An integrated cost model for production scheduling and perfect maintenance

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Abstract: Production scheduling deals with scheduling production jobs on a machine (single or multiple) in order to optimise a specific objective such as total weighted completion times or total weighted tardiness. The assumption that machines are always available for processing jobs is generally used in the production scheduling literature. In reality, machines often are unavailable due to preventive maintenance activities or machine failure. Production scheduling and preventive maintenance planning are interrelated, but are most often treated separately. This interdependency seems to be overlooked in the literature. This work integrates, simultaneously, the decisions of preventive maintenance and job order sequencing for a single machine. The objective is to find the job order sequence and maintenance decisions that would minimise the expected cost.

Keywords: production scheduling; maintenance; deteriorating production process; integrated models; unreliable machine.


Biographical notes: Laith A. Hadidi received his BSc in Mechanical Engineering and MSc in Industrial Engineering from the University of Jordan in 2002 and 2005, respectively. Currently, he is a PhD candidate in the Department of Systems Engineering at King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. His areas of interest include preventive maintenance planning, operations sequencing and scheduling, production systems design and simulation.
1 Introduction

The integration between production elements has received attention in recent years. This integration is strongly justified by significant savings in operations costs; however, integrated models are not easy to solve. This difficulty is due the fact that integrated models deal with multiple conflicting objectives. In this work, production scheduling and maintenance operations, for a single machine, are integrated at the shop floor level. It is a common practice to schedule both of them independently. This independent planning is done through separate functional teams. The resulting plans of a specific function may disrupt the other function plans. For example, the maintenance function assigns scheduled shut-down intervals. These intervals will be communicated to the production unit. The suggested maintenance intervals may maximise the machine availability, but they will affect production plans. Similarly, production schedulers may have the tendency to utilise machines to their full capacity to meet demand. Under this condition, productivity may increase, but machine availability will decrease, due to having more breakdowns.

This work integrates, simultaneously, the decisions of preventive maintenance and job order sequencing for a single machine. The motivation for this work is encouraged by the need of many real life applications, for example, the automotive industry (Aksoy and Ozturk, 2010). In addition, this integration is expected to provide a reduction in the total expected cost. Each job order \( i \) consists of serving \( Q_i \) work pieces on a single machine. The manufacturer is expected to deliver \( n \) job orders with different processing times \( P_i \). The machine is subjected, upon failure, to minimal repair action where each repair will cost the manufacturer \( c_m \) and restores the machine to work with no improvement in its condition. The manufacturer can pick a preventive maintenance major action only before the start of serving a job order. This action will cost the manufacturer \( c_p \) and will restore
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the machine to an ‘as good as new’ condition reducing the chance of machine breakdown during operation. All job order work pieces are released to the shop floor at time 0, hence; the manufacturer should consider the holding cost during the scheduling horizon. The Holding Cost (HC), Minimal Repair Cost (MRC) and Preventive Maintenance Cost (PMC) compromise the expected cost. Each job order sequence will change the average expected cycle costs. The objective is to find the job order sequence and maintenance decisions that would minimise the expected cost.

In the literature, some work has been done to investigate the integrated optimisation problem of preventive maintenance and production scheduling. Cassady and Kutanoglu (2005) investigated this integration, for a single machine, by conducting an extensive experimental study using small scheduling instances. The integrated objective was to minimise the total weighted expected completion times for a single machine. Similarly, Cassady and Kutanoglu (2003) investigated the benefits of integration with the objective of minimising total weighted expected tardiness for a single machine. In Yulan et al. (2008), five objectives, including minimising maintenance cost, makespan, total weighted completion time of jobs, total weighted tardiness, and maximising machine availability, are simultaneously considered to optimise the integrated problem of preventive maintenance and production scheduling introduced by Cassady and Kutanoglu (2005). In order to minimise the total tardiness, Kuo and Chang (2007) proposed a solution method to find the optimal integrated problem for a single machine under a cumulative damage process.

As observed in Cassady and Kutanoglu (2003), integrated models had proved an expected reduction in cost that might reach up to a 30% savings, over treating the two problems independently. However, no analytical solutions were provided for these integrated models. Instead, a full enumeration for small size problems was suggested to solve the integrated problem (Cassady and Kutanoglu, 2003, 2005). Hence, the need for more analytical or near optimal solutions is highly appreciated. The previous models do not capture the direct costs associated with machine downtime (e.g., labour, parts, etc.) of machine failures and machine maintenance. Hence, the need for cost based models of the integrated problem is justified (Gribkovskaia et al., 2010; Khanra et al., 2010; Goswami et al., 2010; Sun-Lee and Yoon, 2010; Karamatsoukis and Kyriakidis, 2010). The use of mathematical modelling for the purpose of production scheduling or preventive maintenance planning is well established in the literature (Diaby, 2010a, 2010b). Typically, preventive maintenance planning models are stochastic models designed to either maximise equipment availability or minimise equipment maintenance costs.

The focus of this work is to formulate a cost model that simultaneously considers, maintenance, minimal repair and holding costs for several production jobs with the objective to minimise the expected total costs. The maintenance is assumed to be perfect and restores the machine to an ‘as good as new’ condition. Hence, after each maintenance action a new cycle will commence without being affected by the machine condition before maintenance. The expected cycle length includes: average holding time, average maintenance time, and average minimal repair time. Similarly, total expected cycle cost includes: expected maintenance cost, expected minimal repair cost and expected holding cost. The organisation of the rest of this paper is as follows. Section 2 defines, in detail, the integrated problem and its assumptions.
Sections 3 and 4 provide notations and formulation of the integrated model as a mathematical program. Section 5 presents and solves an example for the integrated problem. Section 6 derives independent maintenance model and compares it with the integrated solution found in Section 5. Finally, concluding remarks and future directions for research are shown in Section 7.

2 Problem definition

Consider \( n \) job orders to be processed on a single machine that is subject to a Preventive Maintenance (PM) requirement. These job orders are available at time zero with no precedence constraints. Each job order consists of processing \( Q_i \) work pieces. Moreover, the machine is available continuously along the time horizon unless a machine breakdown occurs during job processing. If so, a minimum repair is conducted that restores the machine to its condition before breakdown. Also, the job that was interrupted by the breakdown should resume the remaining portion of the job, after machine repair. Each breakdown will delay completion time of successive jobs by the needed time to repair \( t_r \) (assumed to be constant). To reduce the chance for machine breakdown, a PM activity can be performed before starting any job. If so, it will restore machine condition to ‘as good as new’. This PM activity will delay successive jobs by the time of PM \( t_p \) (assumed to be constant). The machine may or may not fail, causing the completion time for each job to be stochastic. The PM decisions affect the stochastic process governing machine failure. Hence, change the expected value of job completion time \( E(c_i) \).

Each maintenance action costs a fixed PM cost \( c_p \). Similarly, each break down will cost a fixed minimal repair cost \( c_m \). It is assumed that all jobs are released to the shop floor ready to process at the start of the schedule. Hence, given that \( Q_i \) is the average work-in-process inventory, each job order \( i \) waiting on the shop floor will incur a holding cost per unit time \( hQ_i \) till the time when the machine completes job order \( i \), that is, \( E(c_i) \) and the total holding cost will be \( hE(c_n) \sum_{i=1}^{n} Q_i \). The details of these costs will be discussed in Section 4. The problem would be to identify a set of PM decisions, as well as a set of job sequencing decisions in a way that reduces the total weighted expected completion times. Prior to the job starting, a decision has to be made whether to perform PM or not. If so, PM will take a constant time that will delay consequent jobs by such time \( t_p \). If not, the job will start at the completion time of the previous job. However, the single machine can fail during job processing. The number of failures, during job processing, is strongly affected by the machine age i.e., when the machine ages it has a higher probability to fail. Furthermore, failures are randomly distributed over machine operation. Let \( N(\tau) \) be the number of machine failures in \( \tau \) time units of machine operation. Given that \( \tau_{P_i} \) is the time units of machine operation over \( P_i \), then, \( E[N(\tau_{P_i})] \) is the expected number of failures during machine operation for job \( i \). For job \( i \), the PM will reduce \( E[N(\tau_{P_i})] \), but it will delay the start of job \( i \) by \( t_p \). This situation can be represented by a binary variable \( y_i \). Let

\[
y_i = \begin{cases} 
1 & \text{if PM is performed before job } i \\
0 & \text{Otherwise} 
\end{cases}, \quad i = 1, 2, \ldots, n. 
\]
Without loss of generality, Figure 1 represents a Gantt chart for job sequence $j_1, j_2, \ldots, j_{n-1}, j_n$ with hatched area representing PM decisions $y_it_p$.

**Figure 1**  Expected cycle costs for the integrated problem (see online version for colors)

$$E(Cost) = c_1 \sum_{i=1}^{n} |y_i| + c_2 \sum_{i=1}^{n} E[N(\tau_{j_i})] + hE(c_n) \sum_{i=1}^{n} \tilde{Q}_i$$

The following assumptions are considered in the problem:

1. Jobs cannot be preempted for PM
2. Jobs interrupted by failure can be resumed after repair without additional time penalty
3. Machine has increasing hazard rate
4. Upon failure, minimum repair is conducted and machine will resume with the same age
5. PM restores the machine to a ‘as good as new’ condition
6. Repair times are deterministic and known in advance
7. The number of machine breakdowns is unknown (random variable)
8. The number of breakdowns does not depend on job type
9. Machine breakdowns are independent
10. Raw material for all job orders are released at the start of the schedule.

### 3 Notations

- **PM**: Preventive maintenance
- $n$: Number of jobs to be scheduled
- $P_j$: Processing time of job $j$
- $x_{ij}$: Job sequencing decision variable
- $P_i$: Processing time of the $i$th job in the sequence
- $c_{i}$: Completion time of $i$th job in the sequence (deterministic case)
- $\beta$: Weibull shape parameter for probability distribution of $T$
- $\eta$: Weibull scale parameter for probability distribution of $T$
4 Formulation

Consider a single machine in a manufacturing system that is required to process a set of \( n \) jobs, and suppose that preempting one job for another is not permitted. The purpose of production scheduling is to choose an optimal sequence for the jobs. Let

\[
x_{ij} = \begin{cases} 
1 & \text{if the } i \text{th job performed is job } j \\
0 & \text{Otherwise}
\end{cases}
\]

then

\[
P_i = \sum_{j=1}^{n} (P_j x_{ij}) \quad \forall \quad i = 1, \ldots , n.
\]  

Two logical sets of constraints come out; the first set of constraints states that job \( i \) cannot seize two positions at the same time i.e.,

\[
\sum_{j=1}^{n} x_{ij} = 1 \quad \forall \quad i = 1, \ldots , n.
\]

The second set of constraints states that one position cannot hold more than one job i.e.,

\[
\sum_{i=1}^{n} x_{ij} = 1 \quad \forall \quad j = 1, \ldots , n.
\]

**Example:** Suppose a single machine has two jobs needed to be scheduled at with the following parameters: \( P_1 = 2 \) and \( P_2 = 4 \) then the single machine can have two possible sequences: either \( J_1 \) follows \( J_2 \) or \( J_2 \) follows \( J_1 \).
Sequence 1-2 where \( J_1 \) follows \( J_2 \) is expressed with following variables:
\[ x_{11} = 1, x_{12} = 0, x_{21} = 0, x_{22} = 1 P_{[1]} = 2 P_{[2]} = 4 \] (see Figure 2)

Sequence 2-1 where \( J_2 \) follows \( J_1 \) is expressed with following variables:
\[ x_{11} = 0, x_{12} = 1, x_{21} = 1, x_{22} = 0 P_{[1]} = 4 P_{[2]} = 2 \] (see Figure 3).

In addition to choosing a job sequence, it is also necessary to decide whether or not to perform PM prior to each job. The integrated problem is further complicated by the fact that completion times for the jobs are stochastic, because the machine may or may not fail during each job, and PM decisions change the stochastic process governing machine failure. The completion time of a job is a random variable that depends on

- the age of the machine prior to processing the job
- the completion time for previous jobs
- the time to complete PM, and the PM decision
- the job’s processing; the minimal repair time
- the number of machine failures during the job.

PM is assumed to restore the machine an ‘as good as new’ condition i.e., machine age after PM will reduce to 0. Let

\[ a_{[i]} \] be the machine age at the completion of \( i \)th job

then

\[ a_{[i]} = (1 - y_i) a_{[i-1]} + P_{[i]} \quad i = 1, 2, \ldots, n. \]
Suppose the machine used to process the jobs is subject to failure, and the time to failure for the machine, is governed by a Weibull probability distribution, having shaping parameter $\beta$ greater than 1. When the machine fails, it is assumed that the machine is minimally repaired i.e., the machine is restored to an operating condition, but machine age is not altered. This implies that, upon machine failure, the machine operator does just enough maintenance to resume machine function.

The assumption that PM restores the machine to ‘as good as new’ implies that PM is a more comprehensive action than repair that may include the replacement of many key parts in the machine. Also, the operation and maintenance of the machine (between two successive PM’s) can be modelled as a renewal process. Due to the minimal repair, the occurrence of failures during each ‘cycle’ of the renewal process can be modelled using a non homogeneous Poisson process. Given that the job $i$ starts at age $(1 - y_i)a_{[i-1]}$ and ends at age $a_i$, then

$$E[N(\tau P_i)] = \int_{(1-y_i)a_{[i-1]}}^{a_i} z(t)dt$$

$$= \int_{(1-y_i)a_{[i-1]}}^{a_i} \frac{\beta}{\eta} t^{\beta-1}dt$$

$$= m(a_i) - m((1 - y_i)a_{[i-1]})$$

$$= \left(\frac{a_i}{\eta}\right)^{\beta} - \left(\frac{(1 - y_i)a_{[i-1]}}{\eta}\right)^{\beta}$$

(6)

where $z(t)$ corresponds to the hazard function to the underlying Weibull probability distribution and $m(\tau) = \int_0^\tau z(t)dt = \int_0^\tau \frac{\beta}{\eta} t^{\beta-1}dt = \left(\frac{\tau}{\beta}\right)^{\beta}$. Figures 4 and 5 present the effect of maintenance decision $y_i$ on the $E[N(\tau P_i)]$, where it will be reduced from $\int_{(1-y_i)a_{[i-1]}}^{a_i} z(t)dt = m(a_i) - m((1 - y_i)a_{[i-1]})$ (if $y_i = 0$) down to $\int_0^{a_i} z(t)dt = m(P_i)$ (if $y_i = 1$). However, the PM decision, $y_i = 1$, will delay completion time of job $i$ by $t_p$ time units.

The objective is to provide jobs sequence, as well as PM schedules to minimise the expected cost $E(Cost)$.

Figure 4  Job $i$ with PM, $y_i = 1$ (see online version for colours)
The expected cycle cost includes: expected Holding Cost \((HC)\), expected Minimal Repair Cost \((MRC)\), and PM cost \((PMC)\). Each job order will represent processing one batch of size \(Q_i\). Raw materials for all job orders are assumed to be released to the shop floor at time 0. Figure 6 shows the inventory level for job order \(i\). The average inventory for a job order \(i\) is a sequence dependent. Hence

\[
Q_i = \sum_{j=1}^{n} (Q_j x_{ij}) \quad \forall \ i = 1, \ldots, n. \tag{7}
\]

The average holding quantity is equal to the sum of areas I and II (in Figure 6) over the \(E(c_i)\), i.e., \(Q_i = \frac{I + II}{E(c_i)}\)

\[
Q_i = \frac{Q_i \left( \frac{E(c_i)}{E(c_i^2)} \right) - \left[ \frac{P_i + t_r \left[ m(a_{i1}) - m((1 - y_i)a_{i-1}) \right]}{2E(c_i)} \right]}{E(c_i)}
\]

\[
= Q_i \left( 1 - \frac{[P_i + t_r [m(a_{i1}) - m((1 - y_i)a_{i-1})]]}{2E(c_i)} \right). \tag{8}
\]

Figure 7 shows the expected completion times for job 1, 2, \ldots, \(n\).

The expected completion times for \((\text{Job } i = 1, 2, \ldots, n)\) can be found by substituting in a recursive manner as follows

\[
E[c_{i1}] = t_p y_1 + P_{[1]} + t_r [m(a_{i1}) - m((1 - y_1)a_{0})]
\]

\[
E[c_{i2}] = E[c_{i1}] + t_p y_2 + P_{[2]} + t_r [m(a_{i2}) - m((1 - y_2)a_{i1})]
\]

\[
\vdots
\]

\[
E[c_{in}] = E[c_{i[n-1]}] + t_p y_n + P_{[n]} + t_r [m(a_{in}) - m((1 - y_n)a_{i[n-1]})]. \tag{9}
\]
A general formula for \( E[c_{i}] \) can be found as follows

\[
E[c_{i}] = \sum_{k=1}^{i} \left\{ t_p y_k + P_k + t_r E[N(\tau_{P_k})] \right\} \quad i = 1, 2, \ldots, n
\]

= \sum_{k=1}^{i} \left\{ t_p y_k + P_k + t_r \left[ m(a[k]) - m((1 - y_k)a_{[k-1]}) \right] \right\} \quad i = 1, 2, \ldots, n.

(10)

Hence,

\[
\bar{Q}_{[i]} = \frac{Q_{[i]} \left( E(c_{[i]}) - \frac{[P_{[i]} + t_r (m(a_{[i]}) - m((1 - y_i)a_{[i-1]})]}}{2} \right)}{E(c_{[i]})} = Q_{[i]} \left\{ 1 - \frac{[P_{[i]} + t_r (m(a_{[i]}) - m((1 - y_i)a_{[i-1]})]}}{2 \sum_{k=1}^{i} (t_p y_k + P_k + t_r (m(a[k]) - m((1 - y_k)a_{[k-1]})]} \right\}.
\]
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The holding cost is usually estimated by holding cost unit \( h \), which is expressed in monetary unit per workpiece unit per time unit. Let \( E(c[n]) \) be the expected time to complete all \( n \) jobs. Then, the expected holding cost will be

\[
HC = h \times E(c[n]) \times \sum_{i=1}^{n} Q[i]. \tag{12}
\]

Given that \( c_m \) is the expected minimal repair cost per failure. Then expected minimal repair will be

\[
MRC = c_m \sum_{i=1}^{n} [m(a[i]) - m((1 - y_i)a[i-1])]. \tag{13}
\]

Given that \( c_p \) is the cost for each PM. Then expected PM cost

\[
PMC = c_p \sum_{i=1}^{n} y_i. \tag{14}
\]

The expected total cost will be

\[
E(C_{ost}) = HC + MRC + PMC
= hE(c[n]) \sum_{i=1}^{n} Q[i] + c_m \sum_{i=1}^{n} [m(a[i]) - m((1 - y_i)a[i-1])]
+ c_p \sum_{i=1}^{n} y_i. \tag{15}
\]

Finally, the resulting mathematical program for the integrated problem is given as follows

\[
\begin{aligned}
\text{Min} & \left\{ hE(c[n]) \sum_{i=1}^{n} Q[i] + c_m \sum_{i=1}^{n} [m(a[i]) - m((1 - y_i)a[i-1])] + c_p \sum_{i=1}^{n} y_i \right\} \\
\text{Subject to} & \\
P[i] &= \sum_{j=1}^{n} (P_j x_{ij}) \quad i = 1, 2, \ldots, n \\
Q[i] &= \sum_{j=1}^{n} (Q_j x_{ij}) \quad i = 1, \ldots, n \\
a_{[0]} &= \text{Initial machine age} \\
a_{[i]} &= (1 - y_i)a_{[i-1]} + P[i] \quad i = 1, 2, \ldots, n \\
E(c[n]) &= \sum_{k=1}^{n} \left\{ t_p y_k + P[k] + t_r [m(a[k]) - m((1 - y_k)a[k-1])] \right\}
\end{aligned}
\]
\[ Q_{[i]} = Q_{[i]} \left\{ 1 - \frac{[P_{[i]} + t_r [m(a_{[i]}) - m((1 - y_i)a_{[i-1]})]]}{2 \sum_{k=1}^{i} (t_p y_k + P_{[k]} + t_r [m(a_{[k]}) - m((1 - y_k)a_{[k-1]})])} \right\} \]

\[ i = 1, 2, \ldots, n \]

\[ \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n \]

\[ \sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n \]

\[ x_{ij} \text{ binary } i = 1, 2, \ldots, n \quad j = 1, 2, \ldots, n \]

\[ y_i \text{ binary } i = 1, 2, \ldots, n. \]

5 Solving the integrated problem

The formulation shown in Section 4 can be solved through one of the mathematical programming languages. In the following example GAMS language was used to input the model and the BARON solver was used to reach the optimal solution. The BARON solver is a computational system designed for solving non-convex NLP optimisation problems to global optimality. When \( \beta > 1 \), it may be practical to perform preventive maintenance on the machine in order to reduce the increasing risk of machine failure.

Table 1 considers processing 3 job orders consisting of \( Q_1 = Q_2 = Q_3 = 500 \) work piece. Job order 1 needs 6 min for each work piece. Job order 2 needs 3 min for each work piece. Job order 3 needs 2 min for each work piece. The machine age is \( a_{[0]} = 88 \text{ h} \). The preventive maintenance time is \( t_p = 5 \text{ h} \). The machine failure rate follows a Weibull distribution with the following parameters \( \beta = 2, \eta = 100 \).

Upon failure, a minimal repair is conducted with a repair time \( t_r = 15 \text{ h} \). For the Weibull distribution \( m(t) = \int_0^t \frac{\beta}{\eta} t^{(\beta-1)} dt = \left( \frac{t}{\eta} \right)^\beta \). \( h = 1.50 \) $/work piece/hour, \( c_m = $1500 \) and \( c_p = $500 \).

<table>
<thead>
<tr>
<th>Job order</th>
<th>Job order size (work piece)</th>
<th>Processing time per work piece (minute)</th>
<th>Processing time per job order (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>2</td>
<td>16.66</td>
</tr>
</tbody>
</table>

\[
\text{Min} \left\{ h E(c_{[3]}) \sum_{i=1}^{3} Q_{[i]} + c_m \sum_{i=1}^{3} [m(a_{[i]}) - m((1 - y_i)a_{[i-1]})] + c_p \sum_{i=1}^{3} y_i \right\}
\]

Subject to

\[ P_{[1]} = 53x_{11} + 26x_{12} + 18.67x_{13} \]
\[ P_1 = 53x_{21} + 26x_{22} + 18.67x_{23} \]
\[ P_2 = 53x_{31} + 26x_{32} + 18.67x_{33} \]
\[ Q_1 = 500x_{11} + 500x_{12} + 500x_{13} \]
\[ Q_2 = 500x_{21} + 500x_{22} + 500x_{23} \]
\[ Q_3 = 500x_{31} + 500x_{32} + 500x_{33} \]
\[ a_0 = 88 \]
\[ a_1 = P_1 + (1 - y_1)88 \]
\[ a_2 = P_2 + (1 - y_2)\left[P_1 + (1 - y_1)88\right] \]
\[ a_3 = P_3 + (1 - y_3)\left[P_2 + (1 - y_2)P_1 + (1 - y_1)88\right] \]
\[ E(c_3) = \sum_{k=1}^{3} \left(t_p y_k + P_k + t_r [m(a_k) - m((1 - y_k)a_{k-1})]\right) \]
\[ Q_1 = \frac{Q_1}{2} \left[1 - \frac{[P_1 + t_r [m(P_1) + (1 - y_1)88] - m((1 - y_1)88)]}{(t_p y_1 + P_1 + t_r [m(P_1) + (1 - y_1)88] - m((1 - y_1)88))} \right] \]
\[ Q_2 = \frac{Q_2}{2} \left[1 - \frac{[P_2 + t_r [m(P_2) + (1 - y_2)a_{k}]) - m((1 - y_2)a_{k})]}{2\sum_{k=1}^{2} (t_p y_k + P_k + t_r [m(a_k) - m((1 - y_k)a_{k-1})])} \right] \]
\[ Q_3 = \frac{Q_3}{2} \left[1 - \frac{[P_3 + t_r [m(P_3) + (1 - y_3)a_{k}]) - m((1 - y_3)a_{k-1})]}{2\sum_{k=1}^{3} (t_p y_k + P_k + t_r [m(a_k) - m((1 - y_k)a_{k-1})])} \right] \]

\[ x_{11} + x_{12} + x_{13} = 1 \]
\[ x_{21} + x_{22} + x_{23} = 1 \]
\[ x_{31} + x_{32} + x_{33} = 1 \]
\[ x_{11} + x_{21} + x_{31} = 1 \]
\[ x_{12} + x_{22} + x_{32} = 1 \]
\[ x_{13} + x_{23} + x_{33} = 1 \]

The effect of six parameters (\( \beta, t_p, t_r, c_m, c_p, h \)) over the expected cost is shown in the two Tables 2 and 3. A 2^6 = 64 factorial design was used to generate 64 trials (32 trials for \( \beta = 2 \) and another 32 trials for \( \beta = 3 \)). Table 2 shows that the cost will increase with \( c_p, c_m \) and \( h \) increase. The need for more PM can be seen with \( t_r = 30 \) h compared to \( t_r = 15 \) h (trial 1 and 9 in Table 2). The machine will have more failures with the increase of \( \beta \), hence, PM is more economically justified with higher machine failures. This can be shown by comparing the costs with \( \beta = 3 \) and \( \beta = 2 \) where costs are less in Table 3.

The use of mathematical modelling for the purpose of production scheduling or preventive maintenance planning is well established in the literature. Typically,
preventive maintenance planning models are stochastic models designed to either maximise equipment availability or minimise equipment maintenance costs. The next section will show how the integrated solution compares to the independent solution. The PM problem will be treated independently of the production sequence. The optimal PM interval is derived such that the machine availability will be maximised. The PM interval will be then superimposed on the production schedule that was found in Example 1. The resulted cost will be compared to the integrated scheduling cost.

Table 2  Parameters effect on expected cost (β = 2)

<table>
<thead>
<tr>
<th>Trial</th>
<th>β</th>
<th>tp</th>
<th>tr</th>
<th>cm</th>
<th>cp</th>
<th>h</th>
<th>PM</th>
<th>Production jobs</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
<td>15</td>
<td>1500</td>
<td>500</td>
<td>1.5</td>
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### 6 Solution analysis

In Section 4, it was shown how maintenance decision $y_i$ can affect the $E[\text{cost}]$. This section will try to show and investigate the benefits of integrating the job scheduling and PM decisions. This can be done by solving independently for the optimal PM such that the availability of the machine is maximised and see how does it compare to the optimal integrated job sequence.
Machine availability $A(\tau)$ is defined as the ratio of machine uptime $[0, \tau]$ to the total time that includes: uptime, repair time and PM time.

$$A(\tau) = \frac{\text{uptime}}{\tau + t_p + t_r N(\tau)} = \frac{\tau}{\tau + t_p + t_r (\frac{z}{n})^\beta}$$

Hence, to find $\tau^*$, $A(\tau)$ is differentiated with respect to $\tau$ and equal it to zero.

$$\frac{dA(\tau)}{d\tau} = \frac{\tau + t_p + t_r (\frac{z}{n})^\beta - \tau (1 + \frac{\beta t_r}{n} \tau^{(\beta-1)})}{(\tau + t_p + t_r (\frac{z}{n})^\beta)^2} = 0$$

$$\tau + t_p + t_r (\frac{z}{n})^\beta - (\tau + \beta t_r (\frac{z}{n})^\beta) = 0$$

$$\tau = \frac{t_p}{t_r (\beta - 1)} (\frac{z}{n})^\beta.$$

For example 1, $\tau^* = 100 \left[ \frac{5}{15} \right] (\frac{4}{12}) = 57.7$ h. This indicates that the machine should be maintained at age 57.7 h. Since the machine initial age is 88 that is higher than $\tau^*$ then PM is scheduled at time 0 then it will start processing job 3 at age 0. The machine will finish job 3 at age 16.66 and job 2 at age 41.66. Now, if the machine reaches the age of 57.7 h during job 1. In this case, there are two options:

- Schedule PM after job 1. The machine will finish job 1 at age 91.66 h (see Figure 8)
- Schedule another PM before job 1. The machine will finish job 1 at age 53.75 h (see Figure 9)

**Figure 8** 1-0-0 PM, for production sequence $J_3 - J_2 - J_1$
Figure 9 1-0-1 PM, for production sequence $J_3 - J_2 - J_1$

For the given sequence $(J_3 - J_2 - J_1)$ there are $2^3 = 8$ PM decisions that are shown in the following table.

In Table 4, the cost for the schedule in Figure 8 is $182,480$ that is $(\frac{182,480 - 178,030}{178,030} = 2.50\%)$ higher than the optimal integrated solution found. The cost for the schedule in Figure 9 is $181,250$ that is $(\frac{181,250 - 178,030}{178,030} = 1.8\%)$ higher than the optimal integrated solution found.

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Independent planning may provide optimal performance at the level of a specific function. Management usually looks at the production system as a whole and separate optimal solutions may not provide optimal solution for the whole system. Usually, there is a global optimal that includes all major functions in the production system. This global optimal can only be achieved by integrating models for all different functions. Integrated production models are expected to deal with multiple objectives with a conflicting nature. Hence, planning these elements independently will cause conflicts between functions. This disturbance can be avoided through integrated modelling.
7 Conclusions

In this work, production scheduling and maintenance operations, for a single machine, were integrated at the shop floor level. The objective was to find the job order sequence and maintenance decisions that would minimise the expected cost. The contribution of this work was to study the effect of production and maintenance scheduling on WIP inventory. The problem was formulated as a mixed integer program. The integrated solution provided, simultaneously, the production and maintenance schedule.

Integrated modelling is expected to provide better savings over independent models. Hence, integrated modelling has recently gained momentum. However, integrated models are sophisticated and not easy to solve. Research in integrated modelling still has great potential to contribute; justified by the expected savings provided by integrated modelling.

In this work, the repair and maintenance times are assumed to be constant. As an extension to this work, this assumption can be generalised where repair and preventive maintenance times are random variables following a general distribution. The model can be studied under the effect of different repair and preventive maintenance time random distributions. Another extension also can be suggested; it was assumed that the failure hazard rate is not dependent on the production jobs. However, in many cases the machine is drastically affected, by the type of job it performs. This effect can be investigated in future work.

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