Total and Merchantable Stem Volume Equations for Midrotation Loblolly Pine (*Pinus taeda* L.)

Joshua R. Sherrill, Bronson P. Bullock, Timothy J. Mullin, Steven E. McKeand, and Robert C. Purnell

ABSTRACT

Combined-variable (D^2H) and exponential-ratio equations are presented for predicting stem volume of loblolly pine in the Atlantic Coastal Plain of the southeastern United States. The equations were derived using 985 stems in a replicated study with first- and second-generation open-pollinated families across four levels of weed control and fertilization treatments. Prediction equations include inside- and outside-bark total volume and inside- and outside-bark merchantable volume to any top-limit diameter. Each equation has an R^2 of 0.962 or higher and root mean square error of 0.354 ft^3 or lower. The diversity of genetic backgrounds and silvicultural treatments gives the equations broad applicability in situations such as first thinning cruises, tree improvement progeny tests, silviculture research trials, and continuous forest inventory.

Keywords: tree improvement, silviculture, thinning, exponential-ratio equation, combined-variable equation

Loblolly pine (*Pinus taeda* L.) is a commercially important species in the southeastern United States. Loblolly pine seedlings make up 84% of the planting stock planted in the Southeast annually (McKeand et al. 2003). It is often necessary to predict stem volumes of loblolly pine in operational and research situations. These scenarios can include thinning cruises, midrotation cruises, genetic trials, and silviculture research tests.

One common equation used for total stem volume prediction is the combined-variable equation. This equation requires measurement of dbh (D) and total stem height (H), combined in a single variable, D^2H. The combined-variable equation has been shown to sufficiently predict total stem volume when fit to data sets with 50–100 sampled stems (Spurr 1952). Combined-variable equations have been used successfully in previous models to estimate total volume (Warner and Goebel 1963, Van Deusen et al. 1981, Amateis and Burkhart 1987) and total green weight (Bullock and Burkhart 2003) of loblolly pine. In addition, exponential-ratio equations have been used to predict merchantable volume and weight of loblolly pine with the flexibility of predictions for a specified top-limit diameter (Van Deusen 1981, Tasissa et al. 1997, Bullock and Burkhart 2003).

The objective of this study is to present widely applicable stem volume prediction equations for midrotation age loblolly pine trees. These equations predict inside- or outside-bark volumes and total stem volume or merchantable volume to a specified diameter along the stem. They are based on data from representative open-pollinated (OP) families grown using a range of common silvicultural practices. Many of these families have been used in plantations established from the 1970s through today and are diverse in terms of growth performance and genetic background. The equations should be used in place of past equations based on plantations of unimproved seed sources, strictly first-generation families, or situations where little is known about the genetic origin. These equations should prove useful to forestland managers in the Atlantic Coastal Plain of the Southeast.

Materials and Methods

The study site was located on International Paper Company's Southland Forest, Bainbridge, GA (latitude 30.903°N and longitude 84.575°W) and was classified as an Orangeburg soil series. All plots received an application of 0.5 pint/ac imazapyr, an application of 1.0 pint/ac glyphosate, a prescribed burn before planting, and a release treatment with imazapyr at age 5. The five replications used were planted in December 1991 on an 8 × 10-ft spacing (545 trees/ac). This study was a two-by-two factorial of herbicide and fertilizer treatments in a split-plot design. The main-plot factor was cultural treatment, and the subplot factor was OP genetic family. The herbicide treatment consisted of early woody and herbaceous competition control through age 5, and the fertilization treatment consisted of ground applications through age 9 (Table 1). There were 25 OP Atlantic Coastal Plain loblolly pine families arranged as six-tree noncontiguous subplots in a randomized complete-block design. Three OP families were from top first-generation selections, and 22 were from second-generation selections. Some pedigree relationships were known to exist among parents. In total, 985 trees were felled in the 13th growing season, approximately 10 trees per family per treatment. The purpose of the destructive sample was to obtain accurate inside- and outside-bark diameter measurements

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Joshua R. Sherrill (josh.sherrill@rayonier.com), Rayonier, Inc., P.O. Box 819, Yulee, FL 32041; Fax: 904-225-0370. Bronson P. Bullock (bronson_bullock@ncsu.edu) Department of Forestry and Environmental Resources, North Carolina State University, Box 8008, Raleigh, NC 27695-8008. Timothy J. Mullin (tim.mullin@biosylve.com) BioSylve Forest Science NZ Ltd., 58 Huntington Dr. Hamilton 3210, New Zealand. Steven E. McKeand (steve_mckeand@ncsu.edu) Department of Forestry and Environmental Resources, North Carolina State University, Box 8002, Raleigh, NC 27695-8002; Robert C. Purnell (robert.purnell@weyerhaeuser.com) Genetics Research Weyerhaeuser Co., 810 Whittington Ave., Hot Springs, AR 71901. *We thank the North Carolina State University Cooperative Tree Improvement Program for financial support. In addition, gratitude is extended to International Paper Company for the use of the study site. For technical support or advice, the authors would like to thank Fred Haines, Richard Bryant, Steve Henry, Larry Stubbs, Daniel Gräns, Ben Smith, and John Adams.*

Table 1. Herbicide and fertilizer treatments applied in the study.

Herbicide		Fertilizer	
Application	Age	Application	Age
0.75 oz./ac Metsulfuron methyl	1	3 oz./tree 20-4-15	1, February
Sulfometuron methyl and atrazine tank mix	2	3 oz./tree 20-4-15	1, June
Hack and squirt with triclopyr and diesel	Periodic	1,000 lbs/ac 20-5-15 with micronutrients	4
Directed spray, 2% glyphosate	1–5	1,000 lbs/ac 20-5-15 with micronutrients	5
Aerial release, imazapyr	5	1,000 lbs/ac 20-5-15 with micronutrients	9

Table 2. Descriptive statistics for the loblolly pine data set.

Measurement	n	Mean	Standard deviation	Minimum	Maximum
H (ft)	985	42.0	5.9	16.0	56.2
D (in.)	985	6.5	1.4	1.5	10.6
D^2H (in^2 × ft)	985	1933	973	42	5956
Double bark thickness (in.) at breast height	985	0.6	0.1	0.1	1.0
Total volume, i.b.[a] (ft^3)	985	3.4	1.8	0.1	11.1
Total volume, o.b. (ft^3)	985	4.8	2.3	0.1	15.2
Merchantable volume, i.b. (below 3-in. o.b. top) (ft^3)	985	3.2	1.8	0.1	10.9
Merchantable volume, o.b. (below 3-in. o.b. top) (ft^3)	985	4.5	2.3	0.1	15.0

[a] i.b., inside bark; o.b., outside bark.

along the main stem so that volume could be accurately determined. For more details on the results of genetic trait analysis, see Sherrill et al. (2008).

Once felled, each stem was measured at heights of 0.5, 2, 4.5, and 8 ft, and every 4 ft above the 8-ft height to a 3-in. outside-bark (o.b.) top. Branches were avoided by moving the measurement point 2 in. above the branch swell. At each height, o.b. diameter (to the nearest 0.04 in.) and bark thickness (to the nearest 0.05 in.) were measured with aluminum calipers and punch-style bark gauges, respectively. Two caliper and two bark-gauge measurements were made at 90° angles at each height. Each pair of measurements was averaged arithmetically, and the average bark thickness was doubled and subtracted from the average o.b. diameter to yield an inside-bark diameter. Diameters were used in Smalian's log volume equation (Avery and Burkhart 2002) to calculate the volume of each stem section from a 0.5-ft stump to a 3-in. o.b. diameter. The top-section volume above a 3-in. o.b. diameter top was estimated using the volume formula for a cone (Avery and Burkhart 2002). The total stem volume was then obtained from summing all of the sectional volumes.

The general form of the combined-variable equation used in this research is given in the equation

$$V_t = \beta_0 + \beta_1(D^2H) + \varepsilon, \qquad (1)$$

where V_t is total stem volume (ft^3), β_0 and β_1 are coefficients to be estimated, D is tree dbh (4.5 ft above ground line) in inches, H is total stem height in feet, and ε is the error term. To evaluate the volume estimation models, the coefficient of determination (R^2) and root mean square error (RMSE) were used to assess the relative accuracy of the models.

Exponential-ratio equations were fit for both inside- and outside-bark merchantable volumes to a specified top-limit diameter outside bark (Van Deusen et al. 1981, Tasissa et al. 1997),

$$V_m = (V_t)\exp\left(\beta_1\left(\frac{d^{\beta_2}}{D^{\beta_3}}\right)\right) + \varepsilon, \qquad (2)$$

where V_m is merchantable stem volume (ft^3 below diameter d), β_1, β_2, and β_3 are coefficients to be estimated, and d is the top-limit o.b. diameter. Pseudo-R^2 (Schabenberger and Pierce 2002) and RMSE were used to assess the accuracy of the models. A ratio form (Burkhart 1977) was also considered for merchantable inside- and outside-bark volume, but was not used because it resulted in higher RMSEs.

Outliers were evaluated, and as none were identified, all 985 observations were used in each model. Treatment effects were determined to be minor factors in estimating the model parameters. There were minimal differences among OP families in combined-variable models (Sherrill et al. 2008). Models were fit to data for each silvicultural treatment to determine whether the treatment affected the parameter estimates. The parameter estimates were similar, and therefore, treatment-specific models were unnecessary. Table 2 shows descriptive statistics for the data set used to fit the equations.

Results

Four prediction equations are presented for estimation of volume in ft^3 based on measured D^2H. Equation 3 was derived to estimate total stem inside-bark volume and had an R^2 of 0.962 and RMSE of 0.345 ft^3. A graph of the actual and predicted total stem inside-bark volume versus D^2H is presented in Figure 1.

$$\hat{V}_{tib} = -0.06906 + 0.00178(D^2H) \qquad (3)$$

Equation 4 estimates merchantable inside-bark volume to a specified o.b. top. It had a pseudo-R^2 of 0.995 and RMSE of 0.203 ft^3. Figure 2 shows the data points and the predicted values for Equation 4.

$$\hat{V}_{mib} = (\hat{V}_{tib})\exp\left(-1.0973\left(\frac{d^{4.8801}}{D^{4.7596}}\right)\right) \qquad (4)$$

Equation 5, derived to estimate total stem o.b. volume, had an R^2 of 0.977 and RMSE of 0.354 ft^3. See Figure 3 for a graphical representation of the data and predictions.

$$\hat{V}_{tob} = 0.20571 + 0.00237(D^2H) \qquad (5)$$

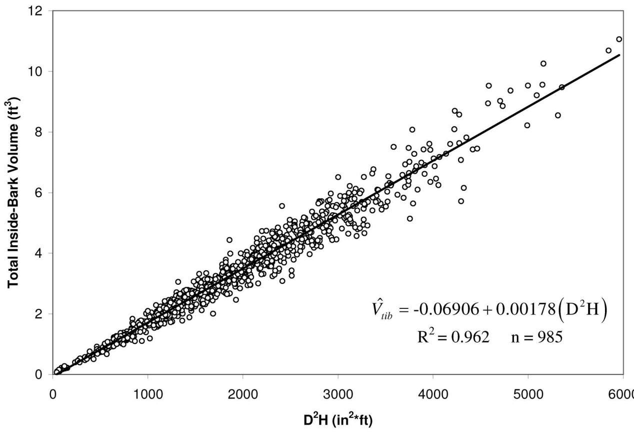

Figure 1. Total inside-bark volume measured (circles) and predicted (line) values.

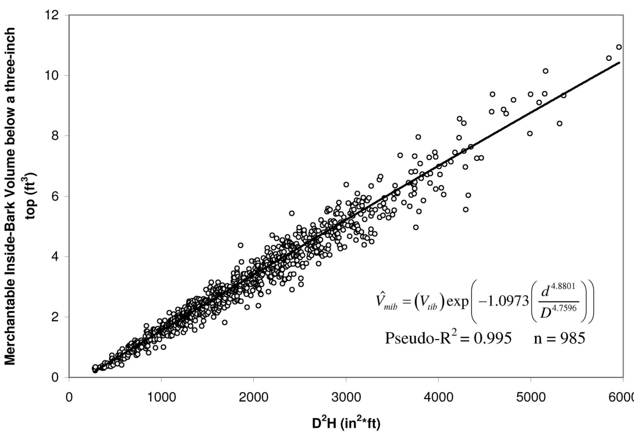

Figure 2. Merchantable inside-bark volume to a 3-in. o.b. top (*d* = 3) for measured (circles) and predicted (line) values. Although all 985 stems were used to fit the equations, 15 stems are not shown because they were less than 3 in. in dbh and therefore did not have a meaningful merchantable volume below a 3-in. top.

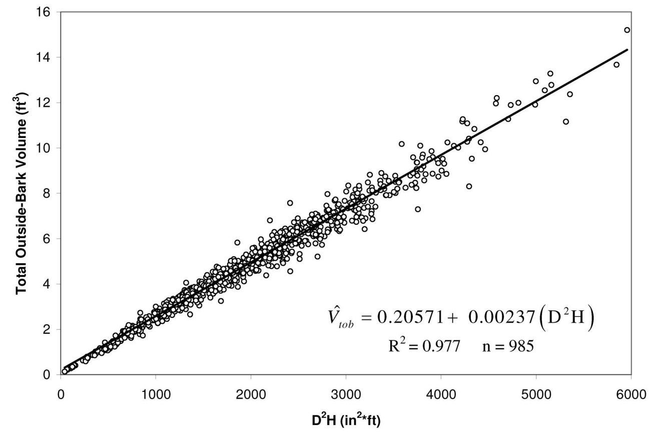

Figure 3. Total outside-bark volume measured (circles) and predicted (line) values.

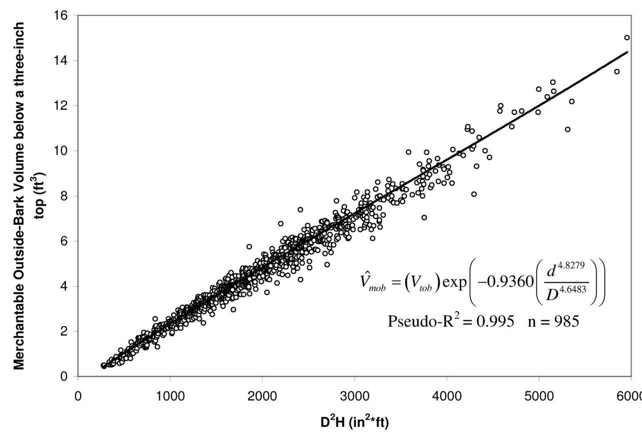

Figure 4. Merchantable outside-bark volume to a 3-in. o.b. top (*d* = 3) for measured (circles) and predicted (line) values. Although all 985 stems were used to fit the equations, 15 stems are not shown because they were less than 3 in. in dbh and therefore did not have a meaningful merchantable volume below a 3-in. top.

Equation 6 estimates the merchantable o.b. volume to a specified o.b. top. It had a pseudo-R^2 of 0.995 and RMSE of 0.275 ft^3. A prediction line of this equation is shown in Figure 4.

$$\hat{V}_{\text{mob}} = (V_{\text{tob}})\exp\left(-0.9360\left(\frac{d^{4.8279}}{D^{4.6483}}\right)\right) \tag{6}$$

A comparison with the presented total inside-bark equation and previous equations (Burkhart 1977, Van Deusen et al. 1981, Amateis and Burkhart 1987) revealed differences of greater than 5% in predicted volume across the range of stem sizes described in Table 2. The differences are important and suggest that the equations presented are unique.

Application

For example, a loblolly pine tree is measured in a midrotation stand where total and merchantable volume to a 3-in. o.b. top is desired. The dbh is 6.1 in., and the total height is 41.6 ft. Total inside-bark volume is calculated using Equation 3, as

$$\hat{V}_{\text{tib}} = -0.06906 + 0.00178(6.1^2 \times 41.6) = 2.686 \text{ ft}^3.$$

Equation 4 is used to calculate merchantable volume to a 3-in. o.b. top:

$$\hat{V}_{\text{mib}} = (2.686)\exp\left(-1.0973\left(\frac{3^{4.8801}}{6.1^{4.7596}}\right)\right) = 2.574 \text{ ft}^3.$$

Next, total o.b. volume is calculated using Equation 5:

$$\hat{V}_{\text{tob}} = 0.20571 + 0.00237(6.1^2 \times 41.6) = 3.874 \text{ ft}^3.$$

Equation 6 is used to calculate o.b. merchantable volume to a 3-in. o.b. top:

$$\hat{V}_{\text{mob}} = (3.874)\exp\left(-0.9360\left(\frac{3^{4.8279}}{6.1^{4.6483}}\right)\right) = 3.714 \text{ ft}^3.$$

Conclusions

Combined-variable and exponential-ratio equations are simple and effective when using *D* and *H* measurements. Four models are presented that fit the data set well and can easily be applied to common situations. Although the data were derived from stems at only one location, the broad diversity in OP families and silviculture treatments give these models applicability in many circumstances. These equations should be used to replace equations derived from unimproved, only first-generation, or unknown genetic origins.

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