Velocity anisotropy of an industrial robot

Karl Gotlih*, Denis Kovac, Tomaz Vuherer, Simon Brezovnik, Miran Brezocnik, Andrejka Zver

University of Maribor, Faculty of Mechanical Engineering, Smetanova 17, SI-2000 Maribor, Slovenia

**Abstract**

Industrial robots are part of production systems and it is important to place them into the system according to their properties and behaviour. The information, obtained from the technical sheets of robots, about workspace (its dimensions and shape) is insufficient for designing the production system. The information about mobility is missing. To represent the behaviour of the robot in the workspace, velocity anisotropy of the robot is introduced and defined as the length of the shortest velocity ellipsoid axes, which can be constructed for any position of robot in its tool centre point. The position of a tool centre point is equivalent to the point in the workspace. A graphical representation of the 3D workspace with included velocity anisotropy is then performed and an example for a design of a robotised welding production system is given. In this example the benefits of the graphical representation of the workspace with included velocity anisotropy are presented and discussed.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

When we design robotised production systems, we recognise some difficulties, which result from the data sheets of industrial robots. Technical manuals offer insufficient information about the robot's workspace. The given data are restricted to two layouts, which give us just rough information about the dimension and the shape of the workspace. From this data we cannot comprehend the manipulability and the real velocity levels of the tool centre point (TCP) of the robot in an arbitrary point in the workspace. So it can happen that some difficulties appear after designing of the robotised production system within the first testing run. In the testing phase we can recognise some, for the robot, inaccessible points on the work piece or there are some parts on the produced work piece where the robot cannot perform the prescribed technological requirements.

The robot workspace analysis has different goals. The visualization on the basis of workspace computation based on geometric sweep of spatial elements, representing partial workspaces is presented in [1]. Each workspace is generated by iteratively rotating or translating workspace \((i+1)\) with respect to joint axes \(i\) between the joint limits. This approach achieves workspace representation since it can, in most cases generate the analytical representation of all boundary surfaces. The capabilities for a two-arm DLR humanoid robot are investigated in the article of Zacharias [2]. The aims are to define places in the workspace which are easily reached with the robot's hands, and reposition of robot's torso to enable optimal manipulation. In this article the internal structure of the workspace is visualized and manipulability measures and reachability index are introduced for a discrete set of points in the workspace. The article by Ottaviano [3] deals with the workspace topologies. A level-set reconstruction is used to analyse workspace topology by using algebraic expressions. The two-parameter set of curves is used to characterize the workspace cross-section and it gives an interesting insight of the internal structure of the workspace boundary obtained as an envelope of generating circles. An interesting approach is given by Lenarcic and Bajd [19]. In their work they define the robot's workspace as the reachable space when moving all robots' axes within their limits. A subset of the reachable workspace is introduced as the dexterity workspace. This subset is a set of points in which the robot can perform all possible orientations of the wrist. The analysis of the workspace singularities for three-axes positioning manipulator is given in the work of Angeles [32]. The author shows that the workspace boundary can be calculated with a system of linear homogeneous equations. The workspace boundary of general N-revolute manipulators is given in the article of Ceccarelli [4]. The workspace boundary is obtained from the envelope of a torus family, which is traced by the parallel circles cut in the boundary of a revolving hyper ring. The formulation is a function of dimensional parameters in manipulator chain and especially of the last revolute joint angle. An article which is close to the presented work is given by Gotlih [5]. The aim of the work is giving the designers of production cells a tool for proper placement of robots into the cell. This is done according to the shape of the task space and the prescribed threshold smallest singular value of the robot's Jacobian matrix in each discrete point of the task space. The procedure is developed for a 2D SCARA
robot mechanism with two degrees of freedom and a rectangular 2D task space.

In the presented articles the work is focused on the analytical formulation of the boundary of the workspace and just the works of Zacharias [2] and Gotlih [5] give the possibility to analyse velocity relations inside the workspace. The aim of the present paper is to give the planning process of robotised production cells a useful tool for correct decision where to put the robot in the cell with respect to technologically conditioned tasks. The developed tool enables 3D representation of the velocity anisotropy of the industrial robot in the workspace. The velocity anisotropy is defined as the normalized length of the shortest velocity ellipsoid axes, which can be constructed for any position of robot in its TCP. The presented paper is structured as follows: The theory of manipulability and different approaches to this matter are discussed in Section 2. In Section 3 the decision for choosing the shortest velocity ellipsoid axes as the velocity anisotropy measure is explained. The workspace of a typical welding robot according to the velocity anisotropy is developed in Section 4. In this section the 3D graphic representation of the velocity anisotropy for the welding robot is given and also the calculation of the Jacobian matrix and the manipulability for the robot is presented. An application for a designed and produced welding robotised production system with the welding robot is given in the discussion in Section 5. It is shown what can happen when the velocity anisotropy of the workspace is not taken into consideration while designing the production system. At the end of Section 5 the benefits of the presented approach are emphasised.

2. Measure of manipulability

The TCP trajectories of industrial robots can be divided into two main groups: the trajectories for simple manipulation (PTP programming—just the time for passing the trajectory or the minimal power consumption is important) and the technologically conditioned trajectories (CP programming – there are additional constraints – the exact tracking of the prescribed trajectory with additional velocity and/or force constraints is important).

This work deals with technologically conditioned trajectories, where besides correct tracking also the velocity profile on it is important. To overcome the difficulties which result from the description of the position and the orientation of the TCP [6–9] and because the transmission of kinematic and kinetic quantities from the actuators to the TCP depends in general on the first three "positional" degrees of freedom, the wrist degrees of freedom which are responsible for the orientation of the TCP are not taken into consideration. The parameter for determining the velocity anisotropy in each point in the robot's workspace depends on the momentary robot's position in the space.

The transmission of motions, from each actuator to the TCP, will not guarantee equal velocities of the TCP in all points in the space. Each point in the workspace $x \in U$, Fig. 1, represents one position of the robot mechanism $\mathbf{x} = \mathbf{TCP}$. The transmission of the motion and the force from each actuator to the TCP is dependent on the actual position of the mechanism [10–13].

In literature several approaches for calculating the manipulability or the dexterity of robot mechanisms are presented. Most of them are developed from the well-known Jacobian matrix of the robot's kinematical structure. We analyse some measures to find the best of them that represent the velocity anisotropy.

2.1. The manipulability index

Yoshikawa [14] was one of the first scientists who investigated the problem of manipulability of robot mechanisms and other authors continued his work [15–19]. The definition of manipulability index was given in Eqs. (1) and (2).

$$w = \sqrt{\det(J(q)J^T(q))}$$

(1)

where $q$ is the position of the mechanism in the configuration space and $J(q)$ is the Jacobian matrix of the mechanism. It represents the linear transformation of velocities from the configuration space coordinates $\dot{q}$ to the task space coordinates ($\mathbf{TCP}$'s workspace) $\mathbf{x}$. Eq. (1) is used when $m \neq n$, where $n$ denotes the number of degrees of freedom of the robot and $m$ is the number of needed parameters for exact determination of the position and the orientation of the TCP. When $m = n$, Eq. (1) transforms into

$$w = \det(J(q))$$

(2)

Serial manipulators with five or six degrees of freedom are mostly used industrial robots. The position part of the structure of such robot is given in Fig. 2.

The configuration space coordinates are $\mathbf{q} = (q_1, q_2, q_3)^T$ and the task space coordinates of the TCP are $\mathbf{x} = (x, y)^T$. For the structure in Fig. 2 the Jacobian matrix is

$$J(q) = \begin{bmatrix}
-s_1(t_2s_2 + l_3s_23) & c_1(t_2c_2 + l_3c_23) & c_1l_3c_23 \\
c_1(t_2s_2 + l_3s_23) & s_1(t_2c_2 + l_3c_23) & s_1l_3c_23 \\
0 & -l_2s_2 + l_3s_23 & -l_3s_23
\end{bmatrix}$$

(3)

and the manipulability index

$$w = l_2l_3 |(l_2s_2 + l_3s_23)s_3|$$

(4)

Fig. 1. The position of the robot and a point in the workspace.
2.2. The shortest axis’s length of the velocity ellipsoid

The velocities which the robot’s TCP can produce in an arbitrary point of the workspace are different not just from point to point in the workspace, but also differ in different directions in a point. So the velocity is anisotropic in robot’s workspace. This can be clearly represented with a velocity ellipsoid [20–22]. The velocity ellipsoid in the workspace of the robot can be constructed in the TCP as the result of transformation of velocity hyper-sphere from configuration space into an ellipsoid in the task space, Fig. 3.

\[
\dot{x} = J \ddot{q}
\]

\[
\ddot{q} = q_1^2 + \cdots + q_n^2 \leq 1
\]

(5)

The position of the mechanism \( q_i \) corresponds with the task space coordinates \( \hat{x}_i \), Fig. 4. The deviation from this position in configuration space coordinates \( dq \) results in the deviation of task space coordinates \( dx \) [23–25]. If the deviations in configuration space are equal in all directions, then \( dq \) for an \( n \)-dimensional sphere is

\[
\tau
\]

(6)

Vector \( dx \) in task space coordinates forms an ellipsoid. This ellipsoid is called the velocity ellipsoid.

2.3. Singular values of the Jacobian matrix

Singular values of the Jacobian matrix are calculated for each robot’s position numerically. Singular values of matrix \( J \), if the matrix is regular, are connected with the eigenvalues of the matrix [26,27] with

\[
\sigma_i = \sqrt{\lambda_i}, \quad i = 1, 2, \ldots, m
\]

(7)

The number of non-zero singular values \( \sigma_i \) of the matrix defines the range of the Jacobian matrix. The singular value decomposition gives the diagonal matrix Eq. (8a):

\[
S = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_m)
\]

(8a)

with

\[
\sigma_1 > \sigma_2 > \cdots > \sigma_m
\]

(8b)

where \( \sigma_1 \) is the greatest singular value and \( \sigma_m \) the smallest non-zero singular value.

If just one singular value is zero, the Jacobian matrix is singular and the mechanism is in a singular position. The product of singular values defines the volume of the velocity ellipsoid and through this the manipulability index of the serial mechanism

\[
M = \sigma_1 \sigma_2 \cdots \sigma_m
\]

(9)

2.4. Condition number

The relation between the minimal and the maximal singular value is the condition number

\[
K = \frac{\sigma_m}{\sigma_1}
\]

(10)

It is a normalized quantity which defines the roundness of the velocity ellipsoid. The ellipsoid is close to a sphere when \( K \) is near to 1 and flat, when \( K \) is near to zero. If \( K = 0 \), the Jacobian matrix is singular, and if \( K = 1 \) it is far from a singular position, the mechanism is in a velocity isotropic position.

2.5. The monotone manipulability

Authors Huy and I-Ming [28] investigated the robot’s workspace and the manipulability. They found out that the manipulability should be defined with two parameters, the degree of manipulability and the type of manipulability. But these two parameters make it hard to compare robot’s mechanisms with different dimensions. So
they introduce a new relation 

\[ M = \sqrt{\det(J^T J)} \]  

(11) 

and 

\[ M_{\text{rot}} = \sqrt{\det(J^T J)}, \]  

(12) 

where \( m \) in Eq. (12) represents the number of degrees of freedom of the robot. Huy and I-Ming [28] and the other authors [29–31] introduced the parameter (the monotone manipulability), which is defined with the relation between the minimal and maximal manipulability 

\[ U = \frac{M_{\text{min}}}{M_{\text{max}}} \]  

(13) 

\( M_{\text{min}} \) and \( M_{\text{max}} \) are the minimal and maximal values of the manipulability in the whole workspace of the robot.

3. The velocity anisotropy parameter 

Quantities which can be used as parameter to describe the velocity anisotropy are shown in the previous chapter. We decided to use the length of the shortest axes of the velocity ellipsoid as the measure of the velocity anisotropy because it directly corresponds with the distance to a singular point. The smaller the shortest axis is, the closer to a singular point TCP is. For the calculation of this parameter we need the robot's structure and its geometrical data. From the structure and the geometrical data the Jacobian matrix can be calculated in symbolic form. The singular values are then calculated numerically. The singular values, if the matrix is regular, represent the lengths of all three axes of the velocity ellipsoid and the shortest of them is our parameter of velocity anisotropy.

The motivation to choose the shortest axis of the velocity ellipsoid as the velocity anisotropy measure is in the nature of it. The velocity ellipsoid is most compressed in the direction of the shortest axis. The velocity of the TCP which can be produced in this direction is always the smallest of all. In comparison with the manipulability index – the volume of the ellipsoid – the shortest axis of the ellipsoid is better indicator of the nearness to singularity because the volume of the velocity ellipsoid is in the close surrounding of the singularity still big enough and it suggests a wrong impression that the TCP is far from singularity.

If the manipulability index is zero, the mechanism is in a singular position. This index should be, for the points of the workspace, as big as possible, so the robot can follow a prescribed trajectory without any difficulties caused by singularities. If we observe the shortest axis's length of the velocity ellipsoid, we come to the same conclusion. This parameter should be greater than the prescribed threshold value in as many points of the workspace as possible. If the required TCP trajectory is programmed through this subset of the workspace, no problems caused with the singularity will appear and the robot TCP will follow the programmed trajectory with the prescribed velocity.

Graphical representation of the velocity anisotropy is important for the design purposes of the robotised production system. The workspace is a set of infinite number of points therefore a discretization procedure is introduced to pick-up from this infinite number of points a set of finite number of points which are distributed through the workspace equally. Thus, each discrete point in the workspace is defined with three coordinates and an additional coordinate which describes the normalized parameter of velocity anisotropy Eq. (14) in it [29–31]. So we obtain the vector space with four dimensions:

\[ u_i = \frac{\sigma_{m,i}}{\sigma_{m,max}} \]  

(14) 

In Eq. (14) \( u_i \) is the normalized parameter of velocity anisotropy in the discrete workspace point “i”, \( \sigma_{m,max} \) is the greatest of all \( \sigma_m \) singular values from Eq. (8b) for any point in the workspace and \( \sigma_{m,i} \) is the actual non-zero singular value in the point “i”. The normalized parameter of velocity anisotropy in an arbitrary point “i” in the workspace can obtain the values between 0 and 1. Zero stands for a singular position and 1 is for the position in the space where the shortest axes of the velocity ellipsoid is the greatest for all short axes in the workspace.

The four-dimensional space can be represented with a projection of each point in 3D space and with a sphere, which radius and colour depend on the parameter of the velocity anisotropy. Fig. 5. Warm colours represent the subset of the workspace of the robot where the parameter is near to 1. The robot’s TCP will in this subset of the workspace move to all directions with reasonable high velocities. Colder colours represent the subset of the workspace where difficulties in
manipulability or producing required velocities on trajectories can be expected.

In the used software package it is also possible to develop different cross-sections through the workspace, Fig. 6.

4. Example of the typical welding industrial robot

The robot OTC DAIHEN AX-V4 [32] is used as a typical example of wide-used welding robots. The speciality of this robot structure is a hollow arm, which is used to carry through the required installations and so enables better flexibility for welding because no wires and pipes are installed outside the robot structure. The payload of the robot is 4 kg. In Fig. 7(a) two layouts of the workspace of the robot are given. Both are from the data sheets of the robot.

The robot’s kinematical structure is given in Fig. 7(b) and the TCP coordinates are

\[
\begin{align*}
    x &= \cos q_1 (l_1 + l_3 \cos q_2 + l_4 \cos(q_2 + q_3)) \\
    y &= \sin q_1 (l_1 + l_3 \cos q_2 + l_4 \cos(q_2 + q_3)) \\
    z &= l_2 + l_3 \sin q_2 + l_4 \sin(q_2 + q_3)
\end{align*}
\]  

(15)

And the Jacobian matrix is

\[
J = \begin{bmatrix}
    -\cos q_1 (l_3 \sin q_2 + l_4 \sin(q_2 + q_3)) & -\sin q_1 (l_3 \sin q_2 + l_4 \sin(q_2 + q_3)) & -l_4 \cos q_1 \sin(q_2 + q_3) \\
    \cos q_1 (l_1 + l_3 \cos q_2 + l_4 \cos(q_2 + q_3)) & -\sin q_1 (l_3 \sin q_2 + l_4 \sin(q_2 + q_3)) & -l_4 \sin q_1 \sin(q_2 + q_3) \\
    0 & -l_3 \cos q_2 - l_4 \cos(q_2 + q_3) & -l_4 \cos(q_2 + q_3)
\end{bmatrix}
\]  

(16)

Fig. 6. Cross-sections through the workspace.
The determinant of this matrix or the manipulability index is
\[
\text{det} J = \frac{9}{c_0 l_3 l_4 (l_2 + l_3 \cos q_2 + l_4 \cos (q_2 + q_3)) \sin q_3}
\] (17)

With the introduced procedure for developing the workspace with included parameter of velocity anisotropy we get Fig. 5, where isometric projection of the workspace is given. Additionally to these isometric projection two cross-sections in the \(x-y\) and \(x-z\) plane through the robots base coordinate origin are shown in Fig. 6 (b) and (c).

5. Discussion and conclusion

In the present paper a procedure for analyzing and 3D representation of robot’s workspace is given. Additionally a parameter for velocity anisotropy determination in each point of the workspace is introduced. The velocity anisotropy is defined with the normalized length of the shortest axis of the velocity ellipsoid for each point of the workspace.

To show the benefits of the presented procedure an analysis of a realized robotised production system for MAG welding is presented, Fig. 8. In the presented figure it is clearly shown that the upper edge of the welded part (point d, Fig. 8) is close to the singular position of the robot. In this region of the workspace the welding on a straight line was impossible. The robot’s TCP could reach just the starting and the ending point of the trajectory, but it was impossible to follow the whole trajectory.

With the use of the introduced graphical representation of the anisotropy, Fig. 8, this difficulty can be recognised in time and the position of the robot can be displaced before the production system is finished and put into work.

From Fig. 5 we can see that the structure of the robot’s workspace is not homogeneous. The bigger circles with warm colours are inside the workspace, the anisotropy parameter is higher inside the workspace than on the margins of it. The robot can track the prescribed trajectories without problems when positioning the trajectories of the TCP in the subset of the workspace where the anisotropy parameter is high. If the trajectories for arc welding are extended to the part of the workspace where the anisotropy parameter is low, difficulties in tracking can be expected.

The isometric representation of the workspace, Fig. 5, gives insufficient insight into the space, but with virtual cuts through the workspace in any arbitrary plane, Figs. 6 (b) and (c), it is also possible to see the anisotropy distribution inside the space.

From the designer’s point of view the visualization of the velocity anisotropy is an important and useful tool, because it gives the possibility of over-viewing the robot’s behaviour in the production system in the phase of designing and virtual simulations before the system is really produced.

The introduced tool helps the designer of production systems with additional data, which are not available from the producer’s data sheets. With the given tool it becomes possible to avoid difficulties which appear when the robot is not properly positioned into the production system.

Today it is impossible to think about design without software tools which do not support 3D representation. The presentation of the velocity anisotropy of the robot’s workspace is in 3D with the possibility of making arbitrary cross-sections through it to get better insight into the behaviour of the velocity anisotropy inside the workspace.

Our further research is focused on the implementation of the introduced graphical tool, on commercial available software for designing the robotised production system and on preparing 3D maps of workspaces with velocity anisotropies for different robot families.

With the given knowledge we are also developing an algorithm for optimal embedding the produced parts into the
robot's workspace if the trajectories on the part have to fulfill additional constraints on manipulability, velocities or forces.

References