Bet Option Model

A bet option is a bet on a basket of stocks. There are multiple reset periods before the maturity of the option. At the end of each period, if all the stocks in the basket are above their respective strikes, the option will payout a rebate amount for this period at maturity.

We propose a model for pricing bet option based on Monte Carlo simulation.

Let *m* be the number of assets in a given basket, $S_j(t)$ be the price process of the *j*th underlying asset in the basket and $1 \le j \le m$. Let $\{t_1 < \cdots < t_n\}$ be a set of reset dates and $T \ge t_n$ be a payoff settlement date or maturity. The bet option on the basket of stocks is a European style derivative security whose matured payoff at the settlement date *T* is given by

$$\sum_{i=1}^{n} R_e \times I_{\{(S_1(t_i) \ge K_1) \cap \cdots \cap (S_m(t_i) \ge K_m)\}}$$

where R_e is the rebate amount, *I* is the indicator function, and $K_i, i = 1, \cdots, m$ is the strike for each individual stock in the basket. The bet option defined above is essentially a series of multivariate digital options with a common payoff settlement date.

Let *t* be the current value date, then the current value of this derivative security can be written as

$$df(t,T) \times E_t\left[\sum_{i=1}^{n} R_e \times I_{\{(S_1(t_i) \geq K_1) \cap \cdots \cap (S_m(t_i) \geq K_m)\}}\right]$$

where $df(t,T)$ is the discount factor at the value date. The above formulae are in a world that is forward risk-neutral with respect to a specific currency C_p. If an underlying asset *j* is measured in another currency C_U, the governing price dynamics of this underlying asset in the risk-neutral world of C_p should be written as

$$dS_j(t) = (r_U - q_j - \rho\sigma_U\sigma_s)S_j(t)dt + \sigma_s S_j(t)dW_t^j, j = 1, \cdots, m$$

where r_U is the short rate of C_U, q_j is the dividend yield of the asset, ρ is the correlation coefficient between the asset price and the cross-currency exchange rate, σ_s is the volatility of the asset price, σ_U is the volatility of the exchange price, and W_t^j is the Wiener process. Asset prices in the basket are correlated with $[dW_t^j, dW_t^k] = \rho_{j,k}dt$ where $\rho_{j,k}$'s are constant correlation coefficients between the logarithmic asset prices. All these parameters are assumed deterministic.

Our test is based on several trades or transactions. In this sample transaction, Monte Carlo simulation associated with stratified sampling variance deduction is employed to evaluate the option. There are three underlying assets whose prices are correlated, and five reset dates. We get market data from FinPricing (https://finpricing.com/lib/IrCurve.html)

The initial price for each asset is 100, and the strike for each asset is 100. The calculated option value is **14.0547** when 2000 simulations and 2000 samples in stratified sampling are used. Keeping the 2000 stratified samples unchanged, calculations with more simulations are conducted. Table 1 shows the simulated results.

Table 1. **Impact of Simulation Numbers on the Value of Option**

By Using Monte Carlo Method with Stratified Sampling Variance Reduction

No. of Simulation	2,000	10,000	50,000	100,000	300,000
Option Value	14.0547	13.9455	13.9963	14.0122	13.9980

We also built an independent pricing model for this derivative security by using crude Monte Carlo simulation. The results are illustrated in Table 2.

Table 2. **Impact of Simulation Numbers on the Value of Option**

By Using Crude Monte Carlo Method

No. of Simulations	2,000	10,000	50,000	100,000	300,000
Option Value	14.1291	14.0391	14.0272	14.0036	13.9841

We also tested cases with different strike levels. Table 3 presents the results of these cases. Relative errors are calculated for different models.

Table 3. **Comparison of advanced Monte Carlo Model and crude Monte Carlo Model**

Strike			Model	Number of Simulations				
Asset 1	Asset 2	Asset 3		2,000	10,000	50,000	100,000	300,000
110	115	120	Advanced	8.2025	8.1986	8.2606	8.2452	8.2274
			Crude	8.3078	8.2863	8.2588	8.2560	8.2253
Relative Error (%)				-1.27	-1.06	0.02	-0.13	0.03
90	95	92	Advanced	17.4457	17.2574	17.2986	17.3173	17.3075
			Crude	17.4331	17.3807	17.3438	17.3196	17.3082
Relative Error (%)				0.07	-0.71	-0.26	-0.01	0.00

It can be seen from Tables 1, 2 and 3 that the results from the two models converge as the number of simulations becomes large in all cases. The differences are within the tolerable range.