The goal of our research is to develop interpolation techniques which preserve or enhance the local structure critical to image quality. In this paper, we present preliminary results which exploit either the properties of vision or the properties of the image in order to achieve our goals. Few algorithms systematically take advantage of both. The median filter, based solely on properties of data, removes statistical outliers and thus isolated errors in images, and preserves approximately the sharpness of isolated image transitions. The visual quality of the resulting images is problematic. Further, the edge preserving property of the median filter does not extend to corners or other two dimensional structures.

In this paper, we consider directional image interpolation, based on a local analysis of the spatial image structure. We also consider the extension of techniques for the design of linear filters based on properties of human perception reported previously to enhance the perceived quality of interpolated images.

I. A BRIEF DISCUSSION AND OVERVIEW OF INTERPOLATION TECHNIQUES

Interpolation is one of the fundamental signal processing operations. For digital images, interpolation is necessary when the display density of images is changed, except in the case of the subsampling by an integer. Interpolation is also required in any geometric transformation or warping of images, even for the same spatial sampling density. Interpolation is finally one of the intermediate operations in the multirate processing of images.

Although interpolation may be considered with reference to the design of a low pass filter based on the frequency content or bandwidth of images, such an approach is seldom fruitful in image processing applications. First, the extent of images is generally small, so that the use of large support filters creates large artifacts at the boundary of images. Second, the design philosophy for low pass filters which is based on the approximation of an ideal low pass frequency characteristic is inappropriate for images [1]. Finally, if one is interested in preserving the whole range of detail available in the original sample, the sampling is then exactly at the Nyquist rate and formal filter design specifications cannot be formulated.

Our interest is in preserving, or even extending the detailed information content in the image. In that context, the classical or common interpolation schemes are pixel replication, bilinear interpolation and bicubic interpolations [4]. Bilinear and bicubic interpolations are small support operations which attempt to preserve the detail by providing a very high bandwidth. Of course, they result in significant aliasing errors, observed most commonly as staircasing for high contrast edges, or moire patterns for high detail parallel lines or streaks.

II. DIRECTIONAL FILTERING AND INTERPOLATION

Directional interpolation recognizes that high detail areas in images most often have a definite geometric structure or pattern, such as in the case of edges. In such cases, interpolation in the low frequency direction, along on edge, is much better than interpolation in the high frequency direction, across the edge. Thus, a directional interpolation scheme has to perform a local analysis of the image structure first, and then base the interpolation on that local structure if a low frequency direction does exist. A number of techniques have been developed through the years, which perform image filtering by either analyzing the local image structure or by performing operations which preserve some types of local structure. The best known method which preserves a specific image structure is the median filter [5]. The median filter is best suited to remove outliers in a local distribution of pixels within a data window. Because of the use of exactly the mid value or median of the distribution, it will also preserve a high contrast edge. For such an edge, the distribution is bimodal and the median will transition, with a single pixel shift of the data window, from one mode of the distribution to the other. The median filter will not preserve other local structures and perform quite poorly for random noise.

Among methods for directional filtering based on image analysis, directional smoothing does a local analysis of the image and generates a direction dependent set of estimates for each central pixel, from which the final estimate is chosen optimally [6]. Other methods for edge-preserving smoothing filters are presented and discussed in [7,8]. In a recent publication, an edge preserving interpolation method has been reported [9]. The method first detects the presence of a high contrast edge, then estimates its location and orientation, and finally bases the interpolation on that edge estimate. The method assumes a two level edge with no transition width. As an alternative to this ideal edge model, the algorithm reverts to a bilinear interpolation. We first consider a simple generalization of the model of an image transition which works well for isolated edges.

A. Interpolation based on a planar transition model

In this work, we have restricted our attention to the doubling of sample densities both in the horizontal and vertical directions. We detect the local areas of the image which can be modeled as a planar model for an isolated transition. For such a simple model, we can perform a detection test by evaluation of the local gradient and Laplacian.

Directional Interpolation using gradients: Consider the image $I(j,k)$. At pixel location $j_0,k_0$ evaluate the gradient vector $G = G \cdot G$ using a gradient $3 \times 3$ operator such as the Sobel Operator. Let $G$ have components $G_x,G_y$. The
direction perpendicular to the gradient corresponds to isointensity contours on the tangent plane to the surface. Let $V_L$, $V_G$, $V$ has for components $V_x = G_y$, $V_y = G_x$ and has a direction $\theta$ in the $xy$ plane

$$\begin{align*}
I_1 & \quad I_2 \\
I_\Delta & \quad I_0 \\
I_\Delta & \quad I_3
\end{align*}$$

Values $I_1, I_2, I_3$ and $I_\Delta$ are known. We estimate $I_0$ by directional interpolation of the four known values for $-\pi/2 < \theta < \pi/2$.

We compute $I_{\Delta_1}, I_{\Delta_3}$ and take

$$I_0 = \frac{I_{\Delta_1} + I_{\Delta_3}}{2}$$

If we have an exact isointensity line, then $I_{\Delta_1}, I_{\Delta_3}$ should be the same, otherwise, averaging them is reasonable.

Again, to compute $I_{\Delta_1}, I_{\Delta_3}$ we perform a linear interpolation between the adjacent values on the grid.

$$I_{\Delta_1} = \alpha I_1 + (1-\alpha)I_3,$$
$$I_{\Delta_3} = \alpha I_3 + (1-\alpha)I_1,$$

with $\alpha = \frac{1}{2} - \frac{1}{2} \tan \theta$, and $\tan \theta = \beta$, from which we obtain

$$I_0 = \frac{1}{4} \left[ I_1 + I_2 + I_3 + I_4 + \beta (I_2 + I_3 - I_1 - I_4) \right]$$

For $\frac{\pi}{2} > |\theta| > \frac{\pi}{4}$ the figure is no longer valid. The intersected values are on different axes. By symmetry, it is clear that we have then

$$I_0 = \frac{1}{4} \left[ I_1 + I_2 + I_3 + I_4 + \beta (I_2 + I_3 - I_1 - I_4) \right]$$

Thus, if we perform the test $\tan \theta = \beta \leq 1$. For $\beta > 1$ we use equation (1) and for $\beta < 1$ we shall use equation (2). The test $\beta \leq 1$ is equivalent to testing $|G_x| \leq |G_y|$.

We have examined only one case of interpolation, when the unknown sample $x$ is at the center of the square formed by the four known pixels. The only other case for a 2:1 interpolation is shown below

$$\begin{align*}
I_1 & \quad I_2 \\
I_\Delta & \quad 26.560
\end{align*}$$

The approach is completely similar with two exceptions.

a) The nearest pixel changes for $\tan \theta = \pm \frac{1}{2} \tan \theta / 26.56^\circ$.

b) as $\theta > \frac{\pi}{2}$ the nearest neighbors change again from $I_2, I_3$ to $I_2, I_1$. This second case is also applicable with obvious symmetric transpositions, for horizontal interpolation between known pixels.

### Estimation of the gradient

We have assumed that the gradient is determined by a $3 \times 3$ gradient operator. However, in the case of interpolation it is not clear which $3 \times 3$ array is used to estimate the gradient. As before, there are 2 cases

**Case 1:** Averaging Pixels:

![Averaging Pixels Diagram]

The averages #1 etc form $3 \times 3$ arrays which can be used to estimate the gradient.

**Case 2:** Averaging Pixels: We now have the situation shown below

![Averaging Pixels Diagram]

use nine averages as shown in #1, #2. Because the figure is not symmetric, we use either 4 pixel or 2 pixel averages. An alternate method is to evaluate two gradients on the two $3 \times 3$ arrays which straddle the unknown pixel and average the results. The Laplacian $V^2$ is estimated by using the common operator

The planar model is accepted if

$$|G_x| \geq T_1 \quad \text{and} \quad V^2 \leq T_2$$

where $T_1$ and $T_2$ are determined empirically. When the test fails, bilinear interpolation is applied. Results of this method based on a planar model are shown in Figure 1.

The method turns out to be fairly similar to what was reported in [9] in many of its details, except that a specific test for the magnitude of the Laplacian is made here, while in [9] only the edge magnitude is used. The results for our method seem to be somewhat better visually.

### B. Interpolation for other local image structures. Use of Quartiles.

Other local image structures commonly encountered, for which image analysis may provide an improvement in the interpolation scheme, are streaks and corners. Streaks or lines are local linear structures which are narrower than the analysis window width. Wedges or corners also have a definite local structure characterized by two intersecting directions within an analysis window. Thus, three types of local structures, edges, streaks and corners, are worth detecting to improve interpolation. They are all characterized by a bimodal distribution within the analysis window. These two modes are not necessary symmetric with respect to the median as in the case of an isolated edge. This observation suggest the use of rank order statistics, within, the analysis window, to detect the occurrence of one of the three structures of interest. Once the presence of any of
these three structures is detected, a spatial analysis has to be performed to determine the type of structure and its orientation. Because of the small analysis window, we limit our study to the use of quartiles. Within an analysis window, say 3x3, we group the eight exterior pixel values into quartiles, $Q_1, Q_2, Q_3, Q_4$. In each quartile there are two ordered values $Q_{ij}$, $i=1,2,3,4$, $j=1,2$. A preliminary test for the detection of a local structure is

$$Q_1 + Q_2 < T_3$$

where $T_3$ is a threshold determined experimentally. Thus, the statistic of (4) is an indicator of a bimodal distribution, which allows for asymmetric modes. Preliminary experiments show that such a detection scheme has promise for the detection of the high contrast local structures including streaks and corners.

Spatial Analysis: When the test of (4) indicates that a local structure may be present, we analyze the distribution of pixel values in 8 possible directions. Instead of pixels, we also use a statistic in each of the 8 directions, such as the mean of a 2 or 4 pixel cluster. As indicated earlier, we expect that the spatial structure of interest is characterized by a bimodal distribution, but the modes are not always distributed about the median. Thus, we classify the pixels into two classes, High (H) and Low (L) using a threshold between the adjacent quartiles with the largest separation, i.e. the largest of $Q_{i+1,j-Q_i,j}$, $i=1,2$. Thus the number of high and low within an analysis window is no longer the same. We have, for example, the patterns shown below:

\[
\begin{array}{ccc}
  H & L & L \\
  L & H & L \\
  L & H & L & L & L
\end{array}
\]

Case 1 Case 2

Case 1 corresponds to possible streaks and case 2 to a possible corner. To confirm the presence of a streak we analyze further the immediate neighbors of $x$ so as to classify $x$ as H or L and perform an interpolation in the appropriate direction. For a corner, a local analysis about $x$ is used to resolve whether $x$ should be estimated only from the high values within the window. Experimental evaluation is not complete, but preliminary tests indicate that such a method complements the interpolation schemes discussed earlier and provides useful results for streaks, which are often encountered in images.

C. Directional enhancement of interpolated images based on visual properties.

In [1] we have developed an approach to the design of FIR filters based on properties of visual perception. This formulation results in an optimization in both the spatial and frequency domain, so as to achieve some desired frequency behavior, while maintaining the quality of the image in the vicinity of the edge. This second condition requires a spatial domain constraint so as to avoid excessive ripples, common in filters with sharp frequency domain transitions. In [2,3], we have extended this approach to image enhancement in the horizontal and vertical directions while controlling the noise variance increase due to the enhancement process. Here we use the same basic formalism to provide for selective enhancement in one of four possible directions. Because we wish to maintain directionality, we now design one dimensional filter of horizontal, vertical or diagonal orientations. Thus, we process the image with four distinct directional FIR filters to enhance the quality of the image. Since the image will be modified by the enhancement filters, this step in the processing will be performed as a last step on an image interpolated to the final display resolution.

D. Some Experimental Results

We show in Figure 1 some of the results obtained by the methods reported in this paper. The original, Figure 1a, is a 512x512 image. The other images are originally 256x256 interpolated to 512x512. Figure 1b shows bilinear interpolation with significant visual artifacts due to aliasing errors. Figure 1c shows the result of directional interpolation using the planar model of Section II.A. Figure 1d illustrates the results of applying a directional enhancement as discussed in Section II.C. The results obtained are quite good in the removal of all remaining artifacts due to aliasing errors along horizontal, vertical or diagonal high contrast edges. The enhancement filter cannot improve portions of the images where the directional image structure has not been preserved by directional interpolation.

III. DISCUSSION AND CONCLUSIONS

In this paper we have presented some results on image interpolation which are based on an analysis of the structure of images in a small window. Because of the sparsity of data which is available for such analysis, we have focused on simple high contrast directional structures, such as edges, streaks and corners. In the absence of such structures, we have examined a simple planar transition model which performs fairly well on our test images. In order to detect streaks and corners, we propose a local analysis of images which group pixel values into quartiles. Finally, we have applied some of our previous work on the design of filters based on properties of human perception to the design of directional filters which enhance structure. Our results are encouraging and indicate that this is a promising approach to an area of research with a number of applications in high quality imaging.

References:

Figure 1: Some Experimental Results