Delay-probability-distribution-dependent robust stability analysis for stochastic neural networks with time-varying delay

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Abstract

The delay-probability-distribution-dependent robust stability problem for a class of uncertain stochastic neural networks (SNNs) with time-varying delay is investigated. The information of probability distribution of the time delay is considered and transformed into parameter matrices of the transferred SNNs model. Based on the Lyapunov–Krasovskii functional and stochastic analysis approach, a delay-probability-distribution-dependent sufficient condition is obtained in the linear matrix inequality (LMI) format such that delayed SNNs are robustly globally asymptotically stable in the mean-square sense for all admissible uncertainties. An important feature of the results is that the stability conditions are dependent on the probability distribution of the delay and upper bound of the delay derivative, and the upper bound is allowed to be greater than or equal to 1. Finally, numerical examples are given to illustrate the effectiveness and less conservativeness of the proposed method.

Keywords: Delay-probability-distribution-dependent; Stochastic neural networks; Time-varying delay; Linear matrix inequality

1. Introduction

The stability analysis problem for delayed NNs has received considerable research attention in the last decade, see Refs. [1–3], where the delay type can be constant, time varying or distributed, and the stability criteria can be delay dependent or delay independent. Since delay-dependent methods make use of information on the length of delays, they are generally less conservative than delay-independent ones. However, in a real system, time delay often exists in a random form, that is, if some values of the time delay are very large but the probability of the delay taking such large values is very small, it may lead to a more conservative result if only the information of variation range of the time delay is considered. In addition, its probabilistic characteristic, such as the Bernoulli distribution and the Poisson distribution, can also be obtained by statistical methods. Therefore, it is necessary and realizable to investigate the probability-distribution delay. Recently, the stability of discrete NNs and discrete SNNs with probability-distribution delay are investigated in Refs. [4,10], respectively. But neither of them considers the information of the delay derivative.

It has been known that there are two kinds of disturbances that are unavoidable to be considered when one models the NNs. One is parameter uncertainty, the other is stochastic disturbance. For the stability analysis of SNNs with parameter uncertainty, some results related to this problem have recently been published, see Refs. [5–12]. However, Refs. [5,6,10,11] do not consider the information of the delay derivative. In Refs. [7–9], the information of the derivative is taken into consideration, but the upper bound \( \mu \) of the derivative must be smaller than 1. In the case of \( \mu \geq 1 \) the results in the aforementioned literatures either cannot be applicable [9] or discard the information.
of the derivative of time delays [7,8], which is obviously unreasonable. Therefore, it is essential and significant to investigate the problem of how to eliminate the constraint on the upper bound of the delay derivative in SNNs.

In this paper, we investigate the robust stability problem for a class of uncertain SNNs with time-varying delay. The information of delay-probability distribution is introduced into the SNNs model and a new method is proposed to eliminate the constraint on the upper bound of the delay derivative. Less conservative stability criteria are presented into the SNNs model and a new method is proposed to

\[
\tau(t) = \begin{cases} \tau(t), & t \in \Omega_1, \\ \bar{\tau}_1, & t \in \Omega_2, \end{cases} \quad \text{and} \quad \tau_2(t) = \begin{cases} \tau(t), & t \in \Omega_1, \\ \bar{\tau}_2, & t \in \Omega_2, \end{cases}
\]

(3)

\[
\tau_1(t) \leq \mu_1 < \infty, \quad \tau_2(t) \leq \mu_2 < \infty
\]

(4)

where \( \tau_0 \in [0, \tau_M] \), \( \bar{\tau}_1 \in [0, \tau_0] \) and \( \bar{\tau}_2 \in [\tau_0, \tau_M] \).

It is easy to know \( t \in \Omega_1 \) means the event \( \tau(t) \in [0, \tau_0] \) occurs and \( t \in \Omega_2 \) means the event \( \tau(t) \in [\tau_0, \tau_M] \) occurs. Therefore, a stochastic variable \( x(t) \) can be defined as

\[
x(t) = \begin{cases} 1, & t \in \Omega_1 \\ 0, & t \in \Omega_2 \end{cases}
\]

(5)

**Assumption 2.** \( x(t) \) is a Bernoulli distributed sequence with

\[
\text{Prob}\{x(t) = 1\} = \mathbb{E}\{x(t)\} = x_0,
\]

\[
\text{Prob}\{x(t) = 0\} = 1 - \mathbb{E}\{x(t)\} = 1 - x_0
\]

where \( 0 < x_0 < 1 \) is a constant and \( \mathbb{E}\{x(t)\} \) is the expectation of \( x(t) \).

**Remark 1.** From Assumption 2, it is easy to know

\[
\mathbb{E}\{x(t) - x_0\} = 0, \quad \mathbb{E}\{(x(t) - x_0)^2\} = 2x_0(1 - x_0).
\]

By Assumptions 1 and 2, the system (1) can be rewritten as

\[
dx(t) = [-A(t)x(t) + W_0(t)f(x(t))] + [C(t)x(t) + DX(t - \tau(t))]dt + [Cx(t) + x(t)DX(t - \tau_1(t))]d\xi(t)
\]

(6)

\[
x(t) = \xi(t), \quad t \in [-\tau_M, 0]
\]

which is equivalent to

\[
dx(t) = [-A(t)x(t) + W_0(t)f(x(t))] + [C(t)x(t) + DX(t - \tau_1(t))]dt + \xi(t)DX(t - \tau_2(t))d\xi(t)
\]

(7)

\[
x(t) = \xi(t), \quad t \in [-\tau_M, 0]
\]

**Assumption 3.** The neural activation function \( f_i(x_i) \) satisfies

\[
L_i \leq f_i(x_i) - f_i(y_i) / x_i - y_i \leq L_i^+, \quad i = 1, 2, \ldots, n
\]

(8)

which implies that

\[
(f_i(x_i) - L_i^+x_i)(f_i(x_i) - L_i^-x_i) \leq 0
\]

(9)

where \( L_i^+, L_i^- \) are some constants.

**Assumption 4.** The parameter uncertainties \( \Delta A(t), \Delta W_0(t) \) and \( \Delta W_1(t) \) are of the forms:

\[
[\Delta A(t)\Delta W_0(t)\Delta W_1(t)] = [EF(t)H_1H_2H_3]
\]

(10)

where \( E, H_1, H_2 \) and \( H_3 \) are known constant matrices, \( F(t) \) satisfies \( F^T(t)F(t) \leq I \), for \( \forall t \in \mathbb{R} \).

Let \( V(x, t) \in C^2(\mathbb{R}^n \times \mathbb{R}^+) \) be a positive function which is continuously twice differentiable in \( x \) and once
differentiable in $t$. Thus, the stochastic derivative operator $\mathcal{L}$ acting on $V(x(t), t)$ is defined by
\[
\mathcal{L}V(x(t), t) = V_r(x(t), t) + V_x(x(t), t)g(t) + \frac{1}{2} \left[ \sigma^T(t) V_{xx}(x(t), t) \sigma(t) \right]
\]
where
\[
V_r(x(t), t) = \frac{\partial V(x(t), t)}{\partial t}, \quad V_x(x(t), t) = \left( \frac{\partial V(x(t), t)}{\partial x_1}, \ldots, \frac{\partial V(x(t), t)}{\partial x_n} \right),
\]
\[
V_{xx}(x(t), t) = \left( \frac{\partial^2 V(x(t), t)}{\partial x_i \partial x_j} \right)_{n \times n}.
\]
\[
g(t) = -Ax(t) + W_0 f(x(t)) + \omega_0 W_f(x(t - \tau_1(t))) + (1 - \omega_0) W f(x(t - \tau_2(t))) + (x(t) - \omega_0) (\sigma(t) - \omega_0).
\]
\[
\sigma(t) = Cx(t) + \omega_0 Dx(t - \tau_1(t)) + (1 - \omega_0) Dx(t - \tau_2(t)) + (x(t) - \omega_0) (Dx(t - \tau_1(t)) - Dx(t - \tau_2(t))).
\]

**Definition 1.** For system (7) and any $\xi \in L^2_p([-\tau, 0]; \mathbb{R}^n)$, the trivial solution is robustly, globally, asymptotically stable in the mean-square sense for all admissible uncertainties, if
\[
\lim_{t \to \infty} \mathbb{E}[x(t, \xi)^2] = 0
\]
where $x(t, \xi)$ is the solution of system (7) at time $t$ under the initial state $\xi$.

**Lemma 1.** [14] For any $G \in \mathbb{R}^{n \times n}$, $G > 0$, scalars $\beta$ and $k > 0$, and vector function $\omega : [\beta - k, \beta] \to \mathbb{R}^n$ such that the integration in the following is well defined, then
\[
\left( \int_{\beta - k}^{\beta} \omega(s) \, ds \right)^T G \int_{\beta - k}^{\beta} \omega(s) \, ds \leq k \int_{\beta - k}^{\beta} \omega^T(s) G \omega(s) \, ds
\]

**Lemma 2.** [15] Let $U, V(t), W$ and $Z$ be real matrices of appropriate dimensions with $Z = Z^T$, then $Z + U V(t) W + W^T V^T(t) U^T < 0$, for all $V^T(t) V(t) \leq I$ if and only if there exists a scalar $e > 0$ such that $Z + e^{-1} U U^T + e W^T W < 0$

**3. Main results**

For presentation convenience, in the following, we denote $L_1 = \text{diag}(l_1, l_1, \ldots, l_n, l_n), L_2 = \text{diag}(\frac{l_1 + l_2}{2}, \ldots, \frac{l_n + l_n}{2})$.

We firstly investigate the stability of nominal SNN without parameter uncertainties.

**Theorem 1.** For given scalars $\tau_0 \geq 0, \tau_M > 0, \mu_1, 0 < \delta_0 < 1$ satisfying $\delta_0 \mu_1 < 1$, the SNN described by (7) is globally asymptotically stable in the mean-square sense, if there exist positive matrices $P > 0, Q_j > 0$ ($j = 1, 2, 3$), $R_1 > 0$, $R_2 > 0$, $S_1 > 0$, $S_2 > 0$, positive diagonal matrices $K_j > 0$ ($j = 1, 2, 3$) and real matrices $M_i, N_i$ ($i = 1, 2, \ldots, 6$) of appropriate dimensions, such that the following LMI holds:
\[
\begin{bmatrix}
\Psi_{1,1} & M & M & M & N & N & A \\
* & -S_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & -S_1 & 0 & 0 & 0 & 0 \\
* & * & * & -S_2 & 0 & 0 & 0 \\
* & * & * & * & -S_2 & 0 & 0 \\
\end{bmatrix} < 0
\]

where
\[
\Psi_{1,1} = Q_1 + Q_2 + Q_3 + M_1 + M_1^T - N_3 A - A^T N_3^T - K_1 L_1
\]

$\Psi_{1,2} = -M_1 + M_2^T$

$\Psi_{1,7} = P - A^T N_6^T - N_5$

$\Psi_{1,8} = -M_1$

$\Psi_{1,13} = N_3 W_0 + K_1 L_2$

$\Psi_{1,14} = 2 \zeta_0 N_3 W_1$

$\Psi_{1,15} = (1 - \zeta_0) N_3 W_1$

$\Psi_{2,2} = -(1 - \zeta_0 \mu_1) Q_1 - M_2 - M_2^T + M_3 + M_3^T$

$\Psi_{2,3} = -M_3 + M_4^T, \Psi_{2,8} = -M_2, \Psi_{2,9} = -M_3$

$\Psi_{3,3} = (1 - \zeta_0) D^T P D - M_4 - M_4^T + M_5 + M_5^T - K_2 L_1$

$\Psi_{3,4} = -M_5 + M_6^T$

$\Psi_{3,5} = -\zeta_0 (1 - \zeta_0) D^T P D$

$\Psi_{3,9} = -M_4, \Psi_{3,10} = -M_5$

$\Psi_{3,14} = K_2 L_2$

$\Psi_{4,4} = -Q_2 - M_6 - M_6^T + N_1 + N_1^T$

$\Psi_{4,5} = -N_1 + N_2^T, \Psi_{4,10} = -M_6, \Psi_{4,11} = -N_1$

$\Psi_{5,5} = (1 - \zeta_0) D^T P D - N_2 - N_2^T + N_3 + N_3^T - K_3 L_1$

$\Psi_{5,6} = -N_3 + N_4^T, \Psi_{5,11} = -N_2, \Psi_{5,12} = -N_3$

$\Psi_{5,15} = K_3 L_2$

$\Psi_{6,6} = -Q_3 - N_4 - N_4^T$

$\Psi_{6,12} = -N_4$

$\Psi_{7,7} = \tau_0 R_1 + (\tau_M - \tau_0) R_2 - N_6 - N_6^T$
\[ \mathcal{L} V_1(x_i, t) = 2x^T(t)P\sigma(t) + \sigma^T(t)P\sigma(t) \] (14)

Furthermore, we can get

\[ \mathcal{L} V_2(x_i, t) \leq x^T(t)(Q_1 + Q_2 + \Omega_1)x(t) - (1 - a_0\mu_1)x^T(t - a_0\tau_1(t))Q_1x(t - a_0\tau_1(t)) - x^T(t - \tau_0)Q_2x(t - \tau_0) - x^T(t - \tau_M)\Omega_1x(t - \tau_M) \] (15)

\[ \mathcal{L} V_3(x_i, t) = g^T(t)[\tau_0R_1 + (\tau_M - \tau_0)R_2]g(t) - \frac{1}{\tau_0} \int_{t - \tau_0}^{t} \int_{t - \tau_0}^{t} g^T(s)g(s) ds \times \int_{t - \tau_0}^{t} \int_{t - \tau_0}^{t} g^T(s)g(s) ds - \frac{1}{\tau_0} \int_{t - \tau_0}^{t} \int_{t - \tau_0}^{t} g^T(s)g(s) ds \times \int_{t - \tau_0}^{t} \int_{t - \tau_0}^{t} g^T(s)g(s) ds \] (16)

\[ \mathcal{L} V_4(x_i, t) \leq \sigma^T(t)[\tau_0S_1 + (\tau_M - \tau_0)S_2]\sigma(t) - \int_{t - \tau_0}^{t} \sigma^T(s)S_1\sigma(s) ds - \int_{t - \tau_0}^{t} \sigma^T(s)S_1\sigma(s) ds - \int_{t - \tau_0}^{t} \sigma^T(s)S_1\sigma(s) ds - \int_{t - \tau_0}^{t} \sigma^T(s)S_1\sigma(s) ds - \int_{t - \tau_0}^{t} \sigma^T(s)S_1\sigma(s) ds - \int_{t - \tau_0}^{t} \sigma^T(s)S_1\sigma(s) ds \] (17)

For arbitrary matrices \( M_i, N_i \) (i = 1, 2, ..., 6) with compatible dimensions, we have
\[ x^T(t)M_1 + x^2(t - z_{\alpha_1}(t))M_2 = 0 \] \[ x^T(t) - x^T(t - z_{\alpha_2}(t)) - \int_{t - z_{\alpha_2}(t)}^{t} g(s)ds - \int_{t - z_{\alpha_2}(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t - z_{\alpha_2}(t))M_3 + x^2(t - \tau_1(t))M_3 = 0 \] \[ x^T(t - \tau_1(t)) - x^T(t - \tau_2(t)) - \int_{t - \tau_1(t)}^{t} g(s)ds - \int_{t - \tau_1(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t - \tau_1(t))M_3 + x^2(t - \tau_2(t))M_3 = 0 \] \[ x^T(t - \tau_1(t)) - x^T(t - \tau_2(t)) - \int_{t - \tau_1(t)}^{t} g(s)ds - \int_{t - \tau_1(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t - \tau_1(t))N_1 + x^2(t - \tau_2(t))N_2 = 0 \] \[ x^T(t - \tau_1(t)) - x^T(t - \tau_2(t)) - \int_{t - \tau_1(t)}^{t} g(s)ds - \int_{t - \tau_1(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t)N_2 + x^2(t)N_3 = 0 \] \[ x^T(t) - x^T(t - z_{\alpha_3}(t)) - \int_{t - z_{\alpha_3}(t)}^{t} g(s)ds - \int_{t - z_{\alpha_3}(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t) - x^T(t - \tau_1(t)) - \int_{t - \tau_1(t)}^{t} g(s)ds - \int_{t - \tau_1(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t) - x^T(t - \tau_2(t)) - \int_{t - \tau_2(t)}^{t} g(s)ds - \int_{t - \tau_2(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t) - x^T(t - \tau_3(t)) - \int_{t - \tau_3(t)}^{t} g(s)ds - \int_{t - \tau_3(t)}^{t} \sigma(s)dw(s) = 0 \] \[ x^T(t) - x^T(t - \tau_4(t)) - \int_{t - \tau_4(t)}^{t} g(s)ds - \int_{t - \tau_4(t)}^{t} \sigma(s)dw(s) = 0 \]}

\[ \Sigma_2 = \int_{t - \tau_1(t)}^{t - \tau_2(t)} \sigma(s)dw(s), \quad \Sigma_3 = \int_{t - \tau_2(t)}^{t - \tau_3(t)} \sigma(s)dw(s), \quad \Sigma_4 = \int_{t - \tau_3(t)}^{t - \tau_4(t)} \sigma(s)dw(s) \]

From (9), for any matrices \( K_i = \text{diag}(k_{i1}, k_{i2}, \ldots, k_{im}) \geq 0, i = 1, 2, 3 \), it is easy to obtain

\[ -\sum_{j=1}^{n} k_j \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} l_j^T \lambda_j e_j & \frac{l_j^T \lambda_j}{2} e_j e_j^T \\ \frac{l_j^T \lambda_j}{2} e_j e_j^T & e_j e_j^T \end{bmatrix} \begin{bmatrix} x(t - \tau_1(t)) \\ f(x(t - \tau_1(t))) \end{bmatrix} \]

\[ \times \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} - \sum_{j=1}^{n} k_{(i+1)} \begin{bmatrix} x(t - \tau_1(t)) \\ f(x(t - \tau_1(t))) \end{bmatrix}^T \begin{bmatrix} l_j^T \lambda_j e_j & \frac{l_j^T \lambda_j}{2} e_j e_j^T \\ \frac{l_j^T \lambda_j}{2} e_j e_j^T & e_j e_j^T \end{bmatrix} \begin{bmatrix} x(t - \tau_1(t)) \\ f(x(t - \tau_1(t))) \end{bmatrix} \]

\[ = \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -K_1 L_1 & K_1 L_2 \\ K_1 L_2 & -K_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \]

\[ + \sum_{i=1}^{2} \begin{bmatrix} x(t - \tau_1(t)) \\ f(x(t - \tau_1(t))) \end{bmatrix}^T \begin{bmatrix} -K_{i+1} L_1 & K_{i+1} L_2 \\ K_{i+1} L_2 & -K_{i+1} \end{bmatrix} \begin{bmatrix} x(t - \tau_1(t)) \\ f(x(t - \tau_1(t))) \end{bmatrix} \geq 0 \]

By Remark 1, it is easy to know

\[ E\{\sigma^T(t)(P + \tau_0 S_1 + (\tau_m - \tau_0) S_2)\sigma(t)\} \]

\[ = E\{[C(x(t) + \tau_0 Dx(t - \tau_1(t)) + (1 - \tau_0)] + 2(x(t) - \tau_0) [C(x(t) + \tau_0 Dx(t - \tau_1(t)) + (1 - \tau_0] \}

\[ + 2(x(t) - \tau_0) [C(x(t) + \tau_0 Dx(t - \tau_1(t)) + (1 - \tau_0] \}

\[ \times \frac{1}{2} \left( [D(x(t - \tau_1(t)) - D(x(t - \tau_2(t))) + (x(t) - \tau_0)^2} \right)

\[ \times \left( [D(x(t - \tau_1(t)) - D(x(t - \tau_2(t))) + (x(t) - \tau_0)^2} \right)

\[ - D(x(t - \tau_2(t))) \right)^2 \right)

\[ - D(x(t - \tau_2(t))) \right)^2 \right)

\[ \times \left( [D(x(t - \tau_2(t)) - D(x(t - \tau_3(t))) + (x(t) - \tau_0)^2} \right)

\[ \times \left( [D(x(t - \tau_2(t)) - D(x(t - \tau_3(t))) + (x(t) - \tau_0)^2} \right)

\[ - D(x(t - \tau_3(t))) \right)^2 \right)

Since

\[ E\left\{ \int_{t - \tau_1(t)}^{t} \sigma^T(s)dw(s)S_1 \int_{t - \tau_1(t)}^{t} \sigma(s)dw(s) \right\} \]

\[ = E\left\{ \int_{t - \tau_1(t)}^{t} \sigma^T(s)S_1 \sigma(s)ds \right\} \]

\[ E\left\{ \int_{t - \tau_1(t)}^{t} \sigma^T(s)dw(s)S_1 \int_{t - \tau_1(t)}^{t} \sigma(s)dw(s) \right\} \]

\[ = E\left\{ \int_{t - \tau_1(t)}^{t} \sigma^T(s)S_1 \sigma(s)ds \right\} \]
where \( \Psi_i = \Psi_i + \text{diag} \left( \gamma H_1^T H_1, 0, \ldots, 0, \gamma H_2^T H_2, \gamma H_3^T H_3, \gamma H_4^T H_4 \right) \),
\[
\rho = \sqrt{2 + z_0^2 + (1 - z_0)^2}, \quad \Sigma^T = \left[ \rho E^T N_5^T 0 \cdots 0 \rho E^T N_6^T 0 \cdots 0 \right],
\]
\( M, M, \tilde{M}, N, \tilde{N}, A, P \) are defined as in Theorem 1.

**Proof.** Replace \( A, W_0, W_1 \) in the LMI (12) with \( A + \Delta A, W_0 + \Delta W_0, W_1 + \Delta W_1 \), respectively, then the LMI (12) can be rewritten as

\[
E + \Theta P^T + \gamma^T P^T E < 0
\]

where \( \Theta = \begin{bmatrix} \Theta_1^T & \ldots & \Theta_2^T & \ldots & 0 \end{bmatrix}^T \),
\[
\Theta_1 = \begin{bmatrix} N_1 E 0 \cdots 0 N_5 E (1 - \sigma_0) N_3 E 0 \cdots 0 \end{bmatrix},
\]
\[
\Theta_2 = \begin{bmatrix} N_6 E 0 \cdots 0 N_6 E (1 - \sigma_0) N_6 E 0 \cdots 0 \end{bmatrix}
\]
and \( \Pi = \text{diag} \left( F(t), F(t), \ldots, F(t) \right) \text{diag} \left( H_1, 0 \cdots 0, H_2, H_2, \ldots, 0 \right) \).

It follows from Lemma 2 that the matrix inequality (38) is equivalent to the following inequality.

\[
E + \gamma^{-1} \Theta \Theta^T + \gamma^T \Theta < 0
\]

By the Schur complement, (37) is equivalent to (39) for a scalar \( \gamma > 0 \). Then, similar to the proof of Theorem 1, the results of Theorem 2 can be obtained. \( \square \)

**Remark 3.** As a special case, when \( \sigma_0 = 1 \) (or \( \sigma = 0 \)), by setting \( Q_i = R_i = S_1 = 0, \tau = \sigma_0 \) (or \( Q_i = R_i = S_1 = 0 \)) in the Lyapunov–Krasovskii functional of Theorem 2, the robust stability criteria can be obtained easily and the corresponding proof is similar to Theorem 2, which are omitted.

**4. Numerical examples**

**Example 1.** Consider the uncertain SNN (7) with parameters as follows (Example in Ref. [6]):
Theorem 2

\[ L_0 = 0.5I \]

by Assumption 3, \( L_1 = 0 \), \( L_2 = 0.25I \) equivalent to \( L = 0.5I \) in Ref. [6].

For various \( \mu_1 \), the computed upper bound \( \tau_M \), which guarantee the robust stability of system (7), are listed in Table 1. From Table 1, when the information of the delay-probability distribution is considered, for various \( \alpha_0 \) the allowable upper bound \( \tau_M \) is larger than those in Refs. [6–8], where only the variation range of the delay is considered. In addition, when \( \mu_1 \geq 1 \), the stability criteria fail in Ref. [6] and become derivative independent in Refs. [7,8]. However, in this paper, the constraint \( \mu_1 < 1 \) is eliminated for \( \alpha_0\mu_1 < 1 \). Therefore, Theorem 2 in this paper is less conservative than those in Refs. [6–8].

Example 2. Consider the uncertain SNN (7) with parameters as follows:

\[ A = \begin{bmatrix} 7 & 0 \\ 0 & 6 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.4 & -0.7 \\ 0.1 & 0 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -0.2 & 0.6 \\ 0.5 & -0.1 \end{bmatrix}, \]

\[ C = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & -0.5 \\ 0 & 0 \end{bmatrix}, \quad E = [0.1 -0.1]^T, \]

\[ H_1 = \begin{bmatrix} 0.2 \\ -0.3 \\ -0.3 \\ -0.3 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.2 \\ -0.3 \\ -0.3 \\ -0.3 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0.2 \\ -0.3 \\ -0.3 \\ -0.3 \end{bmatrix}. \]

Take the activation function as: \( f_1(x_1) = \tanh(-0.2x_1), \quad f_2(x_2) = \tanh(x_2) \). It is obvious that \( -0.2 \leq \frac{d}{dx_1}\tanh(-0.2x_1) < 0, \quad 0 < \frac{d}{dx_2}\tanh(x_2) \leq 1 \), so \( L_1 = \text{diag}(0,0), \quad L_2 = \text{diag}(-0.1 \quad 0.5) \).

For \( \tau_0 = 0.4 \), various \( \mu_1 \) and delay probability distribution \( \alpha_0 \), the computed upper bound \( \tau_M \), which guarantee the robust stability of system (7), are listed in Table 2.

From Table 2, this paper overcomes the constraint \( \mu_1 < 1 \) for \( \alpha_0\mu_1 < 1 \).

5. Conclusions

The problem of robust stability for uncertain SNNs with probability-distribution-dependent time-varying delay has been addressed in this paper. Some new stability criteria have been proposed to guarantee the robust global asymptotical stability of the SNNs. Probability distribution of time varying delay is introduced into the stability criteria, and the new method eliminates the constraint that the derivative of the delay must be smaller than 1. Numerical examples show the effectiveness and less conservatism of the method.

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