Decision Support

Pricing policies for substitutable products in a supply chain with Internet and traditional channels

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A B S T R A C T

This study considers pricing policies in a supply chain with one manufacturer, who sells a product to an independent retailer and directly to consumers through an Internet channel. In addition to the manufacturer’s product, the retailer sells a substitute product produced by another manufacturer. Given the wholesale prices of the two substitute products, the manufacturer decides the retail price of the Internet channel, and the retailer decides the retail prices of the two substitute products. Both the manufacturer and the retailer choose their own decision variables to maximize their respective profits. This work formulates the price competition, using the settings of Nash and Stackelberg games, and derives the corresponding existence and uniqueness conditions for equilibrium solutions. A sensitivity analysis of an equilibrium solution is then conducted for the model parameters, and the profits are compared for two game settings. The findings show that improving brand loyalty is profitable for both of the manufacturer and retailer, and that an increased service value may alleviate the threat of the Internet channel for the retailer and increase the manufacturer’s profit. The study also derives some conditions under which the manufacturer and the retailer mutually prefer the Stackelberg game. Based on these results, this study proposes an appropriate cooperation strategy for the manufacturer and retailer.

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1. Introduction

The rapid development of Internet technology has facilitated and rendered practical establishment of an Internet channel. Kee-nan and Ante (2002) and Tsay and Agrawal (2004) noted that the use of an Internet channel can potentially reduce costs and lead to an increasing margin. Internet channel sales in the US are estimated to grow from $155.2 billion in 2009 to approximately $248 billion in 2014. According to a recent Forrester research report (Mulpuru et al., 2010), the growth in Internet channel sales will continue to outpace the increase in store channel sales because of lower price and greater convenience. Recognizing the potential of an Internet channel, many manufacturers, including Panasonic, Kodak, and Apple, have shown interest in establishing an Internet channel to complement the retailer's channel (traditional channel) such as Wal-Mart and Best Buy. We surveyed a number of consumer electronic products sold in Wal-Mart. In the digital camera catalog, manufacturers such as Canon and Casio not only distribute products to Wal-Mart, but also use their own Internet channels as an avenue to sell products directly, whereas manufacturers such as Agfa Photo and Samsung only deliver product image and information without selling products through their website. Similar situations have been observed in the DVD player category, where manufacturers such as Sony and Toshiba sell through both distribution channels, whereas other manufacturers such as Magnavox and Samsung sell only through retailers. Therefore, Wal-Mart sells some substitute products, but with different brands, and some of these products may be sold through manufacturers' Internet channels. Retailers such as Walgreens and Best Buy demonstrate similar patterns. Hence, how to manage brand competition and channel competition is an important issue for the manufacturer and retailer. Based on these observations, this study develops a model and explores the pricing strategy for the retailer and manufacturer.

Large retail channels could gradually shift supply chain power from the manufacturer to retailers, challenging the manufacturer to obtain more retailer orders when substitute products are sold in a traditional channel. How to decide retail prices for different information revealing scenarios and how consumer brand loyalty and channel loyalty affect these decisions have become important problems. This study develops a model to answer such questions and to derive managerial insights for retailers and manufacturers. Choi (1991), Lau et al. (2007), Cai et al. (2009), and Edirisinghe et al. (2011) focused on the decision sequence among retailers and manufacturers from different research angles.
Our first step is to analyze the pricing policy because many researchers and empirical studies have found retail price to be a major factor affecting consumer purchasing decision and the principal competitive criterion (Swaminathan and Tayur, 2003; Dumrongsiri et al., 2008). Several recent studies have examined an optimal pricing strategy in a supply chain that consists of a traditional channel and an Internet channel. Chiang et al. (2003) showed that the retailer is induced to reduce the retail price in order to stimulate demand in the traditional channel when an Internet channel is opened. The manufacturer’s profit improves, even if no sales occur in the Internet channel. Their results also showed that introducing an Internet channel to increase channel efficiency is worthwhile for the manufacturer and retailer. Huang and Swaminathan (2009) considered four main pricing strategies, which differ in the degree of autonomy for the Internet channel, when the retailer owns a traditional channel and an Internet channel.

Simchi-Levi et al. (2004) presented an exhaustive review of relevant analytical models. The effect of uncertainty on the supply chain was never ignored (Yue and Liu, 2006; Hsieh and Wu, 2009; Xu et al., 2010). Dai et al. (2005) considered the pricing strategies of multiple retailers competing for consumers and derived the existence and uniqueness conditions for an optimal retail price, when retailers face a deterministic demand or a general stochastic demand. They analyzed the sensitivity of optimal retail prices for the manufacturer’s capacity and cost parameters. Dumrongsiri et al. (2008) analyzed the effect of demand uncertainty on the equilibrium results, including retail price and order decision. They also showed that variability in demand induces the manufacturer to introduce an Internet channel.

Another important issue on pricing policy involves consumer’s purchasing behavior. Several empirical studies have shown that the factors of service value, preference of brand, and sales channel are also determinants of consumer purchasing decision (Bucklin and Lattin, 1991; Rohm and Swaminathan, 2004; Yao and Liu, 2005; Yao et al., 2008). Kumar and Ruan (2006) showed that the degree of brand loyalty and channel loyalty influences wholesale price, retail price, service value, and manufacturer’s decision to complement the traditional channel with an Internet channel. Kurata et al. (2007) identified an optimal pricing decision for brand and channel competition.

The proposed model differs from those of prior studies in the following areas: (1) the retailer not only cares for the competition with the Internet channel, but also considers the competition between two substitute products; (2) a linear demand function is used to model consumer demand, where the demand for a product on a channel depends on retail price, service value, and consumer brand and channel loyalties; (3) this model considers uncertainties in demand and manufacturer’s distribution; and (4) to study the effect of information revealing on optimal retail prices and profits, this study proposes three scenarios for the relative information revealing between a manufacturer and a retailer—a Nash game, a Stackelberg game with the retailer as leader, and a Stackelberg game with the manufacturer as leader. The decision sequence directly indicates the relative information revealing. This study sets the Nash game as a benchmark and investigates the effect of information revealing on optimal retail prices and profits by comparing results from the Nash and Stackelberg games. The proposed model also analyzes the effects of consumer brand loyalty and channel loyalty.

The rest of this paper is organized as follows: Section 2 formulates the proposed model and derives existence and uniqueness conditions for equilibrium solutions of the corresponding Nash and Stackelberg games. This section also analyzes the sensitivity of an equilibrium solution with respect to the model parameters and compares the retail prices and profits for both the manufacturer and the retailer in the Nash and Stackelberg games. Section 3 develops a numerical study to uncover the effects of brand loyalty, channel loyalty, service value, and wholesale price on manufacturer and retailer profits. Finally, Section 4 offers a conclusion with ideas for future research.

2. The model

Single period pricing policies are considered in a supply chain with two manufacturers and one independent retailer. Manufacturer 1 sells a product (Product 1) to the retailer at the wholesale price $w_1$, and sells the product to consumers directly through an Internet channel at the retail price $p_1$. The retailer sells Product 1 through a traditional channel at the retail price $p_1$. However, in addition to Product 1, the retailer also sells a substitute product (Product 2) through the traditional channel at the retail price $p_2$. Product 2 is produced by Manufacturer 2 and is distributed only to the retailer at the wholesale price $w_2$. Because the wholesale price is typically set as part of a long-term contract between the manufacturer and the retailer, this study assumes that wholesale prices $w_1$ and $w_2$ are fixed and exogenous. Dumrongsiri et al. (2008) and Kurata et al. (2007) made similar assumptions. Our study will also relax this assumption in the numerical example to check the robustness of the results and focus on the retail price competition for Manufacturer 1 and the retailer. In the proposed model, Manufacturer 1 decides the retail price $p_0$ and the retailer decides retail prices $p_1$ and $p_2$ of two substitute products. Both Manufacturer 1 and the retailer choose their own decision variables to maximize their respective profits. Fig. 1 shows the supply chain framework.

Compared with the Internet channel, the retailer can provide better service, such as sales explanation, immediate response, and personal interaction to customers through the traditional channel. We assume that the service value $v_i$ added to Product 1 sold through the Internet channel is null. The retailer provides a service value of $v_i > 0$ for Product i sold through the traditional channel, for $i = 1, 2$. Assuming that $p_i > v_i$ (Yao and Liu, 2005) is reasonable. The results of Kurata et al. (2007) and Bucklin and Lattin (1991) demonstrate two types of processes for brand choice in consumer decision making. The first type involves consumers who plan their shopping and make decision based on personal preference, before arriving in a channel. Let $b_i > 0$ represent the cross-brand price sensitivity between Product 1 sold through the Internet channel and Product 2 sold through the traditional channel. The second type involves consumers who make decision based on external information, such as promotions and displays in a channel. The parameter $b_0 > 0$ is understood to be the cross-brand price sensitivity between Products 1 and 2 both sold through the traditional channel. In a real shopping environment, for a certain product, consumers must choose a channel to obtain the product sold through different channels based on the consumer channel preference. The parameter $b_0 > 0$ is understood to be cross-channel.

Fig. 1. Supply chain framework.
price sensitivity between the Internet channel and traditional channel. Kurata et al. (2007) indicated that marketing activities, such as advertising campaigns and improved product displays, may lead to more frequent brand switching, causing \( \beta_1 \) and \( \beta_2 \) to increase. The decreasing channel loyalty for a product leads to more frequent channel switching, causing \( \beta_0 \) to increase.

Linear demand functions are used to characterize consumer demand because linear demand functions are tractable and widely used in supply chain analysis (Dai et al., 2005; Yao and Liu, 2005; Kurata et al., 2007; Yao et al., 2008). The demand function for Product 1, sold through the Internet channel, is denoted by

\[
D_1 = a_1 - b_1 (p_1 - v_1) + \beta_0 p_0 + \beta_2 (p_2 - v_2).
\]

The demand function for Product 1, sold through the traditional channel, is denoted by

\[
D_1 = a_1 - b_1 (p_1 - v_1) + \beta_0 p_0 + \beta_2 (p_2 - v_2).
\]

The demand function for Product 2, sold through the traditional channel, is denoted by

\[
D_2 = a_2 - b_2 (p_2 - v_2) + \beta_0 p_0 + \beta_2 (p_2 - v_2).
\]

where \( a_i > 0 \) represents the potential demand of \( D_i \) and \( b_i > 0 \) denotes the self-price sensitivity of demand \( D_i \). Lakshman and Raj (1991) indicated that brand loyal consumers who fulfill their purchasing needs in a specific brand product exhibit less self-price sensitivity. Randomness is included in the demand model in a multiplicative form (Hsieh and Wu, 2009). The stochastic demand is \( X_i = D_i x_i \), where \( x_i \) is a continuous random variable with cdf \( F_i(\cdot) \) over the interval \( [x_i, 1] \). The expected demand function for the retailer and Manufacturer 1 depend on actual uncertain demands. Manufacturer 1 and the retailer may have different attitudes toward risk. Weng (1999) proved that for any choice of retail price \( p_i \), the ordering quantity \( q_i \) satisfies the required profit \( \rho_i \) of achieving at least the expected profit \( \pi_i \) is given by \( q_i = z_i \times D_i \), where \( z_i \) satisfies \( \rho_i = 1 - F_i(A_i) \) and \( A_i \) is the expected sales quantity involving \( z_i \). In our study, \( z_i \) denotes the stock factors for \( D_i \) and \( q_i = D_i \times z_i \). The decision maker with a small value of \( z_i \) is considered risk-averse, whereas a large value of \( z_i \) is considered risk-prone. However, the quantities of Products 1 and 2 from Manufacturer 1 and 2 remain uncertain when the orders arrive. Consequently, we model the allocation uncertainty in a multiplicative form: \( Y_i = q_i \times x_i \), where \( Y_i \) is the deliverable quantity for the order quantity \( q_i \) and \( y_i \) is a continuous random variable with \( E(y_i) = 1 \) over the interval \( [y_i, 1] \). Manufacturer 1’s profit function \( \pi_{M1} \) including the Internet channel profit \( \pi_{M1(I)} \) and the wholesale profit \( \pi_{M1(W)} \) can be expressed by

\[
\pi_{M1}(p_0, p_1, p_2) = \pi_{M1(I)}(p_0, p_1, p_2) + \pi_{M1(W)}(p_0, p_1, p_2)
\]

where

\[
\pi_{M1(I)}(p_0, p_1, p_2) = E[p_i \min\{X_i, \min(q_i, Y_i)\} - c_0 \min(q_i, Y_i) + m_i \max\{Y_i - q_i, 0\} + s_i \max\{\min[q_i, Y_i] - X_i, 0\} - t_i \max\{X_i - \min[q_i, Y_i], 0\}].
\]

\[
\pi_{M1(W)}(p_0, p_1, p_2) = E[(w_i - c_i) \min\{q_i, Y_i\} + m_i \max\{Y_i - q_i, 0\}].
\]

Similarly, Manufacturer 2’s profit function \( \pi_{M2} \) can be expressed by

\[
\pi_{M2}(p_0, p_1, p_2) = E[(w_2 - c_2) \min\{q_2, Y_2\} + m_2 \max\{Y_2 - q_2, 0\}].
\]

Note that \( c_i \) is the respective unit distribution cost. There is a salvage value \( s_i \) per unit of unsold inventory, a unit shortage cost \( t_i \), and a salvage value \( m_i \) for each unit of unallocated inventory when the allocated capacity exceeds the order \( q_i \). The retailer’s profit \( \pi_R \) including Product 1 sales profit \( \pi_{R1(1)} \) and Product 2 sales profit \( \pi_{R2(2)} \) can be formulated as follows:

\[
\pi_R(p_0, p_1, p_2) = \sum_{i=1}^{2} \pi_{R(i)}(p_0, p_1, p_2),
\]

where

\[
\pi_{R(i)}(p_0, p_1, p_2) = E[p_i \min\{X_i, \min(q_i, Y_i)\} - w_i + c_i(v_i) \min\{q_i, Y_i\} + s_i \max\{\min\{q_i, Y_i\} - X_i, 0\} - t_i \max\{X_i - \min\{q_i, Y_i\}, 0\}].
\]

A strictly convex service cost function \( c(v_i) \) is used to depict the relationship between \( v_i \) and its related service cost. One practical form of the service cost function is \( c(v_i) = \eta_i v_i^2 / 2 \), where \( \eta_i > 0 \) is the service cost parameter (Yao and Liu, 2005; Yao et al., 2008).

For brevity, Eqs. (4) and (5) are rewritten as

\[
\pi_{M1}(p_0, p_1, p_2) = [p_0 b_0 - c_0 z_0 + m_0 (1 - z_0) + s_0 (z_0 - \theta_0) - t_0 (\theta_0 - 0) z_0 D_0 + ([w_1 - c_1] z_1 + m_1 (1 - z_1) ) z_1 D_1,]
\]

\[
\pi_{M2}(p_0, p_1, p_2) = ([w_2 - c_2] z_2 + m_2 (1 - z_2) ) z_2 D_2,
\]

where \( \theta_0 = E[\min\{x_i / z_0, \min(1,y_i)\}] \), \( k_i = E[\max\{x_i / z_0, \min(1,y_i)\}] \), \( \lambda_i = E[\min(1, y_i)] \), \( i = 0, 1, 2. \) Note that \( \theta_0, k_0, \theta_2 \) with \( k_i \geq \lambda_i \) \( \lambda_i \geq \theta_i > 0 \), can be interpreted as the average sales volume, average demand volume, and average quantity delivered per unit of the order quantity \( q_i \), respectively. It is assumed that the parameters \( \gamma_0, \tau_1, \tau_2 \) denote the expected revenue of the manufacturers for each unit order quantity \( q_0, q_1, \) and \( q_2 \) in the following manner:

\[
\gamma_0 = p_0 b_0 - c_0 z_0 + m_0 (1 - z_0) + s_0 (z_0 - \theta_0) - t_0 (\theta_0 - 0) \quad \text{and} \quad \tau_i = (w_i - c_i) z_i + m_i (1 - z_i), \quad i = 1, 2.
\]

The parameters \( \psi_1 \) and \( \psi_2 \) denote the expected revenue per unit order quantity \( q_1 \) and \( q_2 \) for the retailer, respectively, in the following manner:

\[
\psi_1 = p_1 b_1 - \theta_1 + c_1 (1) z_1 + s_1 (\lambda_1 - \theta_1) - t_1 (\lambda_1 - \lambda_1, i = 1, 2.
\]

Finally, based on the demand and profit functions, as shown in Eqs. (1)–(7), the following assumptions are made:

**Assumption 1.**

\[ b_0 > \beta_0 + \beta_1, \quad b_1 > \beta_0 + \beta_2 \quad \text{and} \quad b_2 > \beta_1 + \beta_2. \]

**Assumption 2.**

\[ \gamma_0, \tau_1, \tau_2, \psi_1, \psi_2 > 0. \]

**Assumption 1** is satisfied in most industries: if the retailer and Manufacturer 1 increase their retail prices by one unit, then each demand decreases. Otherwise, both Manufacturer 1 and the retailer can increase demand by simultaneously raising their retail prices (Bernstein and Federgruen, 2004; Dai et al., 2005). **Assumption 2** guarantees the participation of the retailer and those of Manufacturers 1 and 2 in the business. To analyze the effect of power for Manufacturer 1 and the retailer in the supply chain, the pricing schemes are examined in three ways: Nash game, Stackelberg game with the retailer as leader, and Stackelberg game with Manufacturer 1 as leader.

2.1. Pricing policy under Nash game

In the Nash game, Manufacturer 1 and the retailer simultaneously decide their retail prices to maximize their respective profits. When an equilibrium solution does not exist, the outcome...
of the game is unclear. However, if an equilibrium solution exists and is unique, then the decision maker can characterize the optimal action without vagueness and ambiguity. In studying the existence and uniqueness of an equilibrium solution, Chacon and Netessine (2004) compiled useful theoretical tools for researchers. This study uses the following related results, introduced by Topkis (1979, 1998) and Milgrom and Roberts (1990):

**Definition 1.** A twice continuously differentiable payoff function \( p_i(x_1, x_2, \ldots, x_n) \) is supermodular if and only if \( \partial^2 p_i / \partial x_k \partial x_l \geq 0 \) for all \( x \) and \( j \neq l \). The game is termed supermodular if its player payoffs are supermodular.

**Lemma 1.** In a supermodular game, at least one Nash equilibrium exists.

**Lemma 2.** If an equilibrium solution exists and \( |\partial^2 p_i / \partial x_k \partial x_l| > \sum_{i=1}^n |\partial^2 p_i / \partial x_k \partial x_l| \), \( \forall k \), then the equilibrium solution is unique.

The conditions, \( |\partial^2 p_i / \partial x_k \partial x_l| > \sum_{i=1}^n |\partial^2 p_i / \partial x_k \partial x_l| \), \( \forall k \), are known as the “contraction mapping conditions” in the diagonal dominance form that has been extensively used (Bernstein and Federgruen, 2004; Dai et al., 2005). In the meanwhile, an iterative play could be viewed as a contraction mapping. For instance, if prices are public information in the market and the manufacturer announces a decision on the retail price, then the retailer makes a decision based on the manufacturer’s decision and his own best response function. Subsequently, given the retailer’s decision, the manufacturer might make another decision based on his best response function. According to the concept of stability of a Nash equilibrium introduced by Moulin (1986), the outcome of an iterative play with alternating first-movers will converge to an equilibrium solution, when the contraction mapping conditions hold.

We now study the existence and uniqueness of an equilibrium solution in our model.

**Theorem 1.** \( p_M \) and \( p_R \) are supermodular in \( (p_0, p_1, p_2) \).

**Lemma 1** assures the existence of the Nash equilibrium. **Theorem 1** also indicates that the profit increment is increasing in other decision variables because of a rise in the self-decision variable.

**Theorem 2.** The Nash equilibrium is unique if the following condition holds:

\[
\frac{\beta_2}{2b_1 - \beta_2 - \beta_1} < \frac{1}{2b_2 - \beta_2 - \beta_1}.
\]

Eq. (8) is satisfied easily as \( b_1 \) and \( b_2 \) increase, or \( \beta_2, \beta_1, \) and \( \beta_2 \) decrease. Eq. (8) is also reasonable because, if consumers become less self-price sensitive, multiple equilibrium solutions may exist. However, if consumer cross-brand or cross-channel price sensitivity becomes high, the equilibrium solution may be unstable and may not exist. The interval \( \frac{\beta_2}{2b_1 - \beta_2 - \beta_1} \) is nonempty by **Assumption 1**. Henceforth, it is assumed that Eq. (8) holds and the superscript ‘N’ is used to denote the optimal value for the Nash game. The retail prices \( (p_0^N, p_1^N, p_2^N) \) are obtained by solving the following system equations: \( \partial p_M / \partial p_0 = 0 \) \( \partial p_R / \partial p_1 = 0 \) \( \partial p_R / \partial p_2 = 0 \).

**Theorem 3.**

(i) \( p_i^N \) decreases in \( b_i \), and increases in \( \beta_i, i, j = 0, 1, 2 \).

(ii) \( p_1^N \) and \( p_2^N \) decrease in \( b_i \), and they increase in \( \beta_i, i = 0, 1, 2 \).

(iii) \( p_0^N \) decreases in \( b_i \), if and only if \( D_i^N \) decreases in \( b_i, i = 0, 1, 2 \).

(iv) \( p_i^N \) increases in \( \beta_i \), if and only if \( D_i^N \) increases in \( \beta_i, i = 0, 1, 2 \).

**Theorem 4.** In Scenario 1 of the Stackelberg game, the equilibrium solution is unique if the following condition holds:

\[
4b_1z_1b_2z_2\left(b_1 - \frac{p_0^2}{2b_0}\right) - (b_1z_1 + b_2z_2)^2\left(\beta_1 + \frac{\beta_2p_1}{2b_0}\right)^2 > 0.
\]

**Theorem 5.** In Scenario 2 of the Stackelberg game, the equilibrium solution is unique if the following condition holds:

\[
-b_0 + \frac{2b_1z_1b_2z_2(b_2\beta_2^2 + b_1\beta_1^2)}{4b_1z_1b_2z_1b_2 - (b_1z_1 + b_2z_2)^4\beta_1^2} < 0.
\]

2.2. Price policy under Stackelberg game

This study investigates two scenarios of the Stackelberg game to analyze whether Manufacturer 1 and the retailer have different powers in the supply chain. In Scenario 1 of the Stackelberg game, the sequence of moves is that the retailer (leader) announces \( p_1 \) and \( p_2 \) to maximize \( p_M \). In response to \( p_1 \) and \( p_2 \), Manufacturer 1 (follower) chooses \( p_0 \) to maximize \( p_M \). Contrarily, in Scenario 2 of the Stackelberg game, Manufacturer 1 selects \( p_0 \) to maximize \( p_M \). In response to \( p_0 \), the retailer decides \( p_1 \) and \( p_2 \) to maximize \( p_R \). Similar to the Nash game case, it is necessary to derive sufficient conditions to guarantee the existence of a unique equilibrium solution in the Stackelberg game. The following theorems show these conditions for Manufacturer 1 and the retailer to make their decisions:

**Theorem 6.** In Scenario 2 of the Stackelberg game, the equilibrium solution is unique if the following condition holds:

\[
-p_0 + \frac{2b_1z_1b_2z_2(b_2\beta_2^2 + b_1\beta_1^2)}{4b_1z_1b_2z_1b_2 - (b_1z_1 + b_2z_2)^4\beta_1^2} < 0.
\]

Similar to **Theorem 2**, the sufficient conditions of the unique equilibrium are easily satisfied as consumers have higher self-price sensitivity, less cross-brand, and cross-channel sensitivities. We use superscripts ‘S1’ and ‘S2’ to denote the optimal values for Scenarios 1 and 2 of the Stackelberg game. In Scenario 1, Manufacturer 1’s profit-maximizing best response function \( p_0^{S1}(p_1, p_2) \) is obtained by setting \( \partial p_M / \partial p_0 = 0 \) to solve \( p_0 \). Substituting \( p_0^{S1}(p_1, p_2) \) in Eq. (7), the retailer’s profit-maximizing retail prices \( p_1^{S1} \) and \( p_2^{S1} \) are derived by setting \( \partial p_R / \partial p_1 = \partial p_R / \partial p_2 = 0 \) to find \( p_1 \) and \( p_2 \). Following a similar procedure, we can obtain the optimal retail prices \( (p_0^{S2}, p_1^{S2}, p_2^{S2}) \). A closed form expression for the optimal retail prices is not reported here because of its complexity. To render tractable results, the next section considers a special case in which we analyze the sensitivity of equilibrium solutions for the Stackelberg game and comparing individual profits for the Nash and Stackelberg games.
2.3. The identical case

In this case, Products 1 and 2 have identical attributes, including the unit production cost $c = c$, the wholesale price $w_i = w$, and the deliverable quantity uncertainty $y_i = y \in [0, y]$. In addition, there is an identical potential demand $d_i = a$ and an identical demand uncertainty $x_i = x \in [x, X]$. The retailer adds an identical service value $v_i = v$ to Products 1 and 2 with the same service cost factor $n_i = n$. Manufacturer 1 and the retailer have an identical unit short-
age cost $t_i = t$, salvage value $m_i = m$, $s_i = s$, and stock factor $z_i = z$ in their profit functions. This identical case signifies a high substitut-
ability between Products 1 and 2. Consumers have identical price sensitivity parameters $b_i = b$ and $\beta_i = \beta$ for $i = 0, 1, 2$. However, Products 1 and 2 are two differentiable yet substitutable products. Using a superscript, $\sim$, for this special case, the profit functions for Manufacturer 1 and the retailer now become the following:

$$\pi_M(\hat{p}_0, \hat{p}_1, \hat{p}_2) = [p_0 \theta - c \lambda + m(1 - \lambda) + s(\lambda - 0) - t(\kappa - \lambda)]zD_0 + [(w - c)\lambda + m(1 - \lambda)]zD_1,$$

$$\pi_R(\hat{p}_0, \hat{p}_1, \hat{p}_2) = \sum_{i=1}^{2} \left[ p_i \theta - [w + c(v)]\lambda + s(\lambda - 0) - t(\kappa - \lambda) \right]zD_i,$$

where $\theta_i = \theta, \kappa_i = \kappa, \lambda_i = \lambda, i = 0, 1, 2$. Let the expected revenue $\gamma_0 = \gamma$ and $\gamma_i = \gamma_i = \gamma$ in the identical case. Assumptions 1 and 2 are still reasonable and prevent an unrealistic phenomenon in this case. The next theorem summarizes the equilibrium solutions for the Nash game and the Stackelberg game for the identical case.

**Theorem 6.** In the identical case, (i) the equilibrium price for the Nash game is given by

$$\hat{p}_0^N = \frac{(\beta - b)\theta x - \sigma - \tau + \theta v + ab\theta}{2\theta(b - \beta) - \beta^2},$$

$$\hat{p}_1^N = \hat{p}_2^N = \frac{\gamma^2 (\tau - 2\theta v) + (\sigma - \theta v)(2b\beta - 2b^2) + 2ab\theta + ab\beta - b\beta\sigma}{2\theta(2b\beta - b^3)};$$

(ii) the equilibrium price for Scenario 1 of the Stackelberg game is given by

$$\hat{p}_0^S = \frac{-b\sigma + \beta\tau + \theta(a - 2\beta v) + 2b\theta \hat{p}_1^S + \hat{p}_3^S}{2\theta \hat{p}_1^S},$$

$$\hat{p}_1^S = \frac{\beta^2\tau + 2ab\theta + ab\beta - b\beta\sigma}{4\theta(b - \beta - \beta^2)} + \frac{\nu - \phi}{2\theta},$$

(iii) the equilibrium price for Scenario 2 of the Stackelberg game is given by

$$\hat{p}_0^S = \frac{\beta(b - b)(2\phi - \tau + \theta v) - 2ab\theta}{4\theta(b - \beta - \beta^2)} - \frac{\sigma}{2\theta},$$

$$\hat{p}_1^S = \hat{p}_2^S = \frac{\phi\beta + \theta(a + b + \beta v - \beta v) - \phi\beta + \theta\hat{p}_0^S}{2\theta(b - \beta)}.$$

This result in Corollary 1 is similar to that of Theorem 3. The managerial insights mentioned for the Nash game still hold for the Stackelberg game. We also perform an analysis of how the retailer uses service provision to influence the retail price in the Internet channel of Manufacturer 1. First, the marginal cost of the retail service increase is more than one, denoted as “cost-ineffective service,” $\partial c(v) / \partial v > 1$. Otherwise, it is denoted as “cost-effective service.” Second, the definitions of $\theta$ and $\lambda$ show that $\theta / \lambda < 1$. From (i) of Corollary 1, we find that, if the marginal cost of the retail service increase is less than $\theta / \lambda$, that is, the service is cost effective, then Manufacturer 1 has to reduce the retail price to attract more consumers. Customers enjoy a higher level of service from the traditional channel and lower price from the Internet channel. When $\theta / \lambda \leq \partial c(v) / \partial v < 1$, the retailer’s service provision not only attracts more demand through better service, but also induces higher retail prices $\hat{p}_1^S, \hat{p}_0^S, \hat{p}_3^S$, and $\hat{p}_0^S$, which results in a decline in demand in the Internet channel. The retailer has an advantage because he can provide services to eliminate the threat from the Internet channel. Hence, the interval $[\theta / \lambda, 1]$ is denoted as the “service advantage interval.” Discovering whether the profit from increased demand can cover the increasing service cost is interesting. Therefore, Section 3 considers the effect of service changes on retailer profit. The next lemma facilitates analysis and understanding of the effect of pricing scheme changes between the Nash game and Scenario 1 of the Stackelberg game.

**Lemma 3.** In the identical case, the optimal retail prices and demand quantities in the Nash game and Scenario 1 of the Stackelberg game are related in the following manner:

(i) $\hat{p}_0^N - \hat{p}_0^S = \frac{\hat{p}_1^S - \hat{p}_2^S}{\hat{p}_0^S - \hat{p}_3^S},$

(ii) $D_0^N - D_0^S = \left( b - \beta - \frac{\phi}{2\theta} \right) (\hat{p}_0^N - \hat{p}_3^S),$

(iii) $D_0^N - D_0^S = \hat{p}_0^N (\hat{p}_0^N - \hat{p}_3^S).$

Based on (i), it is found that the decision variables (retail prices $\hat{p}_0^S$ and $\hat{p}_0^S$) of the leader can exhibit a larger variance than those of the follower. The retailer has a strong influence on retail prices in the market, and it is worthwhile for the retailer to increase or decrease retail prices on a larger scale to repel or attract consumers. From (ii), to maximize retailer profit, the retailer may raise the retail price to offset the lost demand or reduce the retail price to attract consumers. According to (iii), $D_0^N > D_0^S$ if and only if $\hat{p}_0^N > \hat{p}_3^S$. The reason is that, despite $\hat{p}_0^N > \hat{p}_3^S$, the increment of $\hat{p}_1^S, \hat{p}_1^N - \hat{p}_3^S$, is greater than $\hat{p}_0^N - \hat{p}_3^S$ (by (i)). Hence, the demand increases in the Internet channel. However, from Manufacturer 1’s viewpoint, a trade-off problem exists between $\pi_{M(i)}$ and $\pi_{M(j)}$ when the structure changes for the competition. If $\hat{p}_0^N < \hat{p}_3^S$, $\pi_{M(i)}$ may be greater than $\pi_{M(j)}$, because $D_0^N > D_0^S$. However, if $\hat{p}_0^N > \hat{p}_3^S$, $\pi_{M(i)}$ may be less than $\pi_{M(j)}$, because $D_0^N < D_0^S$. The challenge to Manufacturer 1 is to achieve a balance between them.

**Theorem 7.** In the identical case,

(i) $\hat{p}_0^S > \hat{p}_0^i$ and $\hat{p}_0^S < \hat{p}_0^i$ if and only if $w < w$, where $w$ is a threshold and
\[ w^* = (2b + \beta)(2b - \beta) + c(2b + \beta)(2b - \beta) - 2b(2b - \beta) + [m(1 - \lambda) - c(2b + \beta)(2b - \beta)]^t(3b^2 - 2b^3 + 2b^2) \]n \times [m(1 - \lambda) - c(2b + \beta)(2b - \beta)] + 2\theta v(2b - \beta)(2b + \beta)]/[3(3b^2 - 2b^3 + 2b^2)] \].

(ii) \( \pi_{R}^{0} > \pi_{R}^{N} \).

(iii) \( \pi_{M}^{0} > \pi_{M}^{N} \) if and only if \( (\bar{p}_0^N - \bar{p}_1^N) \geq \max(0, A_1) \), or \( (\bar{p}_0^S - \bar{p}_1^S) \leq \min(0, A_1) \), where \( A_1 = \frac{1}{b}(\beta - 2\beta - 2b\beta) \).

Theorem 7 indicates that \( \bar{p}_0 \) and \( \bar{p}_1 \) is greater than that of the Nash game when \( w \) is less than the threshold \( w^* \). With the fixed wholesale price, the retailer is always better off as the leader in Scenario 1 of the Stackelberg game. The discussion of Lemma 3 and (i) show that an appropriate wholesale price may weaken or damage the follower in the Stackelberg game for Manufacturer 1. When the structure of the competition changes from the Nash game to Scenario 1 of the Stackelberg game, if \( (\bar{p}_0^S - \bar{p}_1^S) \leq \max(0, A_1) \), then the wholesale profit for Manufacturer 1 is less than the threshold \( \bar{p}_1^N \) and \( \bar{p}_1^S \).

Lemma 4 shows that consumers face a smaller variance of \( \bar{p}_1^* \) than that of \( \bar{p}_1 \). Similar to Lemma 3, the leader has a great influence on retail prices in the market. To maximize \( \pi_{(R,W)} \), Manufacturer 1 may raise \( \bar{p}_0 \) to offset the loss demand \( D_0^* \) or reduce \( \bar{p}_0 \) to attract consumers. From (iii), the retailer may prefer to use the Nash game when \( \bar{p}_0^S \geq \bar{p}_0^N \) and the second best game when \( \bar{p}_0^S < \bar{p}_0^N \). Similarly, when \( \bar{p}_1^S \geq \bar{p}_1^N \), \( \pi_{M(W)}^{0} \) may be greater than \( \pi_{M(W)}^{S} \) because \( \bar{D}_1^N \geq \bar{D}_1^{S} \). Otherwise, \( \pi_{M(W)}^{0} \) may be less than \( \pi_{M(W)}^{S} \).

Theorem 8 indicates that in the identical case,

(i) \( \bar{p}_1^0 \geq \bar{p}_1^N \) and \( \bar{p}_1^S \geq \bar{p}_1^N \) if and only if \( w \leq w^* \), where \( w^* \) is a threshold and

\[ w^* = (2b + \beta)(2b - \beta)(2b - \beta) + [m(1 - \lambda) - c(2b + \beta)(2b - \beta)] + 2\theta v(2b - \beta)(2b + \beta)]/[3(3b^2 - 2b^3 + 2b^2)] \]n \times [m(1 - \lambda) - c(2b + \beta)(2b - \beta)] + 2\theta v(2b - \beta)(2b + \beta)]/[3(3b^2 - 2b^3 + 2b^2)] \].

(ii) \( \pi_{R}^{0} \geq \pi_{R}^{S} \).

(iii) \( \pi_{M}^{0} \geq \pi_{M}^{S} \) if and only if \( \bar{p}_1^S \geq \bar{p}_1^N \).

Theorems 7 and 8 show that both Manufacturer 1 and the retailer can benefit from being the leader in the Stackelberg game. However, Manufacturer 1 and the retailer may not always benefit from being the follower. Manufacturer 1 could also reduce the damage as a follower in Scenario 1 and enable the retailer to realize a higher profit in Scenario 2 than in the Nash game by setting an appropriate wholesale price in the contract. Finally, this work provides a summary of some cooperative managerial strategies for Manufacturer 1 and the retailer: (1) If the conditions mentioned in (iii) of Theorems 7 and 8 hold, then both Manufacturer 1 and the retailer can benefit from the Stackelberg game. Otherwise, they avoid the Stackelberg game because both suffer from being a follower. (2) If the condition mentioned in (iii) of Theorem 7 or Theorem 8 does not hold, then the follower is worse off. Therefore, if the leader wants to play the Stackelberg game, the follower may be compensated by side profits from the leader, or the leader may be compensated by the follower if both players decide to play the Nash game. Therefore, Manufacturer 1 and the retailer might not necessarily consider being a follower harmful, but rather a chance to negotiate for their mutual benefits.

3. Numerical examples

The next objective is to further explore managerial insights using numerical examples. To avoid the complex effects of cost parameters, this work considers numerical examples for the identical case. Information on the parameter values are referred to the existing numerical studies (Hsieh and Wu, 2009; Kurata et al., 2007). The parameters are set as follows: \( a_1 = 100, b_1 = 10, \beta = 4, z_1 = 1, m_1 = 0.5, s_1 = 0.5, t_1 = 0.5, c_1 = 6, \eta_1 = 0.5, \) for \( i = 0, 1, 2, w_0 = 9, v_1 = 3, \) and \( \eta_2 = 0.5, \) for \( i = 1, 2. \) The random variables \( x_i \) and \( y_i \) are assumed to be uniformly distributed over \([1 - x_i, 1 + x_i]\) and \([1 - y_i, 1 + y_i]\), where \( x_i \) and \( y_i \) are determined by specifying the coefficients of variation \( c_{x_i} = 0.35 \) and \( c_{y_i} = 0.2 \).

3.1. The effects of \( \beta \) and \( \beta_i \)

The effect of a change in \( \beta \) and \( \beta_i \) is examined. Each parameter varies by ±20%. Fig. 2 shows that the effects of \( \beta \) and \( \beta_i \) on \( \pi_{x_i}^{S} \) are stronger than the effect of \( \beta_0 \) and \( \beta_0 \). Hence, the retailer could develop consumer brand loyalty for Products 1 and 2, improve product displays in the traditional channel, and provide a comfortable shopping environment for consumers to increase profit. Fig. 3 shows that Manufacturer 1 could develop consumer loyalty for Product 1 sold through the Internet channel to decrease \( b_0 \). Finally, consumer channel loyalty to Product 1 is less important for the retailer and Manufacturer 1 because \( \beta_0 \) increases as channel loyalty decreases.
Therefore, consumer brand loyalty is more critical than channel loyalty for both Manufacturer 1 and the retailer. The Stackelberg game shows similar phenomena from Figs. 4–7. Thus, the previously proposed marketing activities remain profitable for Manufacturer 1 and the retailer, regardless of the competition structure.

3.2. The effect of competition structure change

The numerical results for retail prices, demands, and profits for the Nash and Stackelberg games are summarized in Table 1. First, we focus on the results of the Nash game and Scenario 1 of the Stackelberg game. Observations show that $D_0$ in Scenario 1 of the Stackelberg game is greater than that in the Nash game because the increments of $p_1$ and $p_2$ are greater than the increment of $p_0$. The retailer is better off by raising retail prices to cover the lost demand. The increase in $\pi_{M1}$ can compensate for the decline in $\pi_{M1W1}$ in Scenario 1 of the Stackelberg game. The following discusses the changes in competition between the Nash game and Scenario 2 of the Stackelberg game. Manufacturer 1 not only gains a higher Internet profit, but also gains a higher wholesale profit in Scenario 2 of the Stackelberg game than in the Nash game. The higher retail price $p_0$ produces a higher unit margin of Product 1 for Manufacturer 1. However, the increments in $p_1$ and $p_2$ are less than the increment in $p_0$, so there is an increase in the traditional channel’s demand. The rising retail prices and rising demands for Products 1 and 2, sold through the traditional channel, result in a growth in the retailer’s profits. As mentioned, it is always profitable to be a leader in the Stackelberg game. In this numerical setting, Manufacturer 1 and the retailer can also benefit as followers in Scenarios 1 and 2 of the Stackelberg game. Hence, the follower does not always need to view being a follower as harmful, but rather as a coordinating mechanism to benefit both the leader and the follower.

3.3. The effects of $v_1$ and $v_2$

Increased service value may stimulate consumers to purchase products, but an increased retail price may deter some consumers. Hence, the retailer cannot offer an unlimited increase in service value. Focusing on the retailer’s profit in Fig. 8, we see that the retailer’s profit increases with $v_1$ in the beginning. In this portion of the graph, an increase in $v_1$ induces a higher $D_1$ and a higher $p_1$ as shown in Figs. 9 and 10. As the trend continues, beyond a critical value of $v_1$, the retailer’s profit decreases with $v_1$. In our example, this occurs at $v_1 = 1.8$. When service value is higher than the critical
value, there is an immediate sharp decline in $D_1$ because some consumers cannot afford a high $p_1$ for Product 1. Hence, consumers may purchase Product 1 through the Internet channel or purchase Product 2 through the traditional channel. However, the increased $p_1$ and the rising $D_2$ cannot compensate for the decrease in $D_1$, which causes a decrease in the retailer’s profit.

We now focus on the profit of Manufacturer 1 shown in Fig. 8, which slightly drops and gradually rises in conjunction with $v_1$. An increase in $v_1$ results in a decrease in $D_0$ in the beginning, as shown in Fig. 9. The wholesale profit for Manufacturer 1 does not offset

<table>
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<th>Model</th>
<th>Retail price</th>
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<th>Profit</th>
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<td>$p_2$</td>
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Table 1
Numerical results for all models.

Fig. 8. Effect of $v_1$ on profits.

Fig. 9. Effect of $v_1$ on demands.

Fig. 10. Effect of $v_1$ on retail prices.

Fig. 11. Effect of $v_2$ on profits.

Fig. 12. Effect of $v_2$ on demands.

Fig. 13. Effect of $v_2$ on retail prices.

Table 2
Parameters values for the effects of wholesale prices.

<table>
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<th>$a$</th>
<th>$b$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$c$</th>
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</tr>
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</table>
the lost profit of Internet channel in this example. However, as service value continues to grow, it is advantageous for Manufacturer 1 to compete with the retailer’s service by charging \( p_w \), which is smaller than \( p_R \). Figs. 11–13 summarize the effect of an increase in \( p_R \) on profits, demands, and retail prices, respectively. The observations for increased \( p_S \) are similar to those for increased \( p_R \). Therefore, for different marketing parameters, the retailer can add an appropriate service value to the product to erode the Internet channel, attract more consumers, and improve the retailer’s profit. Manufacturer 1 might not necessarily view the increase in service value as harmful, but rather as beneficial.

3.4. The effects of wholesale prices \( w_1 \) and \( w_2 \)

The wholesale prices up to this point have been given as fixed in a contract. This subsection incorporates wholesale prices as decision variables for Manufacturers 1 and 2. The decision procedure unfolds in two stages. In the first stage, both Manufacturers 1 and 2 simultaneously set the wholesale prices \( w_1 \) and \( w_2 \) to maximize their respective profits. In the second stage, the retail prices \( p_0, p_1, \) and \( p_2 \) are set in one of three scenarios as previously discussed. Kumar and Ruan (2006) and Dumrongsiri et al. (2008) adopted similar decision procedures. This subsection conducts a numerical experiment consisting of different parameter combinations and checks if the results and phenomena revealed in Section 2 still hold. Parameter values are chosen according to three levels (low, medium, and high) of a parameter over a reasonable range and control of the relationship \( b = 2/b \) as given in Assumption 1. The combinations yield 4374 instances described in Table 2. The effects of salvage cost and shortage cost are omitted and the remaining parameters are set as the given example in Section 3. Except for 78 instances eliminated, where the corresponding demands or retail prices violate the assumptions of the proposed model, we focus on 4296 effective instances to analyze the performance of the results.

Relaxing the wholesale prices and incorporating them as manufacturer decision variables showed that the results of Theorem 3 and Corollary 1 hold in all instances. Consequently, the managerial insights proposed in Section 2 remain valuable to manufacturers and the retailer. Some interesting results are summarized in Table 3 in which \( \Delta \pi_S = \pi_{S1}^1 - \pi_{S2}^2 \) and \( \Delta \pi_M = \pi_{M1}^2 - \pi_{M2}^2 \). From Table 3, the retailer prefers to be a price leader in all instances (100%); however, Manufacturer 1 prefers to be a retail price leader in 3,564 instances (85%). The retailer is significantly benefited to be a price leader and gains at least 0.645% extra profit in Scenario 1 of the Stackelberg game. However, although Manufacturer 1 does not benefit from being the leader in a few instances, the lost profit is at most 0.017% in Scenario 2 of the Stackelberg game. Thus, being a price leader remains an attractive pricing structure for Manufacturer 1 in most instances.

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