

## **Buckling of Rubber Bearings for Seismic Isolated Structures**

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### **Abstract**

The characteristics of rubber bearings used for seismic isolated structures involve the many unknown factors such as a mechanism at an ultimate stage. In case of design of themselves, experimental results are commonly used. For example, the results of regression analysis of test data are utilized for the design of buckling point, surface pressure, temperature and strain dependence for shear stiffness. Especially for buckling behavior, it is very important to estimate the precise point for more safety.

In this paper, a prediction method of buckling point is proposed by considering the relationship between the stress of interlayer steel plate and shear strain of interlayer rubber. In this prediction method, it is considered that the buckling of rubber bearings is caused by the yield of steel plate around the center hole. To verify the method, finite element analysis (FEA) is carried out for whole the rubber bearing for several cases. From the results of FEA, it is also recognized that the yield of steel plate around center hole causes the buckling of rubber bearing. It is also confirmed that the buckling point predicted by proposed method show a good agreement with the load decreasing point of previous experimental results.

### **Introduction**

Rubber bearings are set between the upper structures and the base to not transmit the earthquake vibration for seismic isolated structures. In these structures, not only damages of themselves but also secondary damages such as stumbles and destructions of furniture are restrained. The rubber bearing is made from the laminates of 4-5mm

thick rubbers and 3-6mm thick steel plates. The stiffness of rubber bearing for vertical direction is very high, and one for horizontal direction is small. It can be possible to support large axial forces of columns and to minimize shear force acting to the structure. For example, in 7 story buildings subjected to 500gal acceleration, the maximum response acceleration on first story and 7th story decreases 150gal and 200gal, respectively. The response of maximum drift angle of first story becomes less than 1/1000 radians. The value of base shear coefficient is 0.06, which is less than 1/5 of ordinary buildings. In Japan, seismic isolated structures have been popular especially after Hyougo-ken Nanbu Earthquake (Kobe Earthquake) in 1995. And the performance of rubber bearings has been improved dramatically.

The characteristics of rubber bearings, however, involve the many unknown factors such as a mechanism at an ultimate stage. In case of design of themselves, experimental results are commonly used. For example, the results of regression analysis of test data are utilized for the design of buckling point, surface pressure, temperature and strain dependence for shear stiffness. Especially for buckling behavior, it is very important to estimate the precise point for more safety.

In this paper, a prediction method of buckling point is proposed by considering the relationship between the stress of interlayer steel plate and shear strain of rubber. In this prediction method, it is considered that the buckling of rubber bearings is caused by the yield of steel plate around the center hole. To verify the method, finite element analysis (FEA) is carried out for whole the rubber bearing for several cases.

## **Predicting Method for Buckling of Rubber Bearings**

### **Factors for Load Decreasing of Rubber Bearings**

The factors for load decreasing of rubber bearings are assumed that the impossibility of equilibrium of forces or yielding (buckling) of internal steel plates (Fig. 1).

Considering the mechanism for the former case, rubber bearings are subjected to shear force and vertical compressive force at the same time. As shown in Fig. 2, compression strut is formed in the bearings. If the horizontal deformation ( $\delta$ ) exceeds outside diameter of bearings ( $D$ ), carrying part for vertical force disappears and then load decreasing takes place. In the experimental results, however, rupture of rubbers often recognized after deformation of  $\delta > D$ . The reason for this phenomenon is assumed that edge of outer rubber deforms toward expansion by local compressive stress and sectional area for vertical force increases outwardly. After that, compression strut can not be formed and load decreasing is observed. In the most experiments, rupture of rubbers takes place before load decreasing.

Considering the mechanism for the later case, the compression strut makes high stress in steel plates around the center hole. It is considered that the load decrease is caused by yielding (buckling) of interlayer plates. Some experimental results indicate this phenomenon as shown in Photo. 1.

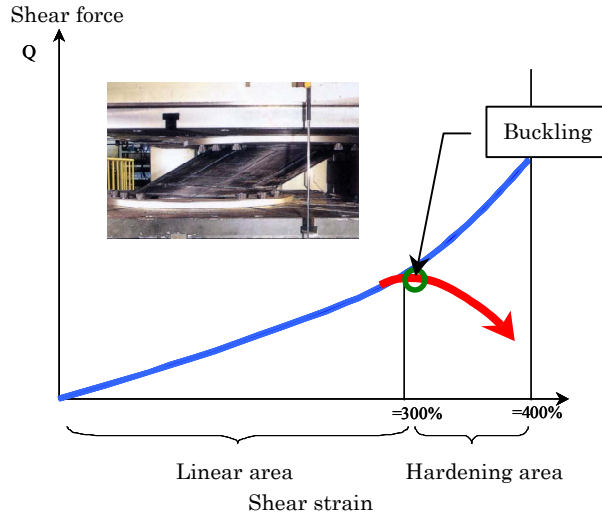


Fig. 1 Buckling point

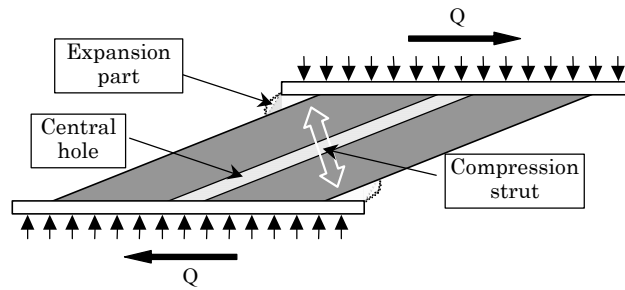


Fig. 2 Compression strut

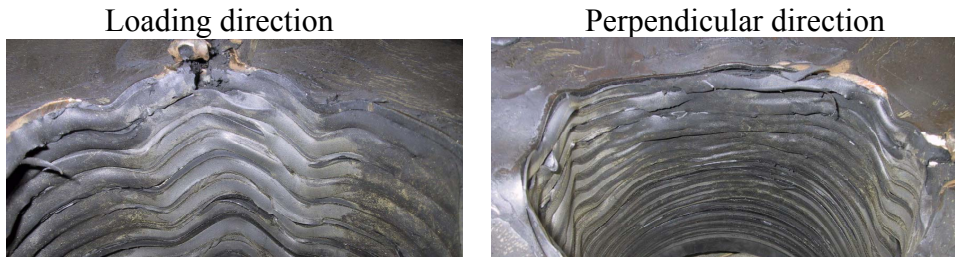


Photo. 1 Deformation of interlayer plates around center hole

### Predicting Method for Stress of Interlayer Plates

Wrapping area of each interlayer steel plate after deformation is defined as effective acting area,  $A_0$  as shown in Fig. 3. Von-Mises stress at the effective acting area can be calculated by Eq. (1). In this equation, the ratio of rubber thickness ( $t_r$ ) to steel plate thickness ( $t_s$ ) is multiplied by the assumption that the effective acting area transmutes vertical force for whole bearing and shear force for only this area. Where, shear force ( $Q$ ) is given by horizontal stiffness ( $K_h$ ) (AIJ 2001).

$$\sigma_{A0} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + (\sigma_1 - \sigma_2)^2}{2}} \cdot \frac{t_r}{t_s} \quad (1)$$

$$\sigma_1 = \frac{1}{2} \left( \sigma_N + \sqrt{\sigma_N^2 + 4\tau^2} \right) \quad (2)$$

$$\sigma_2 = \frac{1}{2} \left( \sigma_N - \sqrt{\sigma_N^2 + 4\tau^2} \right) \quad (3)$$

where,

- $\sigma_{A_0}$  : Von-Mises stress of effective acting area
- $\sigma_N$  : compressive stress of effective acting area =  $P / A_0$
- $P$  : vertical force
- $A_0$  : sectional area for effective acting area
- $\tau$  : shear stress of effective acting area =  $Q / A$
- $Q$  : shear force
- $A$  : effective area

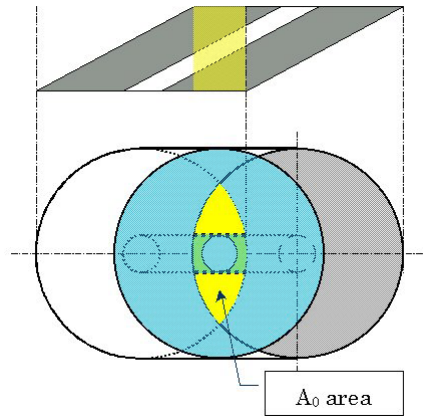


Fig. 3 Effective acting area

### Relationship between Interlayer Plate Stress and Shear Strain

From the previous experimental test results, the relationships between interlayer plate stress for effective acting area and shear strain are shown in Fig. 4 for the specimens failed by buckling (No.2, 3, 4, 5 × 2, 6 and 7), failed by rubber rupture (No.8) and without fail (No.1). The filled circles show buckling points and the triangle shows the maximum point. It is recognized that if the stress of effective acting area exceeds  $270\text{N/mm}^2$ , buckling takes place for the specimens with second shape factor (S2) of less than 4.0 (No.1, 2, 5, 6 and 7), and specimens subjected to high vertical stress with value of S2 of 4.5 (No.3 and 4). For the specimen No.8 having value of S2 of 5.0, the interlayer plate stress is very small comparing with other specimens at same shear strain.

### Predicting of Buckling Point

Based on the assumption that the buckling of rubber bearings caused by yielding of interlayer plates, buckling takes place in the region surrounded by dotted lines shown in Fig. 4. In the region of  $\gamma > 320\%$ , it becomes difficult to cause buckling because stress of interlayer plate decreases by the hardening of rubber. Therefore, the prediction of buckling of rubber bearing is can be calculated by Eq. (4).

$$\sigma_{A_0} \geq \sigma_{sa} = 270\text{N/mm}^2 \quad (\gamma < 320\%) \quad (4)$$

where,

- $\sigma_{sa}$  : tensile strength of interlayer plate (SPHC)

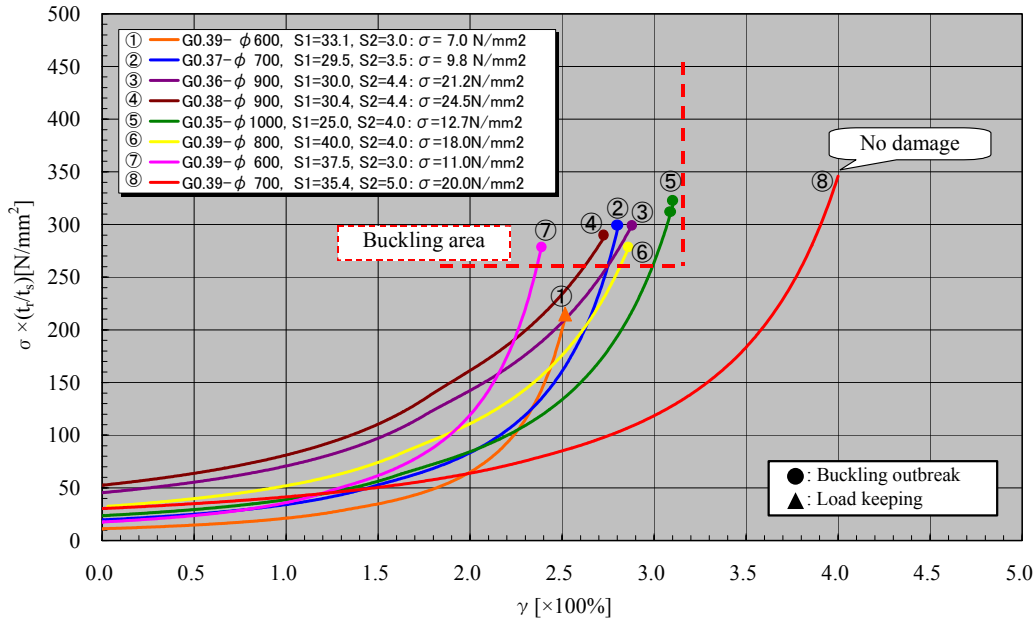


Fig. 4 Relation between interlayer plate stress for effective acting area and shear strain

## Finite Element Analysis for Rubber Bearing

### Finite Element Model

**Rubber** : Rubber is modeled by using strain energy density function ( $W$ ). In this analysis, the cubic polynomial expression considering compressive characteristics of rubber materials is used as shown in Eq. (5).

$$W = \sum_{i+j=1}^3 C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \frac{1}{D_1} (J_{el} - 1)^{2j} \quad (5)$$

where,

$\bar{I}_1$  : primary unbiased of deviation strain =  $\bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2$

$\bar{I}_2$  : secondary unbiased of deviation strain =  $\bar{\lambda}_1^{(-2)} + \bar{\lambda}_2^{(-2)} + \bar{\lambda}_3^{(-2)}$

$\bar{\lambda}_i$  : deviation elongation ratio =  $J^{-1/3} \lambda_i$

$J$  : volume ratio

$J_{el}$  : mechanical elastic volume ratio

Table 1 shows the values of constants,  $C_{ij}$  and  $D_1$ . These values are intended for natural rubber with shear modulus ( $G$ ) of  $0.4\text{N/mm}^2$ . These are decided by the regression analysis using the least square method from the stress – strain curves obtained by tensile test (JIS K6251, #3 test coupnes) and shear test for  $220 \times 15 \times 1\text{mm}$  rectangle test coupnes.

Table 1 Constants for rubber model

$C_{10}$	$C_{01}$	$C_{20}$	$C_{11}$	$C_{02}$
$3.67 \times 10^{-2}$	$-1.92 \times 10^{-2}$	$6.55 \times 10^{-4}$	$-1.09 \times 10^{-3}$	$7.96 \times 10^{-4}$
$C_{30}$	$C_{21}$	$C_{12}$	$C_{03}$	$D_1$
$-2.08 \times 10^{-7}$	0	0	$1.07 \times 10^{-6}$	$5.70 \times 10^{-3}$

**Steel Plate :** Elastic-plastic model are used for interlayer steel plate as shown in Fig. 5. Elastic modulus ( $E$ ) and Poisson's ratio ( $\nu$ ) is  $210000\text{N/mm}^2$  and 0.3, respectively. Perfect plastic occurs at 2% strain.

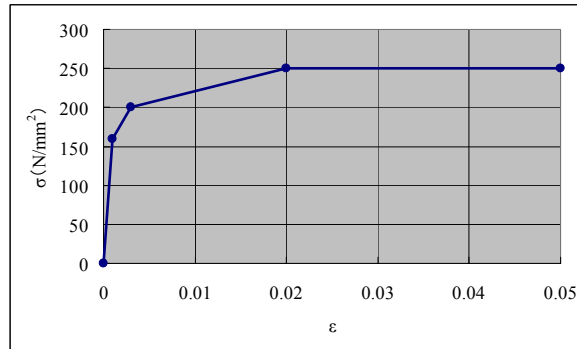


Fig. 5 Steel plate model

**Boundary Conditions :** Freedom of nodes at upper and lower flange unified into two representative nodes. Compressive and shear force are applied between the two nodes.

**Elements and Mesh:** Analyzed specimens are listed in Table 2. Specimen No.1 corresponds to No.2 in Fig. 4, which failed by buckling. Specimen No.2 corresponds to No.6, which failed by rubber rupture. Fig. 6 shows the mesh model for specimen No.1. Elements are divided in nine for radius direction, six for circumference direction and rubber and steel plate are divided two elements for one laminate. Utilized model is solid

Table 2 Shape of specimens

Specimen	Outer diameter (mm)	Inside diameter (mm)	Thickness of rubber (mm)	Number of laminating	Total thickness of rubber (mm)	Thickness of a steel plate (mm)	First shape factor	Second shape factor
No.1	700	110	5.0	40	200.0	2.5	29.5	3.50
No.2	700	35	4.7	30	141.0	3.1	35.4	4.96

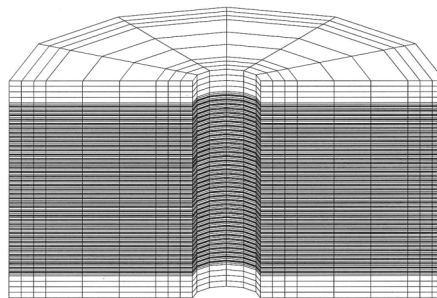


Fig. 6 Mesh for specimen No.1

model with eight nodes. Compressive stress of  $10\text{N/mm}^2$  was applied, and then shear displacement was applied monotonously.

### Results of Finite Element Analysis

Fig. 7 shows effective strain contours of interlayer steel plates of the upper and center part of specimens No.1 and No.2 at shear strain ( $\gamma$ ) of 300%. It is recognized that strains of two specimens are different even if each specimen is applied same vertical stress ( $10\text{N/mm}^2$ ) and same shear strain (300%). In specimen No.1, which is inferior to buckling, strains at the top edge and the center around the hole are bigger than No.2. These agree with the stresses shown in Fig. 4.

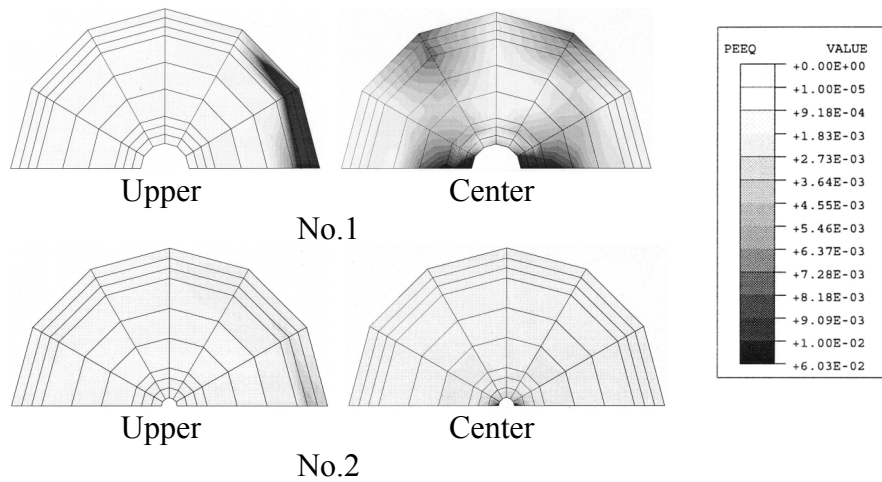


Fig. 7 Effective strain contour of interlayer plate

Deformations of specimens No.1 and No.2 at the central sections are shown in Fig. 8. Deformations of specimen No.1 at the top and bottom edges and center around the hole are big comparing with those of specimen No.2. Fig. 9 indicates the compression strut overdrawn on FEA results of specimen No.1.

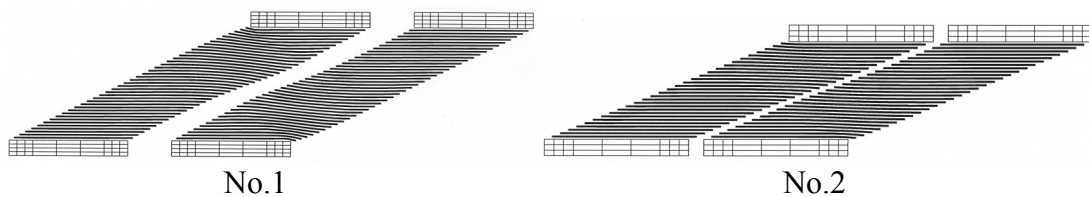


Fig. 8 Deformation at the section

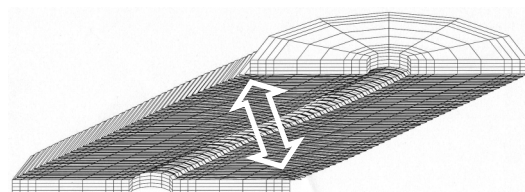


Fig. 9 Compression strut from FEA

Shear force versus shear strain curves are compared between analytical results and experimental results for each specimen as shown in Fig. 10. In specimen No.1, buckling point obtained from FEA almost agrees with experimental result.

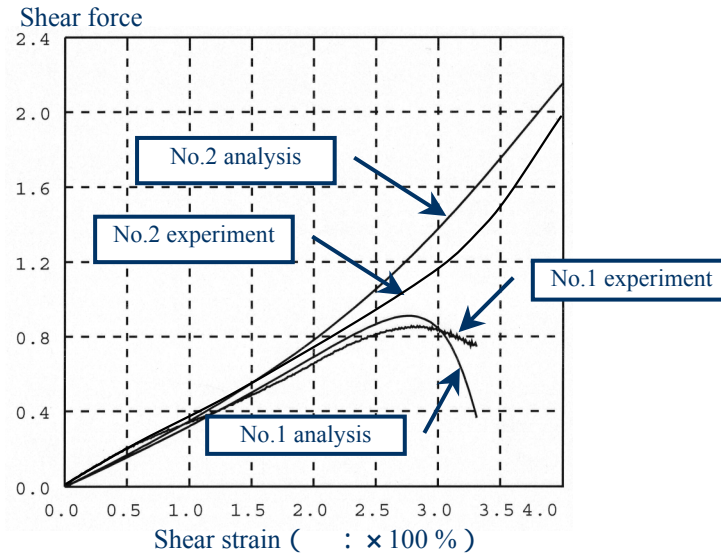


Fig. 10 Comparison between experimental results and FEA results

## Conclusions

A prediction method of buckling point of rubber bearings is proposed by considering the compression strut, of which sectional area is defined by wrapping area of each interlayer steel plate as effective acting area. From the relationship between the Von-Mises stress of effective acting area and shear strain of rubber, the stress exceeds tensile strength of steel plate when rubber bearings fail by buckling. FEA results also show the formation of compression strut and concentration of stress around the center hole. These phenomena are observed in case of inferior shape of rubber bearing. It is considered that the buckling of rubber bearings is caused by the yield of steel plate around the center hole.

## References

Architectural Institute of Japan (AIJ) (2001). Design Guidelines for Isolated Structures, Tokyo, Japan.