Properties of underwater acoustic communication channels in shallow water

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Underwater acoustic channels are band-limited and reverberant, posing many obstacles to reliable, phase-coherent acoustic communications. While many high frequency communication experiments have been conducted in shallow water, few have carried out systematic studies on the channel properties at a time scale relevant for communications. To aid communication system design, this paper analyzes at-sea data collected in shallow water under various conditions to illustrate how the ocean environments (sea surface waves and random ocean medium) can affect the signal properties. Channel properties studied include amplitude and phase variations, and temporal coherence of individual paths as well as the temporal and spatial coherence of multipaths at different time scales. Reasons for the coherence loss are hypothesized. [DOI: 10.1121/1.3664053]

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I. INTRODUCTION

Underwater acoustic communications are useful for many practical applications providing voice/data telemetry between sensor nodes deployed underwater for data gathering or other purposes. For either fixed nodes or mobile nodes (e.g., autonomous underwater vehicles) wireless communications provide substantial saving in cost, shortened deployment time, and convenience in operation compared with a cabled/tethered system. Despite recent research efforts, underwater acoustic communications still face many technical challenges.\(^1\) Compared with radio frequency (RF) communications, underwater acoustic channels are different in many aspects. First, the communication channel is severely bandwidth limited due to sound attenuation by sea water, and interaction with the ocean surface and bottom. Bandwidth efficient communications are generally required (using e.g., phase coherent modulations). Second, sound travels in multipaths with delay spreads covering often hundreds of symbols, causing extensive inter-symbol interference. Third, complex time-varying oceanographic processes and ocean surface waves often produce a channel with short coherence time (or large Doppler spread) making channel tracking difficult. Fourth, Doppler-shift to carrier-frequency ratio in underwater channels is several orders higher than that in the RF channels, due to the relatively low sound speed compared with the speed of light, making symbol synchronization difficult. The reverberant and time-varying nature of the channel (referred to as a doubly spread channel) poses many obstacles to reliable, high-rate communications. The problems are more pronounced at higher frequencies (>10kHz) where the channels are desirable because more bandwidth is available, but are more complex due to increasing (unknown) degrees of freedom that are the subject of this study. We will focus on shallow water in this paper where the problems are least understood.

The goal of robust underwater acoustic communications is to develop efficient algorithms (and acoustic modems) that work for all oceans. Toward such a goal, channel models for various waters are needed to aid the system designer to model the performance and achieve robustness. In reality, one finds that every shallow water is different in some details, and is different from itself at different times, making modeling of the (time variation of the) channel very difficult. (Shallow water here refers to coast water with depth between 10 and 200 m, below 10 m is very shallow water or beach environment.) To understand this, one notes that sound propagation in shallow water is heavily influenced by sound interactions with the inhomogeneities in the water column, and scattering from rough surfaces and bottom. Oceanographic processes in the water column include meso-scale and sub-meso-scale (e.g., front, offshore forcing, surface flux, etc.) processes, (linear and non-linear) internal waves,\(^2\) fine-structure (spice/thermohaline) and micro-fine-structure (turbulence)\(^3,4\) inhomogeneities. These processes are different not only in spatial scale but also in time scale. Some of these (fast varying) oceanographic processes (e.g., turbulence) are partially understood and are currently active research topics in the oceanographic community. On a slow time scale (>tens of minutes), given a measured sound speed profile, one can reasonably model the channel impulse response at low frequencies where the small scale medium fluctuations can be neglected. At high frequencies (>10 kHz), due to the lack of knowledge on the small scale medium fluctuations (their time, range and depth dependence), one has difficulty modeling the channel impulse responses when sound scattering from the inhomogeneities cannot be neglected. Various models have been proposed in the literature to simulate the underwater communications channels.\(^1\) Few have successfully matched the simulation results with real data to the best of our knowledge. None has attempted to model the time evolution of the acoustic channel on the scale of milli-seconds to seconds relevant for communications. From the point of view of the system designer, one important element to performance modeling is the channel temporal and spatial coherence.\(^1\) The ability
to model the temporal coherence is partially successful at low frequencies. Such capability does not exist at high frequencies.

Similar things can be said about acoustic signal scattering from the rough surfaces at high frequencies. While the spectra of large waves are well measured, spectra of small waves and sound speed near the surface are not well known, because of wave splashing, bubbles, etc. Statistical properties (e.g., second moment) of the sound scattering from the surface waves may be modeled, but not the time evolution of the sound field, mainly because the time evolution of the surface waves cannot be predicted.

Thus, while it is desirable to be able to model the channel based on environmental inputs, such an approach is not yet possible due to lack of detailed information about the environment, particularly at high frequencies where fine and micro-fine processes are not well understood. An alternative approach is to gain understanding/knowledge about the channel by studying the temporal and spatial variation of the channel impulse response (CIR), extracted/inverted directly from the acoustic data. While many high frequency communication experiments have been conducted in shallow water, few have attempted to measure and characterize the channel systematically on the relevant scales. Earlier works have reported on the channel variations on the scale of second to minutes. In dynamic oceans, the acoustic channel can vary on the scale of hundreds of milli-seconds or less, since channel coherence time can be that short (see below). To determine whether a (particular) channel equalizer can cope with such variations, knowledge about channel variations on the scale of second to minutes is needed. To our knowledge, no such analysis has been carried out in shallow water. This paper attempts to fill in this gap by presenting experimental data collected from several shallow water experiments. Our purpose is to show how the environment can influence the channel properties, and provide interpretations of the observations based on the underlying physics. Toward this goal, the experiments have been designed in a way such that the effects of the surface waves and ocean inhomogeneities on signal propagation can be largely separated to allow detailed analysis and theoretical interpretations.

Channel analysis is based on communication data collected at sea, where messages are transmitted in packets. From the received data, CIR is estimated as a function of time and depth. From the CIRs, we study the following.

1. Temporal variations of the channel on two (slow and fast) time scales. Temporal correlation between CIRs estimated from different packets, transmitted every 30 s, reveals a slow time scale (30 s to tens of minutes) variation of the channel, referred to as inter-packet channel variation. Fast channel variation is estimated from data within a packet, based on CIRs measured approximately every 0.12 s. Temporal correlation between these CIRs reveals the channel variations within a packet, referred to as the intra-packet channel variations. In dynamic oceans, temporal coherence time can be as short as \(0.2\) s. For these cases, the channels have changed so much that the channels are uncorrelated between the packets.

2. Spatial coherence length of the channel. From the measured CIRs as a function of depth, one obtains an estimate of the spatial coherence of the channel.

3. Individual path statistics. From the measured CIRs, one separates the multipaths and studies the amplitude and phase variations, and temporal correlation of individual paths. The analysis is applied to well-defined sparse path arrivals for a given packet. Note that the paths/arrivals defined for communications (tap coefficients) may, physically, consist of several ray arrivals when their arrival times are close and not resolved by the (available) signal band width. The amplitude statistics of the paths are related to the statistics of the tape coefficients, and the phase-rate statistics are needed to design, and model the performance of, the phase-lock loop. The analysis is applied to an individual packet; it needs to be repeated, in principle, when the channel changes significantly (as when the inter-packet correlation is low). One assumes/hopes that the general characteristics of the distributions vary slowly with time (semi-stationary).

4. The “randomness” of the channel. Within the channel coherence time, the channel can be characterized by a group of dominant arrivals which vary slowly with time (which defines the channel coherence time), and a group of weak scattered arrivals, which are random in nature. The dominant arrivals can be estimated using, for example, a least square method, yielding the so called estimated CIR for that duration of time. From the estimated CIR and transmitted symbols, one obtains the “estimated data.” The mean squared error between the actual and estimated data is referred to as the data estimation error. The data estimation error normalized by the mean signal energy reveals the percentage of the acoustic energy that is random (the scattered arrivals, and noise), and is referred to as the (degree of) randomness of the channel. Generally speaking, the more random the channel the higher the bit error rate.

This paper is organized as follows. The effect of the surface waves on the communication channel is illustrated in Sec. II. The effect of medium inhomogeneities is illustrated in Sec. III. The degree of randomness of the various channels is measured in Sec. IV. Observations from the data are summarized and discussed in Sec. V. Section VI provides concluding remarks. The Appendix expresses the vertical coherence in terms of the normal modes. The results suggest that the vertical coherence is determined more by the vertical structure of the channel (the modal vertical wavenumbers) and less influenced by the ocean inhomogeneities and boundary scattering, as discussed in Sec. V.

II. INFLUENCE OF SURFACE WAVES

In an ocean with either a constant, or an upward refractive (winter) sound speed profile, or a near-surface sound channel in which the source and/or receivers reside, signal
A. Inter-packet channel variations

Each transmitted communication packet contained a probe signal placed at the beginning of each packet for the purpose of symbol synchronization; a linear frequency modulated (LFM) signal was used in this experiment. By matched filtering the received (LFM) data with the transmitted (LFM) signal, an estimate of the CIR is obtained for each packet. By stacking the measured CIRs against transmitted time, one can study the channel variations on the time scale of every 30 s. Figures 1(a) and 1(b) display the measured CIRs under calm and rough sea conditions respectively. Only nine packets of data were collected when the sea was rough [Fig. 1(b)], before the experiment was suspended because of the bad weather.

One notices in Fig. 1(a) that the CIR varies slowly with respect to time under relatively calm sea conditions; note that even under the sea state 0 condition, there exist small waves (with wave height comparable to the acoustic wavelength) that can cause the CIR to vary temporally. Under rough sea conditions, one observes in Fig. 1(b) many more scattered arrivals following the dominant arrivals. These late arrivals are generated presumably by sound scattering from the rough sea surfaces. (The scatters usually occur at steeper angles, thus arrive later than the low grazing angle arrivals) as a function of depth for relatively calm and rough sea conditions. One notes that the CIRs are significantly different across the array, not only in the multipath

<table>
<thead>
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<th>Hydrophone number</th>
<th>Hydrophone depth (m)</th>
<th>Distance to the first phone (m)</th>
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</thead>
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<tr>
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<td>0.04 0.76 1.06 1.39 1.82</td>
</tr>
<tr>
<td>AUVFest07 Rough Sea</td>
<td>20.02</td>
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<tr>
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<td>39.6</td>
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<tr>
<td>TREX04</td>
<td>17.2</td>
<td>0.04 0.76 1.06 1.39 1.82</td>
</tr>
</tbody>
</table>

Fig. 2(a) and Fig. 2(b)] is due to different source-receiver ranges and is well understood.

To characterize the channel variations over time, one calculates the channel temporal coherence function defined by

$$\Gamma(t) \equiv \langle \frac{[p^*(t)p(t+\tau)]}{\sqrt{[p^*(t)p(t)][p^*(t+\tau)p(t+\tau)]}} \rangle,$$

where $p(t)$ is the reference signal time series at geotime $t$, and $p(t+\tau)$ is the signal time series arriving with a lag time $\tau$, the square brackets [ab] denotes the maximum value of the correlation between the a and b time series, and the angular brackets denote ensemble average over the geotime $t$ (using reference signals transmitted at different times).10

From the measured CIRs [Figs. 1(a) and 1(b)], using CIR at different times as the reference signal, one obtains an estimate of the temporal coherence of the channel. The results are shown in Figs. 1(c) and 1(d) for relatively calm and rough sea conditions. One notes that temporal coherence is high ($\geq 0.7$) across (adjacent) packets (30 s apart) under calm sea conditions. High inter-packet temporal coherence means that one can increase the communication packet length (from 25 to 50 s) to improve the average data rate (by eliminating the probe signals and guard time between the packets). In contrast, under rough sea conditions [Fig. 3(d)], the signals are poorly correlated (correlation $\leq 0.5$) between adjacent packets. In this case, each packet travels through a different channel that bears little relationship to and cannot be predicted from the channel encountered by the previous packet.

B. Depth dependence and spatial coherence

The depth dependence of the signal can be studied using the probe signal received on the array. Examples of the measured CIRs (using LFM signals) are shown in Figs. 2(a) and 2(b) as a function of depth for relatively calm and rough sea conditions. One notes that the CIRs are significantly different across the array, not only in the multipath.
arrival time but also in the path amplitudes. The question is whether the multiple signals received by the array are correlated or not. Multi-channel communications use spatial diversity (spatial uncorrelation) to minimize signal fading and inter-symbol interference. When signals are uncorrelated, the probability is small for all signals to suffer a deep fade at the same time. The (minimum) separation distance at which the signals become uncorrelated is referred to as the spatial coherence length. Spatial coherence length is an important parameter for the system design.

Similar to temporal coherence, the spatial coherence function (in this case, vertical coherence) is defined by

$$\Gamma(z_i, \Delta z) \equiv \left\langle \frac{[p_i^*(t)p_j(t)]}{\sqrt{[p_i^*(t)p_i(t)][p_j^*(t)p_j(t)]}} \right\rangle,$$

(2)

FIG. 1. (Color online) CIR determined from LFM signals as a function of time measured from the AUVFest07 data under calm-sea (a) and rough-sea (b) conditions. Color scale in dB. Temporal coherence of the LFM signals under calm-sea (c) and rough-sea (d) conditions.

FIG. 2. (Color online) CIR as a function of depth based on AUVFest07 data under calm-sea (a) and rough-sea (b) conditions. Color scale in dB. Spatial coherence as a function of receiver separation under calm-sea (c) and rough-sea (d) conditions. The solid curve is measured with respect to the top-most receiver, and the dashed curve is measured with respect to the third receiver from the top.
where $p_i(t)$ is the time series on the $i$th receiver at depth $z_i$, $\Delta z$ is the separation between the $i$th and $j$th receivers, and the average is over signal transmitted at different geotime $t$. Spatial coherence is calculated for each LFM signal and then averaged over all (available) LFM signals. The results are shown in Figs. 2(c) and 2(d) using receivers at two different depths as the reference receiver. Note that the vertical structure is in principle inhomogeneous, thus the vertical coherence may be depth dependent. To determine the depth dependence, one needs reference receivers at different depths. One finds little depth dependence based on the current data within the measurement error; this may be due to the fact that the array has limited depth span. Thus, for clarity, only the results using two reference receivers are shown in Figs. 2(c) and 2(d): the solid curves display the spatial coherence measured from the first (top) receiver and the dashed curves display the spatial coherence relative to the third receiver. One notes that because the receiver array has non-even spacing, one obtains diverse spatial sampling (sampling at different spacing) of the vertical coherence using two different receivers as the reference receiver.

One finds that the spatial coherence length evaluated at $\Gamma = 0.8$ (by interpolating the data points) is about 0.16 and 0.09 m for calm and rough sea conditions, and is 0.3 and 0.25 m evaluated at $\Gamma = 0.5$ for the same data. Note that the measurement error (standard deviation) is higher at lower coherence values. For example, the standard deviation is on the order of $\sim 0.02$ m at $\Gamma = 0.8$, and $\sim 0.1$ m at $\Gamma = 0.5$ based on data analysis. Hence, for experimental measurement, one prefers to quote the coherence length at $\Gamma = 0.8$. Based on the arrival time differences, one might associate path 1, 2, 3 with $n = 3, 4, 5$ respectively. But note that identifying the observed paths in Fig. 3(a) with the corresponding ray arrivals is likely to be ambiguous without additional

**C. Intra-packet channel variations**

To study the intra-packet temporal coherence, we use BPSK signals employing a $m$-sequence code. An estimate of the CIR is obtained for each $m$-sequence (approximately every 0.1 s) with little interference from the adjacent $m$-sequences due to the cyclic orthogonality of the $m$-sequence signal. One packet of $\sim 25$ s duration provides $\sim 200$ independent estimates of the CIR as a function of time.

The measured CIRs are shown in Figs. 3(a) for relatively calm sea conditions. One finds that, when the sea surface is relatively smooth (calm sea), the multipaths have stable arrival times over a time interval of 25 s. The dominant path arrivals, identified by the arrows below Fig. 3(a), show little fluctuation in amplitude presumably because they come in at shallow angles and are perhaps totally reflected by the sea surface. Using a ray propagation model, one can estimate the nominal ray arrival times and angles assuming a flat surface, given the source and receiver geometry. One finds that the arrival time difference between the $n$ and $n-1$ ray arrivals are 0.23, 0.7, 1.16, 1.62, 2.08, and 2.54 ms (at a range of 2.3 km) for $n = 1–6$, where $n$ is the number of surface/bottom bounces; $n = 0$ refers to the direct arrival. Based on the arrival time differences, one might associate path 1, 2, 3 with $n = 3, 4, 5$ respectively. But note that identifying the observed paths in Fig. 3(a) with the corresponding ray arrivals is likely to be ambiguous without additional
information such as the absolute travel time or the path arrival angles. Unfortunately, the receiver array does not have the resolution to measure the path arrival angles, and has grating lobes at \( \pm 10^\circ \) grazing angles due to the sparse array element distribution (Table II). The measured beam width is on the order of \(~3^\circ\) in agreement with the calculated beam width. The measured beam power (within \(~10^\circ\) is found to peak at the horizontal direction suggesting that the paths arrive predominantly with small \((< 3^\circ)\) grazing angles which the array cannot resolve. At these angles, these rays are expected to be totally reflected by the (smooth) sea surface. Some weak arrivals remain arriving at a later time implying that they have encountered more bounces with the surface and bottom. They fade in and out with time due to scattering from (small) sea surface waves (at a steeper grazing angle).

When the sea surface is rough, the temporal variation of the CIR within a packet is shown in Fig. 4(a). Arrivals which are not well separated in time are grouped into one path in order to conduct similar statistical analysis similar to that shown above; the paths arrivals are shown by the arrows below Fig. 4(a). To aid path identification, we calculate the (mean) ray arrival-time difference assuming a flat surface. The arrival time-difference between the \( n \) and \( n - 1 \) ray arrivals are 0.11, 0.32, 0.53, 0.75, 0.96, 1.17, 1.39, 1.6, 1.81, and 2 ms (at a range of 5 km) for \( n = 1–10 \), respectively. One finds that identifying the path arrivals in Fig. 4(a) with the ray arrivals is even more challenging than the previous case. The wide spreads of the path arrival times suggest a path in Fig. 4 could involve more-than-one ray arrivals. For example, path 4 could consist of seventh and eighth rays, path 5 could consist of ninth and tenth rays, etc.

When the sea surface is rough, one finds that all path arrivals are being affected by the sea surface. Figure 4(a) shows that all the dominant paths suffer a certain degree of “fading” with respect to geotime, while showing stable arrival (delay) time. This presumably happens when a ray is scattered by the rough surface into an exit angle that is different than the incident angle. As a result, it arrives at the receiver at a different time (different path) with a different energy. For the path in question, it represents significant “fading” (up to 20 dB). Many late (scattered) arrivals are evident in Fig. 4(a). They are more abundant (appearing with higher energy than the calm surface case) because rough surfaces produce longer/stronger reverberation returns.

1. Amplitude and phase fluctuation

For paths having stable arrival (delay) times, one can study their amplitude and phase fluctuations. The path separation is done as follows: one obtains an averaged intensity curve by averaging the intensities of all CIRs, from which one identifies the path arrival time, and their start and end time based on the intensity peaks and their \(~3\) dB points. The start time and end time are used to gate (window) the time series for each path. For each time series, the amplitude of each path is obtained by the square root of the intensity integrated from the start time to the end time of each path. The histograms of the amplitudes, for the three paths indicated in Fig. 3(a), are shown in Fig. 3(b).

One notes that if an arrival is deterministic, the amplitude distribution should have a very narrow width (e.g., path 2). The wider the width, the higher the amplitude fluctuations or, the more random the signal. Ideally, one would like to determine the amplitude distribution in order to build a channel model. In reality, quantitative estimation of the amplitude distribution requires a large amount of data in order to be statistically significant; it is a non-trivial task and is beyond the scope of this paper. From the perspective of channel equalization, the issue is how fast and to what extent does the amplitude vary. The former question can be addressed by investigating the path temporal coherence as discussed later in this section. For the latter question, a rough answer can be obtained by estimating the mean-to-standard deviation (SD) ratio of the distribution. Given that single path amplitude is normally assumed as Rician distributed, one can estimate the mean-to-SD ratio for the Rician distribution to serve as a reference. The Rician distribution is given by

\[
\begin{align*}
px(x) &= \left(\frac{x}{\sigma^2}\right) \exp\left(-\frac{\left(x^2+\sigma^2\right)}{2\sigma^2}\right) I_0\left(\frac{2\sigma^2}{\pi}\right),
\end{align*}
\]

where \( x \) is the amplitude, \( I_0 \) is the zero order modified Bessel function of the first kind, and \( \mu^2/2\sigma^2 \) is referred to as the Rician parameter, representing the power ratio of the deterministic-to-random signal components. The numerical values of the mean-to-SD ratios are given as a function of the Rician parameter in Table III for comparison with data.

From the distributions given Fig. 3(b), one finds that the mean-to-SD ratio is 21.1, 14.8, and 6.3 for the first three paths (Table IV). Assuming that single path amplitude is Rician distributed, one finds that the three paths are mostly deterministic, with a deterministic-to-random ratio greater than 200 for the first two paths, and of the order of 72 for the third path. Note that the Rayleigh distribution is given by setting the Rician parameter to zero. The mean-to-SD ratios suggest that the paths are (definitely) not Rayleigh distributed (not randomly distributed).

For the rough sea data, the amplitude histograms of individual paths are shown in Fig. 4(b), for the five dominant paths shown under Fig. 4(a), numbered 1 to 5 from the left to the right. One finds that the mean-to-SD ratios are 4.2, 2.5, 2.8, 2.9, and 2.7 for path 1 to 5, respectively. These values are significantly lower than that reported above for the calm sea data, consistent with the fact that paths are less deterministic in rough sea than calm sea conditions. Quantitative comparison can be done by comparing the measured mean-to-SD ratios with that given in Table III. One finds that the deterministic-to-random ratios are now less than 32 for all the paths. On the other hand, they all have sufficient deterministic components suggesting that a Rayleigh amplitude distribution will not be a good assumption (as commonly done in the published literature). For example, for path 5, which has the lowest mean-to-SD ratio, the deterministic-to-random ratio is of the order or greater than 8. (The above analysis is limited to data within one packet, since channels have changed significantly between packets under the rough sea conditions. The general features are expected to be representative of other packets.)

The gated time series for each path (within the path start and end time), obtained from CIRs at different times, can be
correlated to determine the temporal correlation of the corresponding path. One plots the maximum of the autocorrelation (time series), normalized by the geometric mean of the path energies, as a function of the lag time between the CIRs as shown in Figs. 3(c) and 4(c) for the calm and rough sea conditions, respectively. One finds that under calm sea conditions, all three paths are consistently and highly coherent over the duration of the packet. Under the rough sea conditions the paths remain coherent (>0.8), except when they suffer from fading. The temporal coherence curves of selected paths as shown in Fig. 4(c).

The phase of each path relative to its initial phase is determined from the phase of the path correlation. The results are shown in Figs. 3(d) and 4(d) for the calm and rough sea conditions. One finds that the phase changes relatively smoothly over time under calm sea conditions and more rapidly with time under the rough sea conditions [note the oscillations of the phase in Fig. 4(d)].

2. Intra-packet temporal coherence

While individual paths may remain temporally coherent, their combination may not, because the relative phases between them could vary rapidly with time. Multipath fluctuation can be measured in terms of the temporal coherence of the received signal, defined in Eq. (1), using measured CIRs from the same packet of data. Using the first CIR as the reference, averaging over many packets of data, the result is shown in Fig. 5(a) and 5(b) as a function of the lag time for calm and rough sea conditions respectively. Noting that the CIRs are measured every $\frac{1}{24}$ s, the temporal coherence functions reported here reveal a “fast-time” variation of the channel as opposed to the “slow-time” variation of the channel determined from the inter-packet correlation, sampled every $\frac{30}{24}$ s, shown in Figs. 1(c) and 1(d).

<table>
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<tr>
<th>$\nu^2/2\sigma^2$</th>
<th>Mean-to-SD ratio</th>
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<tr>
<td>0</td>
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<td>2</td>
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One finds that the temporal coherence of the intra-packet CIRs shows a high value ($\geq 0.8$) over 25 s under relatively calm sea conditions [Fig. 5(a)]. In contrast, the intra-packet temporal coherence drops to below 0.5 in a very short time ($\sim 0.2$ s) under rough sea conditions [Fig. 5(b)]. We observed above that even under the rough sea conditions, individual paths still maintain a high temporal coherence except when the signal fades [Fig. 4(c)], but paths arrive with rapidly varying phases. Hence, we attribute the loss of the multipath temporal coherence under rough sea conditions to both signal fading and rapidly varying (relative) phase between the dominant arrivals, as well as the increased presence of long diffused arrivals.

From Figs. 5(a) and 5(b), one obtains an estimate of the coherence time, $\tau_{0.8}$, defined as the lag time when the coherence drops to 0.8. One finds that $\tau_{0.8}$ is approximately 0.1 s under rough-sea conditions during the AUVFest07 experiment [Fig. 3(b)]. For the calm sea conditions, $\tau_{0.8}$ is about 30 s from Fig. 1(c). As noted before, the temporal coherence varies significantly from packet to packet; the standard deviation of the variation is high when the coherence value is low (e.g., below 0.5). Hence $\tau_{0.8}$ is used in experimental data analyses.

III. INFLUENCE OF MEDIUM INHOMOGENEITIES

In other than winter time, the surface layer is usually warmer than the water below, causing the sound to refract downward. The sound speed profile varies with range and time due to oceanographic processes created by the temperature/density differences inside the water column. To study the effect of random ocean media on acoustic communications, we compare the open ocean data from the 2004 Time Reversal Experiment (TREX04) with data from the 2006 Underwater Network (UNet06) experiment, which was collected in a semi-stationary shallow water environment.
These two environments have approximately the same water depth, and similar downward-refractive sound speed profiles as shown in Fig. 6. The sea surfaces are relatively smooth. The effect of the surface waves on sound propagation can be “ignored” for the source and receivers placed below the layer as most acoustic rays are refracted downward by the warm surface layer.\(^{13}\)

The TREX04 experiment took place on the New Jersey shelf (near the Mid-Atlantic Bight). Internal wave activities have been observed in this area during the summer time based on thermistor string data and high frequency backscattering images.\(^{14}\) Evidence of turbulence generated by shear instability at the edge of internal waves has been previously reported.\(^{15}\) During the TREX04 experiments, no thermistor string was deployed, thus one cannot say positively whether internal waves were present. On the other hand, a SIMRAD EK500 scientific echo sounder was deployed, which reveals abundant evidence of internal-wave-like features from the backscattering images\(^{16}\) (in some areas more than others). The acoustic data analyzed below suggest that the signal encountered rapid temporal fluctuation, consistent with the hypothesis of a highly dynamic environment where internal waves are a potential source contributing to the signal fluctuations.

Source-receiver range and depths, water depths for the experiments are given in Table I. Receiver array configurations are given in Table II. Quadrature phase-shift keying signals were transmitted during these experiments with a carrier frequency centered at 17 kHz, with a bandwidth of 4–5 kHz. The data have high input signal-to-noise ratios of \(~18–24\) dB.

### A. Inter-packet channel variations

An estimate of the CIR is obtained for each packet from the LFM signal every 30 s. Figure 7(a) illustrates the CIR from the (stationary ocean) UNet06 experiment and Fig. 7(b) illustrates the CIR from the (dynamic ocean) TREDX04 experiment. For the UNet06 data [Fig. 7(a)], one observes three dominant arrivals. They are refractive bottom-reflected (RBR) ray arrivals, traveling a large portion of the water column (\(~3\) to 60–65 m depending on the ray angle); the source and receivers are in the upper water column. The first arrival (at \(~5\) ms delay time) shows a relatively stable amplitude with a stable arrival time for the first 15 min and then starts to fade out in the last 15 min. The second arrival (at \(~9\) ms delay time) is the most stable of all. The third arrival (at \(~12\) ms delay time) exhibits more fluctuation in arrival time as well as more frequent signal fading. There are several other unstable arrivals weaker in amplitude (at 2–3, 6, and \(~14\) ms delay time).

For the TREX04 data [Fig. 7(b)], one notes that most of the energy is concentrated in the first 7 ms, followed by long...
and weak arrivals. The CIR exhibits a very different arrival structure than that shown in Fig. 7(a). The different arrival structures are due to the difference in sound speed profiles and source-receiver depths. Note that the TREX04 sound speed profile presents a high gradient near the bottom and in the upper water column. The source and receivers are located close to the sound axis. As a result, sound travels via mostly refractive rays which do not touch the bottom or surface. The refractive rays stay mostly in the middle and lower water column centered around the sound axis. The delay-time differences (path-length differences) between the refractive rays are small compared with that between RBR rays in the UNet06 environment which travel a large portion of the water column.

Given a mean sound speed profile, the mean arrival structure can in most cases be modeled (or fitted by a model). From the point of view of communications, identifying/modeling the mean path arrivals is not the most critical task, since an acoustic modem has no advance knowledge of what the arrival structure will be. As remarked above, it is the time varying nature of the arrivals (how fast and to what extent, random- to-deterministic ratio) which determines the equalizer performance. The random nature of the arrivals is in this case attributed to medium inhomogeneities which are known to generate micro-path rays or coupled modes through simulation studies.17

Figures 7(c) and 7(d) shows the inter-packet temporal coherence based on data from Figs. 7(a) and 7(b). One notes that temporal coherence is high (≥0.7) across packets (30 s apart) for the stationary ocean [Fig. 7(c)] environment, whereas in a dynamic open ocean environment, the signals are poorly correlated (with coherence ≤0.5) between packets [Fig. 7(d)].

B. Depth dependence and spatial coherence

The depth dependence of the CIR can be illustrated using LFM signals received on the vertical line array as shown in Figs. 8(a) and 8(b) for the UNet06 and TREX04 environments. One sees well-defined sparse multipath arrivals in the UNet06 data, showing little depth dependence. In contrast, one observes significant depth dependence in the TREX04 data. Not only are the dominant arrivals depth-dependent, but so are many scattered late arrivals which are random and different from one receiver to the other. (This is evidence of the dynamic nature of the propagation environment.)

Using the first (top) receiver as the reference phone, one can calculate the cross-correlation of the signals across the array. Averaging the normalized correlation over many LFM signals, one obtains the vertical coherence function as shown in Figs. 8(c) and 8(d) (the solid curves) for the UNet06 and TREX04 environments. Similarly, to study the depth dependence of the coherence, one uses another receiver, for example, the third receiver, as the reference receiver, and obtains the vertical coherence function as shown by the dashed curves in Figs. 8(c) and 8(d). The spatial coherence length evaluated at Γ = 0.8 (by interpreting the data points) is approximately 0.15 and 0.1 m for the UNet06 and TREX04 environments respectively, with a standard
deviation of the order of $\pm 0.02$. The spatial coherence length evaluated at $\Gamma = 0.5$ is 0.3–0.4 m and 0.25 m for the Unet06 and TREX04 data. The standard deviation of the measurement is relatively large, of order of 0.1 m at $\Gamma = 0.5$ as remarked before. The above analyzed signal properties are summarized in Table I.

Assuming that the spatial coherence follows an $\exp(-d^2/\rho^2)$ dependence, and using the measurements at $\Gamma = 0.8$, one finds $\rho \sim 0.2$ and 0.32 m (or approximately 2.3 and 3.5 wavelengths) at 17 kHz for the TREX04 and Unet06 environments. The TREX04 result is similar to previous measurements at a lower frequency (1.2 kHz) off the coast of New England, which showed a vertical coherence length of approximately two wavelengths. Both are open ocean environments with reported internal wave activities when a thermocline is present.

C. Intra-packet channel variations

To study the intra-packet channel variations, we divide the data into blocks, each $\sim 0.1$ s long. The CIR is estimated for each block of data using the minimum mean square error (MMSE) channel estimation method. The results have been shown in good agreement with that deduced directly from the m-sequences.

The measured CIRs are shown in Figs. 9(a) and 10(a) using one packet of data from the Unet06 and TREX04 experiment. One notes that for the stationary environment (Unet06), the multipath delay times are quite stable within a packet. In contrast, for a dynamic ocean environment (TREX04), the multipath arrival/delay times vary with time within a $\sim 10$ s packet. Some multipaths are found to split into micro-paths. Many (weak) scattered arrivals are found following the main arrivals.

1. Amplitude and phase fluctuation

Five paths are identified from Fig. 9(a) for the stationary ocean environment as indicated by the arrows below it. Their corresponding amplitude histograms are shown in Fig. 9(b) for the various paths. One notes that this is a minimum phase channel with the first path having significantly higher energy than the rest of the paths. However, even though the multipath arrival can be considered as almost deterministic, with stable arrival time, the energy of the first path varies significantly over a period of $\sim 10$ s. This is shown by the histogram distribution of path 1 in Fig. 9(b). The mean-to-SD ratio is 4.6, 6.8, 4.6, 4.3, and 3.4 for paths 1–5, respectively. Figure 9(c) shows that individual paths maintain a high temporal coherence over the packet length and Fig. 9(d) shows that paths have similar phases, with path 5 showing more phase fluctuations than the other paths. One notes that the wide distribution of path 1 (hence the lower mean-to-SD ratio compared with path 2) is because the amplitude in the second half of the data [Fig. 9(a)] becomes significantly higher than the first half ($\sim 5$ s) of the data. Despite the large amplitude variation, path 1 still retains high temporal coherence.

![Figure 9](https://example.com/figure9.png)

**FIG. 9.** (Color online) (a) Measured CIRs from the Unet06 data showing time evolution of the channel within a packet. Color scale in dB. Paths are indicated by the arrows under the figure, numbered from the left to right. (b) Histogram of amplitude distributions for individual paths identified by arrows under (a). (c) Temporal coherence of selected paths. (d) Phase variation of individual paths.

coherence since the coherence is more sensitive to phase variation than amplitude variation.

For the open sea TREX04 environment, identification of individual paths is difficult because paths are split into micro-paths [Fig. 10(a)]. We divide the dominant arrivals into three groups as indicated by the vertical bars under Fig. 10(a). The later arrivals (following the dominant arrivals) are more difficult to characterize except to say that they are random in nature. For each CIR, one integrates the intensity of the CIR between the time separations (vertical bars) and takes the square-root to determine the amplitudes of each group of paths. The histogram distributions of the amplitudes of the three groups of paths are shown in Fig. 10(b). The histogram distribution of the No. 3 group suggests that group 3 may be a combination of two subgroups each having a distribution similar to group 1 and 2. The mean-to-SD ratios of the group 1 to 3 are 11, 7.2, and 4.2, respectively. Compared with Table III, one finds that group 1 and 2 are highly deterministic. This is an artifact of our path grouping which integrates the energy of micro-paths as if they were one path. The results suggest that ray splitting into micro-paths does not affect the amplitude distribution (assuming no energy loss), if the micro-rays are appropriately grouped.

The temporal coherence curves of each group of arrivals are shown in Fig. 10(c). One finds relatively high value of coherence (∼0.7) for the three groups of paths despite the micro-path nature in each group of arrivals. The phases of each group of arrivals (each normalized by its initial phase) are shown in Fig. 10(d) as a function of transmission time.

One finds that all three groups have similar phases for the first 6 s of data.

2. Intra-packet temporal coherence

The intra-packet temporal coherence is measured using Eq. (1), where \( p(t) \) is the CIR measured from each block of data within each packet of data. Using the first CIR as the reference, and averaging over many packets of data, the result is shown in Fig. 11(a) and 11(b) as a function of the lag time for the UNet06 and TREX04 data. One finds that the temporal coherence of the intra-packet CIRs has a high value (≥0.8) over 10 s for the stationary UNet06 ocean environment [Fig. 11(a)]. In contrast, the intra-packet temporal coherence drops to below 0.6 in a very short time (~0.5 s) for the open ocean TREX04 environment [Fig. 11(b)]. Note that this temporal coherence calculation includes all paths. For the UNet06 data, one notes that despite the large amplitude variations of the first path, the signal remains highly coherent. This is attributed to the almost identical phases between the multipath arrivals. Note that multipath temporal coherence is more sensitive to the phase differences between the paths than the amplitude fluctuations of individual paths. In contrast, the TREX04 data also show high temporal coherence for individual groups of paths, and little phase differences between groups of arrivals (for the first 6 s of data), yet the multipath coherence drops to below 0.5 in a short time. Observe from Fig. 10(a), the split of arrivals into micro-paths and also the time-varying multipath arrival/
delay times. Thus, we attribute the loss of the intra-packet coherence (multipath coherence) in this case to the presence of micro-paths and time-varying multipath arrival/delay times, as well as long diffused arrivals.

The temporal coherence time, \( \tau_{0.8} \), is approximately 0.17 s based on TREX04 data [Fig. 11(b)] and ~20 s for the UNet06 data [Fig. 7(c)]. See Table I for a summary.

IV. CHANNEL ESTIMATION

Many communication algorithms require channel estimation, i.e., estimation of the CIR. For a time-varying channel, channel estimation is meaningful only for a time window shorter than the channel coherence time, within which the channel can be considered semi-stationary. Based on the data, the channel can be represented by a number of dominant arrivals which are stable within the coherence time, plus a number of scattered arrivals which cannot be reliably estimated and can be treated as random noise. See Figs. 3(a), 4(a), 9(a), and 10(a).

For a semi-stationary (time invariant) channel, the CIR can be estimated by minimizing the error between the received and calculated data

\[
\hat{h} = \arg \min_h |r - s^T h|^2, \tag{3}
\]

where \( r \) is the received signal block, and \( s \) is the transmitted signal matrix which is assumed known. Let \( r = s^T h + \eta \), where \( h \) is the true CIR, and \( \eta \) represents the noise. Let \( h = h + \hat{h} \), where \( h \) is the estimated dominant arrivals, and \( \hat{h} \) represents the weak, scattered arrivals which are random (treated like noise). One can define a normalized data estimation error (between the data \( r \) and estimated data \( \hat{r} = s^T \hat{h} \)) as

\[
\varepsilon^2 = \frac{\langle |r - s^T \hat{h}|^2 \rangle}{\langle |\hat{r}|^2 \rangle} = \frac{(h - \hat{h})^T (ss^T) (h - \hat{h}) + |\eta|^2}{h^T (ss^T) h} = \frac{|\hat{h}|^2 + N_0}{|h|^2}, \tag{4}
\]

where angular brackets denote ensemble average; the CIRs are assumed uncorrelated with the noise, the symbol sequences are assumed white and of unit magnitude, and \( N_0 \) is the noise level per symbol. The numerator of Eq. (4) is often referred to as the data estimation error, as it represents the components of the received signal that cannot be reliably estimated. Likewise, we note that \( \varepsilon \) represents the percentage of the data that are random (not \( a \ priori \) predictable), referred to as the degree of randomness of the channel (randomness for short). If \( \varepsilon^2 \sim N_0/|h|^2 \), it indicates that a substantial percentage of the signal is random. On the other hand, if \( \varepsilon^2 \approx N_0/|h|^2 \) it indicates that the signal has little or no random components. If \( N_0 \ll 1 \), then \( \varepsilon^2 = |\hat{h}|^2 / |h|^2 \) also represents the normalized channel estimation error.

The randomness of the channel is estimated for UNet06, TREX04, and AUVFest07 data by dividing data into blocks of a size less than the least of the channel coherence times among the data sets. Using a fixed block size of 500 symbols (~0.1 s long), the data estimation error is shown in Fig. 12 as a function of the block number for the various environments. One finds that the data estimation error is on the order of -16 and -6 dB for the AUVFest07 calm-sea and rough-sea conditions as shown in Figs. 12(a) and 12(b). The data estimation error is found to be on the order of -10 and -6.5 dB for the UNet06 and TREX04 data, respectively, as shown in Figs. 12(c) and 12(d). Channels with randomness ~ -6 dB may be considered “harsh” channels.

Channel randomness impacts the performance of the channel equalizer. Many communications algorithms assume a sparse equalizer, where taps coefficients are assigned only for those discrete arrivals identified in the CIR. The discrete arrivals can be estimated from a given block of data using the least square, or matching pursuit method among other possibilities. The random components of the CIR (the arrivals that are randomly time varying within the block) are treated as noise. Thus, given high input SNR, the degree of randomness of a channel can be interpreted as the degree of non-sparsity of the channel. When a channel is sparse (randomness is low), the sparse equalizer is expected to work well since the sparse arrivals can be adaptively tracked.
When a channel is highly non-sparse, one anticipates that the sparse equalizer will have difficulty in removing the intersymbol interference created by the random arrivals. In this case, improving the bit errors would require help from powerful error encoding/decoding and/or joint channel equalization and decoding (turbo-equalization).

V. SUMMARY AND DISCUSSIONS OF THE EXPERIMENTAL RESULTS

The observed high-frequency channel properties in shallow water are summarized/discussed below, together with some explanations/interpretations.

1) In a stationary environment (calm sea, protected ocean), the CIR can be characterized by sparse arrivals with stable arrival times. Sparse channel may not be a good assumption for the equalizer algorithm design in dynamic environments. Under rough sea conditions, the CIR can be characterized by a group of dominant arrivals followed by a train of diffused, random arrivals, which are interpreted as scattered arrivals from the rough surface waves [Fig. 4(a)]. The dominant arrivals seem to have well-defined arrival times but suffer significant fading (due to scattering by the surface waves). In open ocean environments, the dominant arrivals have varying arrival times and are split into micro-paths (as a result of scattering by the ocean inhomogeneities). They are also followed by many diffused and random arrivals. The diffused and random arrivals effectively increase the delay spread of the CIR. They create inter-symbol interference that are difficult to estimate and track by an equalizer.

2) The amplitude distributions of dominant paths seem to be different for different paths and different environments. The deterministic nature of the channel is indicated by high mean-to-SD ratios of the amplitude histograms. The mean-to-SD ratios tend to decrease with delay time (i.e., smaller for later arrivals) and are lower in dynamic oceans (see Table IV). The results suggest that a Rayleigh amplitude distribution is not a good assumption for underwater acoustic channels. Assuming that single path distribution is Rician distributed, one needs to know the Rician parameters which are path dependent. Simulating/predicting the Rician parameters in a dynamic ocean, even for the early deterministic-like arrivals, would be a worthwhile endeavor.

3) Individual dominant arrivals tend to be highly coherent within (the duration of) a packet. The temporal coherence is often smaller for later arrivals.
(4) Paths encounter phase change as a function of time. The AUVFest07 data indicate that (sound scatterings from) surface waves seem to inject large time-varying phase differences between path arrivals. In contrast, the TREX04 data indicate relatively small, slow-varying phase differences between path groups despite sound scattering from ocean inhomogeneities. It is noted that scattering by rough surfaces could change the acoustic ray angle by a significant amount, hence could produce a large phase change with respect to time due to change of ray path length; the amount of change is different for different paths. On the other hand, scattering by inhomogeneities is not expected to change the ray angle much. The phase change with respect to time is expected to be similar for rays with similar grazing angles.

(5) A major difference between sound channels under calm sea and rough sea conditions is the intra-packet temporal coherence (including all paths). The intra-packet temporal coherence is high (>0.8) over 25 s under calm sea conditions [Fig. 5(a)] whereas the intra-packet temporal coherence drops to below 0.5 in ~0.2 s under rough sea conditions [Fig. 5(b)]. Note that individual paths (of the dominant arrivals) maintain high temporal coherence as stated above. The loss of multipath coherence is attributed to signal fading and rapidly varying phases between the dominant arrivals as well as the presence of many scattered arrivals under rough sea conditions.

(6) A major difference between sound channels between protected and open seas is the intra-packet temporal coherence (for all paths). The intra-packet temporal coherence is high (>0.8) over 10 s for a semi-stationary ocean [Fig. 11(a)]. For an open ocean, the intra-packet temporal coherence drops to below 0.6 in ~0.5 s [Fig. 11(b)]. Note that individual groups (of the dominant arrivals) maintain high temporal coherence as shown above. The decrease of multipath coherence is, in this case, attributed to the presence of micro-paths, long diffused arrivals, and time-varying multipath arrival/delay times.

(7) The vertical coherence length is 2–4 wavelengths for all four environments; similar results were found in earlier measurements at lower frequencies.11 Between rough and calm seas, and between open ocean and protected bays, the vertical coherence length decreases by ~0.05 m at \( \Gamma = 0.8 \) or 30–50%.

The notations in Eq. (5) are as follows: \( a_m \) is the mode amplitude, angular brackets denote ensemble (time) average, \( \psi_m(z) \) is the mode depth function at \( \tilde{z} = (z + i\lambda)/2 \), \( \gamma_m = \sqrt{k_m^2 - k^2} \) is the vertical wave number of the normal modes, where \( k_m \) is the mode horizontal wavenumber, and \( k(\tilde{z}) = \omega/c(\tilde{z}) \). One observes from Eq. (5) that the vertical coherence length is determined to a large extent by the mode vertical wavenumbers. The effect of the random media or surface scattering is to change the mode amplitude spectrum. Such effects when weighted over modes may be limited.

\[
\rho^2 = \frac{1}{4} \frac{\sum m \langle |a_m|^2 \rangle \gamma_m^2(k_m, \tilde{z})}{\sum m \langle |a_m|^2 \rangle \sum m \langle |a_m|^2 \rangle}. 
\]

The numerators of Eq. (6) is the mean of the modal vertical wavenumber squared, and the denominator of Eq. (6) is related to the normalized acoustic intensity at depth \( \tilde{z} = (z + i\lambda)/2 \).

(8) A major obstacle to high data rate communication is the removal of inter-symbol interference due to diffused scattered arrivals that are random in nature and hard to estimate and track. The percentage of the signal energy that is carried by the random noise-like components, referred to as the randomness of the channel, can be characterized by the normalized data estimation error, the mean square error between the data samples and the predicted data samples normalized by the data. The normalized data estimation error is small for the AUVFest07 calm sea condition (~16 dB) and for the UNet06 stationary ocean (~10 dB), suggesting that a sparse equalizer will work well for these environment. The normalized data estimation error is relatively large for the AUVFest07 rough sea condition (~6 dB) and for the TREX04 open ocean (~6.5 dB), suggesting that a sparse equalizer will encounter much higher bit errors in these two environments compared with the two previous environments.

VI. CONCLUSIONS

Underwater acoustic communication channels are poorly understood particularly in shallow water and at high frequencies. Since fine and micro-fine scale ocean inhomogeneities are not well known, and sound scattering from the complex oceanographic processes and rough surfaces waves cannot yet be reliably calculated/predicted at high frequencies,
knowledge about the channel must come from experiment
data. While many high frequency communication experi-
ments have been conducted in shallow water, few have car-
ried out systematic studies on the channel properties.1
Understanding the signal temporal and spatial coherence is
critical to performance modeling in dynamic channels,1 but
there exist little data to conduct such analysis. For the com-
munication system design, knowing the range (extent) of the
parameter variations will be very helpful to the algorithm de-
velopment and system design.

In this paper, data from 20 m of water are analyzed to
study how channel properties change when the sea surface
becomes rough. The effects of the medium inhomogeneities
on channel properties are analyzed by comparing data
between open and protected ocean environments. Channel
variations are analyzed based on measurements of CIR at
different time scales. CIRs are measured from communica-
tion data transmitted in packets. The inter-packet and intra-
packet temporal coherence functions are deduced from data
for a fixed receiver. The spatial coherence function of the
signal is obtained by cross-correlating data over receivers at
different depths. Individual path amplitude and phase vari-
tions and temporal coherence are also measured for individ-
ual packets. Single path temporal coherence is normally high
in all the environments studies, but the multipath temporal
coherence drops rapidly with respect to lag time when the
sea surface becomes rough or when the water column
becomes a random medium. The reasons are attributed to
path fading due to sound scattering from the rough surface,
and ray splitting (micro-paths) and arrival time variation due
to interaction with the volume inhomogeneities, and the
presence of many (late) scattered arrivals. In contrast, the
vertical coherence is found less sensitive to the environ-
tmental conditions. The reason is that the vertical coherence is
determined primarily by the vertical structure of the field.
The vertical coherence length is determined by the mode
intensities and mode vertical wavenumber (squared).

In summary, channel properties are studied in this paper
in terms of the rate, the extent of the channel variations,
and the random-to-deterministic ratio (the randomness) of the
channels. Experimental data are analyzed to show the range of
the “rate,” “extent,” and “randomness” variations between
different oceans to guide system design.

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APPENDIX: VERTICAL COHERENCE IN TERMS OF
NORMAL MODES

The cross correlation of a broadband signal between two
receivers is given by

\[
\int dz p_i(t - \tau)p_j(\tau)
= \int dz p_i^*(\omega, z) p_j(\omega, z) \exp(i\omega t),
\] (A1)

where \(z_i\) and \(z_j\) are receive depth. The maximum of the corre-
lation occurs at \(t = 0\), hence one can express Eq. (2) as

\[
\Gamma(z_i, \Delta z) \equiv \left( \int dz p_i^*(\omega, z) p_j(\omega, z) \right)
\]

\[
\approx \frac{\left( \int dz p_i^*(\omega, z) p_j(\omega, z) \right)}{\left( \int dz |p_i(\omega, z)|^2 \right)^{1/2}}
\approx \frac{\left( \int dz p_i^*(\omega, z) p_j(\omega, z) \right)}{\left( \int dz |p_i(\omega, z)|^2 \right)^{1/2}}.
\] (A2)

where \(\Delta z = z_i - z_j\), and we approximate the integral by the
mean of the spatial coherence of a narrowband signal

\[
\Gamma(\tau, \Delta z) \approx \frac{\left( \int dz p_i^*(\omega, z) p_j(\omega, z) \right)}{\left( \int dz |p_i(\omega, z)|^2 \right)^{1/2}}
\]

\[
= \frac{\sum_m \left| a_m \right|^2 \psi_m(z_i) \psi_m(z_j)}{\sum_m \left| a_m \right|^2 \psi_m^2(\tau)}.
\] (A3)

where we assumed that the mode amplitudes are uncorrelated,
i.e., \(\langle a_m \cdot a_n \rangle = \delta_{mn} \left| a_m \right|^2\). We shall further assume the WKB
approximation for the mode depth function, which expresses
the mode as a summation of up and down traveling rays,

\[
\psi_m(z) = \frac{1}{2\sqrt{i|m|}} \left[ \exp(i\Theta(z)) + \exp(-i\Theta(z)) \right].
\] (A4)

The ray phase is given by \(\Theta(z) = \int_{z_l}^z \gamma_m(k_m, z) \, dz - \pi/4\),
where \(z_l\) is the lower turning point of the ray, \(\gamma_m = \sqrt{k_m^2 - k^2(z)}\), \(k_m\) is the mode wavenumber,
and \(k(z) = \omega/c(z)\). One then finds

\[
\Gamma(\tau, \Delta z) = 1 + \frac{1}{2} \sum_m \left| a_m \right|^2 \left[ \cos(\Delta z \gamma_m(k_m, \tau)) - 1 \right]
\]

\[
\sum_m \left| a_m \right|^2 \psi_m^2(\tau).
\] (A5)

Expressing \(\bar{\Gamma}(\tau, \Delta z) = \exp(-\langle \Delta z \rangle^2/\varphi^2) \approx 1 - \langle \Delta z \rangle^2/\varphi^2\) to first order in \(|\Delta z|^2\), one finds

\[
\rho^{-2} = \frac{\sum_m \left| a_m \right|^2 \gamma_m^2(k_m, \tau)}{\sum_m \left| a_m \right|^2 \psi_m^2(\tau)}.
\] (A6)