period. As such, the detection performance assumed to be achievable in existing sensing optimisation studies may be compromised. In this section we calculate the resulting probability of detection \( P_D \) and probability of false alarm \( P_F \) by modifying the conventional energy detector to consider \( 0 < D < 1 \) under \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \). We then investigate the change in detection performance due to duty cycle using sensing and traffic parameters described in Section II.

A. Revised CFAR Energy Detection

The energy detector (radiometry) has been studied extensively in the literature as a candidate for spectrum sensing [2]. A conventional constant false alarm rate (CFAR) approach calculates \( P_D \) as a function of \( P_F \) for a given SNR \( \gamma \) and number of samples \( L \). Energy detector assumes that each observed sample is i.i.d. normally distributed. The detector is designed such that at minimum PU SNR, the required probability of detection \( P_{D_r} \) is achievable with a tolerable probability of false alarm \( P_{F_r} \). \( P_F \) and \( P_D \) are calculated as [2].

\[
P_F = Q\left( \frac{\lambda - L\sigma_n^2}{\sqrt{2L\sigma_n^2}} \right), \quad (16)
\]

\[
P_D = Q\left( \frac{\lambda - L\sigma_n^2}{\sqrt{2L\sigma_n^2}} \right). \quad (17)
\]

\( \sigma_n^2 \) is the noise power, \( \sigma_n^2 = \sigma_s^2(1 + \gamma) \) is the noise plus signal power, \( \lambda \) is the detection threshold and \( Q(.) \) is one minus the cumulative distribution function of the standard normal distribution. For a CFAR of \( P_{F_r} \), the threshold is calculated as,

\[
\lambda = L\sigma_n^2 + \sqrt{2L\sigma_n^2}Q^{-1}(P_{F_r}). \quad (18)
\]

When a signal is observed with duty cycle \( D \), the number of samples that contains PU signal is \( L_s = LD \) and samples that contains noise only is \( L_n = L(1 - D) \). Thus the test statistic of the energy detector affected by duty cycle is given as \( Y_D = Y_n + Y_s \) where \( Y_n \) is the test statistic of the noise only portion and \( Y_s \) is the test statistic of the signal plus noise portion. Following the same approach of normal approximation in [2], \( Y_n \) and \( Y_s \) are modelled as,

\[
Y_n \sim N(L_n\sigma_n^2, 2L_n\sigma_n^4), \quad (19)
\]

\[
Y_s \sim N(L_s\sigma_s^2, 2L_s\sigma_s^4). \quad (20)
\]

\( Y_D \) can then be modelled as,

\[
Y_D \sim N\left(L\sigma_n^2(1 + D\gamma), 2L(1 + 2D\gamma + D^2\gamma^2)\right). \quad (21)
\]

When a CFAR energy detector neglects duty cycle, it will attempt to compare the threshold calculated in (18) with the test statistic in (21). We denote the duty cycle observed under \( \mathcal{H}_0 \) as \( D_0 \) and the duty cycle observed under \( \mathcal{H}_1 \) as \( D_1 \). Thus the new probability of false alarm and probability of detection, denoted as \( P_F' \) and \( P_D' \) respectively, becomes

\[
P_F' = Q\left( \frac{\sqrt{2LQ^{-1}(P_{F_r})} - LD_0\gamma}{\sqrt{2L(1 + 2D_0\gamma + D_0^2\gamma^2)}} \right), \quad (22)
\]

\[
P_D' = Q\left( \frac{\sqrt{2LQ^{-1}(P_{F_r})} - LD_1\gamma}{\sqrt{2L(1 + 2D_1\gamma + D_1^2\gamma^2)}} \right). \quad (23)
\]

B. Detection Performance Under Duty Cycle Effect

Spectrum sensing parameters from [4]–[6] are used to present the resulting detection performance affected by duty cycle. Please note that we are not judging or comparing the individual performance of optimisation proposed by each authors. This study uses \( P_{F_r} = 0.1 \) and \( P_{D_r} = 0.9 \) similar to [1] as a benchmark to investigate the resulting detection performance using traffic and sensing parameters already considered by the authors. The actual detection performance achieved in the studies varies depending on how optimisation is performed.
Optimisation techniques calculate the optimal sensing period that ensures the resulting $P_D$ and $P_F$ satisfies the constraints and the objective function is maximised. However when $P_D' \neq P_D$ and $P_F' \neq P_F$, there is no guarantee that the resulting performance fulfils the design constraints for the chosen sensing period. Even if by chance the deviation in detection performance is still within the acceptable requirements, the chosen $\tau$ will not be optimised for the resulting $P_D'$ and $P_F'$.

For spectrum sensing optimisation to achieve truly optimised parameters it must consider the effect of PU duty cycle as it reflects practical scenarios. This can be achieved by incorporating the detection performance due to duty cycle into constraints of the optimisation problem. The conventional CFAR energy detector approach must also be redesigned to consider $0 < D < 1$ and aim to achieve $P_F' = P_F$, on average and while providing the true $P_D$ observed.

V. CONCLUSION

In conclusion, an analysis detailing the effect on spectrum sensing performance due to primary user duty cycle is presented. Many spectrum sensing optimisation studies model the primary user activity as a random process and assume that the primary user’s activity within the sensing period is constant. The analysis shows that for a few sensing parameters considered in the literature, it is possible for the primary user to change activity between idle and busy during the sensing period. This leads to the duty cycle problem and the detection performance of spectrum sensing deviates from the designed values. If an optimisation technique neglects the duty cycle problem, there is the potential danger that calculated sensing parameters may invalidate the assumption which the optimisation algorithm is based on, leading to violated interference constraints and non-optimised performances.

REFERENCES


