High speed interconnect data dependent jitter analysis

Tian Xia*, Di Mu
School of Engineering, University of Vermont, VT 05405, USA

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This paper focuses on modeling and characterizing the data dependent jitter (DDJ) in high-speed interconnect. The analysis process is performed based on the Fourier series using the interconnect RLC model. By calculating the pattern dependent delay deviations, the DDJ is characterized. To validate the model accuracy, the analysis results have been compared against Cadence simulations. The interconnect layout optimization is also explored to minimize the DDJ.

1. Introduction

As the microelectronics steps into the deep sub-micron era, the VLSI chip density and the operating speed have been increased tremendously. In high-speed and high-density integrated circuits, the timing accuracy has become a critical issue concerning about the system performance. In high speed interconnects, the properties of driver, transmission line and receiver have great impact on signal integrity. Limited bandwidth and non-ideal termination may aggravate timing jitter, which will degrade the bit error rate (BER). Therefore, in high-speed interconnect, timing jitter analysis is of great importance.

According to the ITU-T O.171 recommendation [1], timing jitter is characterized by the relative time variation of the transitions of the signal across a specific level. Generally, timing jitter is composed of both deterministic and random contents [2,15,16]. Random jitter comes from thermal vibrations of semiconductor crystal structures, material boundaries having less than perfect valence electron mapping due to semi-regular doping density and process anomalies, thermal vibrations of conductor atoms, and many other minor contributors [3]. Deterministic jitter (DJ) is typically caused by crosstalk, EMI, simultaneous switching outputs (SSO), device function dependency and other regularly occurring interferences. For a system with a very small BER specification, i.e. 1e-14–1e-12, it has stringent jitter requirements. For instance, in [18], for a 2.5 Gb/s system of 1e-12 BER, the deterministic jitter is required to be less than 8 ps. Data-dependent jitter (DDJ) is an important component of DJ, which characterizes the delay deviation among different data patterns. Small DDJ, i.e. 1–2% UI (unit interval), can critically degrade the system performance. Therefore, in this paper, we will focus on the DDJ analysis.

In the recent literature, research has been conducted to investigate the data dependent jitter. In [4], the perturbation method is proposed to generalize the analytical expression for DJ. To demonstrate its accuracy, extensive experiments have been conducted. However, the analysis in [4] does not include the transmission line model. Instead it requires instrument set up to measure the transmission channel step response. Hence, it can hardly provide early guidance for high-speed interconnect design, logic synthesis and physical implementation. In this paper, we utilize the Fourier series with the transmission line RLC model to analyze the DDJ. By calculating the pattern dependent delay deviations, DJ can be characterized with good accuracy. This proposed method does not rely on the physical measurement setup. It offers circuit designers a mathematical method to gain early knowledge of DJ.

This paper is organized as follows. In Section 2, the DDJ will be briefly introduced. Section 3 develops the transmission line model and transfer function. In Section 4, the method for periodic pattern delay calculation will be presented. Section 5 presents the DDJ experimental results. Section 6 analyzes the DDJ minimization method. Section 7 concludes the paper.

2. Data dependent jitter

As the signal is transmitted through the transmission line, propagation delay is induced at each transition edge. Due to
transmission line's limited bandwidth and other uncertain effects, such as the electromagnetic reflection, crosstalk effects, different data patterns may experience different transition delays. The delay deviation associated with various data patterns is the data-dependent jitter. On account of the transmission channel memory properties, for a data transition, its threshold crossing time is determined not only by the current bit value but also by the previous bit sequence. The previous data sequence causes the channel output response. As can be seen, different data sequences experience different transition delays. These delay deviations indicate the data dependent jitter.

3. Transmission line model

In this analysis, depending on the circuit operation speed and chip integration density, two uniform transmission line models are employed [11].

In Fig. 2a, the transmission channel is modeled as parasitic R–C segments connected in series. This distributed RC model is widely adopted when signal transmission speed is low.

In circuits and systems with very high operation frequency and high integration density, the interconnect lines not only have parasitic resistance and capacitance but also exhibit significant parasitic inductance effects [6]. Therefore, to accurately characterize the DDJ in these transmission lines, as shown in Fig. 2b, the R–L–C model is employed. In transmission line models, R, L and C are the unit length resistance, inductance, and capacitance, respectively. \( R_t \) represents the source resistance at the driver end, and \( C_t \) represents the load capacitance. As both models show, the transmission line is a two-port network and consists of only linear elements. Hence, it can be described by an \( ABCD \) matrix [5].

Fig. 3 depicts the transmission line abstract model, where \( ABCD \) matrix correlates the input–output terminal voltages and currents

\[
\begin{bmatrix} V_{1}(s) \\ I_{1}(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{\text{out}}(s) \\ I_{\text{out}}(s) \end{bmatrix}
\]

where \( V_{1}(s) \) and \( I_{1}(s) \) are the Laplace transform of voltage and current at the input port, while \( V_{\text{out}}(s) \) and \( I_{\text{out}}(s) \) are the Laplace transform of the output voltage and current. Suppose the transmission line length is \( h \), the corresponding \( ABCD \) matrix [7]

\[
\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & R_{d} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh(\theta) & Z_{0}\sinh(\theta) \\ Y_{0}\sinh(\theta) & \cosh(\theta) \end{bmatrix}
\]

in frequency domain equals

\[
\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & R_{d} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh(\theta) + R_{d}Y_{0}\sinh(\theta) & Z_{0}\sinh(\theta) + R_{d}\cosh(\theta) \\ Y_{0}\sinh(\theta) & \cosh(\theta) \end{bmatrix}
\]

where

\[
Z_{0} = \sqrt{(R + sL)/sC} = \frac{1}{Y_{0}}, \quad \theta = h\sqrt{(R + sL)sC}.
\]

where \( Z_{0} \) is the transmission line's characteristic impedance and \( \theta \) the propagation constant. For R–C modeled transmission line, the inductance parameter \( L \) equals zero.

By extending the matrix to the input driver end, a new \( A'B'C'D' \) matrix is obtained

Fig. 1. Signal waveforms over a transmission line.

Fig. 2. (a) Distributed RC model for interconnect and (b) distributed RLC model for high speed interconnect.

Fig. 3. Transmission line abstract model.
By utilizing the new $A'BCD'$ matrix, the transfer function of the transmission line equals
\[ H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{(1 + R_0C_4s) \cosh \theta + (R_d/Z_0 + Z_0C_3s) \sinh \theta} \quad (4) \]

4. Data dependent jitter analysis

Suppose the channel input signal $V_{\text{in}}(t)$ takes the NRZ form (non-return-to-zero). According to [3], the NRZ data sequence can be represented by
\[ V_n(t) = \sum_{n=-\infty}^{0} a_n p(t - nT_b), \quad (5) \]
where
\[ p(t) = \begin{cases} 1, & 0 < t \leq T_b, \\ 0, & t \leq 0 \text{ or } t > T_b \end{cases} \quad (6) \]
where $a_n$ is the binary value and $p(t)$ the input pulse whose pulse width equals $T_p$. Since the transmission line is a linear time-invariant (LTI) system, with the NRZ data sequence as the input, the channel output can be expressed as
\[ r(t) = \sum_{n=-\infty}^{0} a_n g(t - nT_b), \quad (7) \]
where $g(t)$ is the pulse response function, which equals the convolution of $p(t)$ and the channel impulse response function $h(t)$.

For the input pulse function $p(t)$, it can be described in the closed form
\[ p(t) = u(t) - u(t - T_b) \quad (9) \]
The corresponding Laplace transform is
\[ P(s) = \frac{1 - e^{-Ts}}{s}. \quad (10) \]

Multiplying $p(s)$ with the transmission line transfer function $H(s)$, the pulse response can be obtained in frequency domain
\[ G(s) = P(s)H(s) = \frac{1 - e^{-Ts}}{s} \cdot \frac{1}{(1 + R_0C_4s) \cosh \theta + (R_d/Z_0 + Z_0C_3s) \sinh \theta} \quad (11) \]

By applying the inverse Laplace transform, the time domain pulse response function $g(t)$ can be obtained. Unfortunately Eq. (11) is not a linear polynomial function, therefore it is hard to perform the inverse Laplace transform directly. To solve this problem, the Padé approximation [10] will be utilized.

4.1. Pulse response in the 1st order channel

For the interconnect R–C model in Fig. 2a, by applying the Padé approximation [10,14], its transfer function $H(s)$ can be simplified in the 1st order form,
\[ H(s) \approx \frac{1}{1 + b_1 s} \quad (12) \]
where $b_1$ represents the system time constant and equals
\[ b_1 = R_dC + R_0C_1 + RCH^2/2 + RHC \quad (13) \]

From Eq. (12), the channel $-3$ dB bandwidth $BW$ can be computed,
\[ BW = 1/(2\pi b_1). \quad (14) \]

Using Eq. (12), Eq. (11) is simplified to
\[ G(s) = \frac{1 - e^{-Ts}}{s} \cdot \frac{1}{1 + b_1 s} \quad (15) \]

Applying the inverse Laplace transform to Eq. (15), the pulse response signal $g(t)$ is thus obtained,
\[ g(t) = \begin{cases} 0, & t < 0 \\ e^{-(t-T_b)/b_1} / (1 - e^{-(T_b)/b_1}) (1 - e^{-(t-T_b)/b_1}), & T_b < t \end{cases} \quad (16) \]

Fig. 4 shows the curve of the pulse response signal $g(t)$, and the square-wave is for a single input pulse $p(t)$. As illustrated, when at time instant $T_b$, the pulse response signal $g(t)$ reaches the peak value. After $T_b$ seconds, $g(t)$ decreases exponentially.

Substituting Eq. (16) into Eq. (7), the channel NRZ response function $r(t)$ can be expressed as
\[ r(t) = \sum_{n=-\infty}^{0} a_n g(t - nT_b) = a_0(1 - e^{-(t/T_b)}) \]
\[ + \sum_{n=-\infty}^{1} a_n e^{-(t/T_b)} \left[ e^{-(n+1)(T_b/b_1)} - e^{-(nT_b/b_1)} \right] \quad (17) \]

Eq. (17) shows that at time instant $t$, the channel output voltage $r(t)$ equals the sum of pulse responses to all previous NRZ bits. Suppose $r(t)$ threshold voltage equals $V_{th}$, and its threshold crossing time is $t_c$, thus
\[ r(t_c) = V_{th} = \sum_{n=-\infty}^{0} a_n g(t - nT_b) = a_0(1 - e^{-(t_c/T_b)}) \]
\[ + \sum_{n=-\infty}^{1} a_n e^{-(t_c/T_b)} \left[ e^{-(n+1)(T_b/b_1)} - e^{-(nT_b/b_1)} \right] \quad (18) \]

Eq. (18) illustrates that $r(t)$’s threshold crossing time $t_c$ is affected by all previous bit responses. Since $g(t)$ amplitude decreases exponentially after one bit duration, hence its impact on later bit transition also reduces exponentially.

In the DDJ analysis, we define that at the $k$th bit cycle $(t = kT_b)$, if $g(t)$ amplitude drops below 5% of its previous cycle’s $(t = (k-1)T_b)$ amplitude, then the impact from $g(kT_b)$ is small and can be ignored.

To calculate $g(t)$ depletion rate, in the following analysis, we compare bit responses between two adjacent cycles, where $k=2$ and 3, respectively.
When \( k=2 \), the response signal \( g(t=2T_b) \) equals
\[
g(t)_{t=2T_b} = e^{-(2T_b-b_2/b_1)}(1-e^{-(T_b/b_1)}) = e^{-(T_b/b_1)}(1-e^{-(T_b/b_1)})
\]
\[\text{(19)}\]

When \( k=3 \), the response signal \( g(t=3T_b) \) reduces to
\[
g(t)_{t=3T_b} = e^{-(3T_b-b_3/b_1)}(1-e^{-(T_b/b_1)}) = e^{-(2T_b/b_1)}(1-e^{-(T_b/b_1)})
\]
\[\text{(20)}\]

Comparing \( g(t) \) amplitudes at these two time instants, the pulse response depletion rate is calculated
\[
R_{3/2} = \frac{g(t)_{t=3T_b}}{g(t)_{t=2T_b}} = e^{-T_b/b_1}
\]
\[\text{(21)}\]

Plugging Eq. (14) into Eq. (21), we get
\[
R_{3/2} = e^{-T_b/b_1} = e^{-(T_b/(2\pi BW))}
\]
\[\text{(22)}\]

In a communication system, the transmission channel bandwidth \( BW \) is usually set above 70–80% of the signal bit rate \( [4] \), \( BW \gg (70–80\%)/T_b \). As a result,
\[
R_{3/2} = e^{-T_b/b_1} = e^{-(T_b/(2\pi BW))} < e^{-2\pi (70–80\%)/5\%}
\]

Ratio \( R_{3/2} \) less than 5% illustrates that the amplitude of the pulse response signal \( g(t) \) decreases significantly at the beginning of the 3rd bit cycle. Therefore the channel inter-symbol interference is mainly limited within two bits range, and the impact from the 3rd bit backward can be neglected. In other words, during the data dependent jitter analysis, by considering

\[
g(t) = \begin{cases} 
0 & \text{for } t < 0 \\
\frac{1}{b_2} [k_0 + k_1 \exp(p_1t) + k_2 \exp(p_2t)] & \text{for } 0 < t < T_b \\
\frac{1}{b_2} [k_1 \exp(p_1t) + k_2 \exp(p_2t)] - \frac{1}{b_2} (k_1 \exp(p_1(t-T_b)) + k_2 \exp(p_2(t-T_b))) & \text{for } t > T_b
\end{cases}
\]

the interference from two prior bits, the high analysis accuracy can be ensured.

4.2. Pulse response in the 2nd order channel

In this section, DDJ analysis is conducted to the R–L–C modeled transmission line, whose diagram is shown in Fig. 2b.

For the R–L–C transmission line, by applying the Pade approximation [10], its transfer function can be simplified in the second order form [11]
\[
H(s) \approx \frac{1}{s^2b_2 + sb_1 + 1}
\]
\[\text{(23)}\]

where
\[
b_1 = R_0C_1 + R_0C_2 + C_1R_H + \frac{RCH^2}{2} \quad b_2 = C_1L + \frac{LCh^2}{2} + \frac{R^2C^2h^4}{4!} + \frac{R_2C_2RCH^2}{2!} + \frac{(R_0C_1 + R_0C_2)RCH^2}{2!}
\]

The poles of the transfer function \( H(s) \) are \( p_1 \) and \( p_2 \).
\[
p_1 = -b_1 + \sqrt{b_1^2 - 4b_2} \quad p_2 = -b_1 - \sqrt{b_1^2 - 4b_2}
\]

For an input pulse \( p(t) \), the corresponding output response in frequency domain equals
\[
G(s) = P(s)H(s) = \frac{1-e^{-T_bS}}{S} \frac{1}{s^2b_2 + sb_1 + 1}
\]
\[\text{(24)}\]

Depending on the sign of \( b_1^2 - 4b_2 \), the poles of the transfer function \( H(s) \) can be either real or complex, which will result in different pulse responses and will be analyzed separately as below.

4.2.1. Real poles

When the sign of \( b_1^2 - 4b_2 \) is positive, \( H(s) \) has two real poles \( P_1 \) and \( P_2 \). Performing the inverse Laplace transformation for Eq. (24), the time domain function \( g(t) \) can be obtained
\[
t < 0 \\
0 < t < T_b
\]
\[\text{(25)}\]

where \( k_0, k_1 \) and \( k_2 \) are three coefficients
\[
k_0 = \frac{1}{p_1P_2} \quad k_1 = \frac{-1}{p_1(p_2 - p_1)} \quad k_2 = 1 \frac{1}{p_1(p_2 - p_1)}
\]

Fig. 5 illustrates the pulse response function \( g(t) \) in the 2nd order channel. It is clear to see, after one \( T_b \) duration, the pulse response signal \( g(t) \) depletes exponentially, which is similar to that in the 1st order channel.

To check the \( g(t) \) depletion rate, we also compare the signal amplitudes \( g(t)_{t=T_b} \) and \( g(t)_{t=3T_b} \).

In Eq. (25), when \( t \) is larger than \( T_b \), the response signal \( g(t) \) can be expressed as
\[
g(t)_{t>T_b} = \frac{k_1}{b_2} (1-\exp(-p_1T_b)) \exp(p_1t) + \frac{k_2}{b_2} (1-\exp(-p_2T_b)) \exp(p_2t)
\]
\[\text{(26)}\]

Since \( |p_2| \) is much larger than \( |p_1| \), the second term in the above equation decreases rapidly compared to the first term. Thus, the two-pole response signal \( g(t) \) can be estimated as
\[
g(t)_{t>T_b} \approx \frac{k_1}{b_2} (1-\exp(-p_1T_b)) \exp(p_1t)
\]

When \( t = 3T_b \),
\[
g(t)_{t=3T_b} = \frac{k_1}{b_2} (1-\exp(-p_1T_b)) \exp(p_1(3T_b))
\]

When \( t = 2T_b \),
\[
g(t)_{t=2T_b} = \frac{k_1}{b_2} (1-\exp(-p_1T_b)) \exp(p_1(2T_b))
\]

As a result, the signal depletion rate \( R_{3/2} \) is estimated as
\[
R_{3/2} = \frac{g(t)_{t=3T_b}}{g(t)_{t=2T_b}} = e^{p_1T_b}
\]

\[\text{(26)}\]
Since the amplitude of $P_2$ is much larger than that of $P_1$, the transmission line’s bandwidth $BW$ is mainly determined by the amplitude of $P_1$, and $BW \approx |P_1|/2\pi$. Given that the transmission line bandwidth $BW$ is over 70–80% of the signal bit rate $(1/T_b)$ [4], it is obtained that,

$$R_{3/2} = g(t)|_{t = 3T_b}/g(t)|_{t = 2T_b} = e^{pT_b} < e^{-2n\pi BW T_b} \approx 1.2\% \leq 5\% \quad (27)$$

This ratio is also less than 5%, which implies that, in the 2nd order transmission channel, the impact from the 3rd prior bit is also small and can be ignored. Note, if $|p2|$ is not much larger than $|p1|$, the channel bandwidth $BW$ will be slightly less than $|P_1|/2\pi$, and the ratio $R_{3/2}$ will be larger than 1.2%, however in most cases, it will still be less than 5%. Therefore, during the DDJ analysis, the inter-symbol interference is mainly limited within two bits range.

### 4.2.2. Complex poles

When the interconnect inductance is large and the load capacitance is small, it may be possible that $b_1^2 - 4b_2 < 0$. Thus the channels’ transfer function $H(s)$ will have complex conjugate poles, which can be represented as $P_1 = -\alpha + j\beta$ and $P_2 = -\alpha - j\beta$. By performing the inverse Laplace transformation for the step response function in Eq. (24), the time domain response function $g(t)$ can be obtained as

$$g(t) = \begin{cases} 0 & t < 0 \\ 1 - \exp(-\alpha t) \cos(\beta t/2) + \frac{\alpha}{\beta} \sin(\beta t/2) & 0 < t < T_b \\ -k \exp(-\alpha t) \sin(\beta t/2) + k \exp(-\alpha t) \cos(\beta t/2) & t > T_b \\ \end{cases} \quad (28)$$

Where $\alpha = b_1/2b_2$, $\beta = \sqrt{4b_2 - b_1^2}/2b_2$, $\theta = \tan^{-1}(\beta/\alpha)$ and $k = (\alpha^2 + \beta^2)/\beta$.

As shown in Fig. 6, the step response function $g(t)$ is oscillatory when the channel transfer function has two complex poles. The oscillation signal amplitude decreases exponentially by the factor of $\exp(-\alpha t)$. When $t = 3/\alpha$, signal $g(t)$ will settle within 5% of its final value [17]. In other words, for symbol duration $T_b$, when $2T_b > 3/\alpha$, the inter-symbol interference is within 2-symbol interval for channel transfer function $H(s)$ with complex poles. In the selected 0.18 um CMOS process [9] in this paper, the $T_b$ is calculated to be 127 ps when the load capacitance is 200 F and 131 ps when the load capacitance is 400 F.

### 4.3. Pattern extension

When the impact from two prior bits is considered, there will be totally four data patterns under investigation, which are 001, 101, 110 and 010. For the input signal with the symmetrical rising edge and falling edge, the output response will have the same rising and falling transition delays [4]. Hence during the delay analysis, patterns 001 and 110 can be grouped together for having the same transition delay. For the same reason, patterns 101 and 010 can also be grouped together. By comparing the delay deviation between two pattern groups, the DDJ can be characterized [4]. In this paper, we use the rising transition delay as the investigation object. Patterns 001 and 101 are selected, because they both have rising transitions (‘0’→’1’) in the end.

In the analysis, suppose the transition delays for patterns 001 and 101 are recorded as $t_{c1}$ and $t_{c2}$, respectively, the deviation between $t_{c1}$ and $t_{c2}$ thus represents the data dependent jitter,

$$\Delta DDJ = t_{c1} - t_{c2} \quad (29)$$

Since the transition delay is mainly affected by two prior bits, and the impact from other bits backward are negligible, the data pattern can be reconstructed in the periodic form. In other words, the periodic data pattern can be created for DDJ analysis. By using the periodic data pattern, the calculations of the delay time $t_{c1}$ and $t_{c2}$ can be simplified.

During the data reconstruction, a bit ‘0’ is added in front of ‘101’ to generate the ‘0101’ pattern. By extending the ‘0101’ sequence, a periodic bit stream $P_1 = \ldots 01010101 \ldots$ is created, as shown in Fig. 7. Since the signal bit length is $T_b$, the period of the new periodic signal equals $2T_b$. Similarly, for data sequence ‘001’, a periodic pattern $P_2 = \ldots 001001001 \ldots$ is constructed. As Fig. 7 shows, $P_2$’s period equals $4T_b$. Note, ‘101’ and ‘001’ can also be extended to form other periodic patterns. The reason to generate $P_1$ and $P_2$ here is because their duty cycles are 50%, which will ease the pattern delay analysis.

In the following sections, the Fourier series analysis is used to calculate the periodic patterns delay deviation.

### 4.4. Periodic pattern delay characterization

As described in the above section, the data dependent jitter can be characterized as the delay deviation between periodic patterns $P_1$ and $P_2$. In this section, an analytical model is developed to calculate the periodic signal delay.

Suppose the input periodic signal is $V_{in}(t)$, whose magnitude is $V_{dd}$ and period is $T$. This signal can be written as

$$V_{in}(t) = \begin{cases} V_{dd} & nT \leq t < \left(n + \frac{1}{2}\right)T \\ 0 & \left(n + \frac{1}{2}\right)T \leq t < \left(n + 1\right)T \quad n = 0, 1, 2, \ldots \end{cases} \quad (30)$$

For the periodic patterns $P_1$ and $P_2$, their periods are $2T_b$ and $4T_b$, respectively. The Fourier series extension of $V_{in}(t)$ equals

$$V_{in}(t) = \frac{V_{dd}}{2} + \sum_{m=1,3,\ldots}^\infty A_m \cos(m\omega_0 t + \phi_m) \quad (31)$$
\[ \phi_m = \frac{m \pi}{2}, \quad A_m = \frac{2V_{dd}}{m \pi} \sin \phi_m \]

where \( \omega_0 \) is \( V_{dd}(t) \) fundamental angular frequency, \( \phi_m \) and \( A_m \) are the phase and amplitude of the \( m \)th order harmonics.

The transmission line is a LTI system, thus the overall output response can be calculated using the superposition method with each harmonic as the input. The transfer function \( H(s) \) at a particular angular frequency \( \omega \) can be represented as

\[ H(\omega_j) = H(s)|_{s=j \omega} = A(\omega)e^{j\phi(\omega)} \]

The channel output, therefore, equals

\[ V_{out}(t) = \frac{V_{dd}}{2} + \sum_{m=1,3, \ldots} A_m \cos(m\omega_0 t + \phi_m(t)) \]

where

\[ \phi_m = \beta(m\omega_0) + \phi_m, \quad A_m = A_m(m\omega_0) \]

Since \( A_m \) decreases quadratically as \( m \) increases, \( V_{out}(t) \) can be approximated as the sum of the first several order harmonics. In this paper, all calculations are performed up to the 11th order harmonics. Including more harmonics will increase the computation complexity while only improve the computation accuracy by 1–2% approximately.

To determine \( V_{out}(t) \)’s threshold crossing time \( t_c \), Eq. (34) is set to 50% of the signal magnitude \( V_{dd}/2 \), then the Secant method [8] is applied to solve the resulted polynomial equation. The result equals the signal transition delay.

5. Experimental results

In this experiment, the data dependent jitter is calculated using the proposed modeling method. To validate the model accuracy, Cadence simulations are also conducted. The parameters of the transmission line in the experiments are: length \( h = 2 \) mm, width \( w = 2 \) um, \( R_o = 100 \) Ω, \( R = 9 \) mΩ/um and \( C = 0.18 \) fF/um, which are typical values in 0.18 um CMOS technology [9].

5.1. The 1st order transmission line experiment

For the 1st order transmission line, the signal transmission rates are set relatively low (around 1 Gbps). At these low speeds, the parasitic inductance effect in the transmission line is weak and can be ignored.

Table 1 summarizes the experimental results. In the experiment, several \( C_L \) values are applied to examine DDJ with different capacitive load. To verify the modeling effectiveness, Cadence simulation results are also presented.

In the table, the 1st column shows the load capacitance. The 2nd column describes the bit width \( T_b \), which indicates the transmission bit rate \( 1/T_b \). Columns 3–6 are pattern dependent delays obtained from Cadence simulations and Matlab computations based on the proposed modeling method. \( J_C \) and \( J_M \) are the delay deviations between \( t_{c1} \) and \( t_{c2} \). \( J_C \) is calculated from the Cadence simulation data, while \( J_M \) is calculated from the Matlab data. The last column \( J_F \) is the DDJ measured directly with random input patterns. By comparing from Fig. 8 and Table 1, we can see that the proposed model can estimate the DDJ accurately.

![Fig. 8. DDJ vs. the bit rates.](image)

5.2. The 2nd order transmission line experiment

In the 2nd order transmission line DDJ analysis, the signal bit rates are increased significantly. As Table 2 shows, the bit rates in the experiments range from 3.4 to 6.7 GHz. Under these high frequencies, the transmission line’s parasitic inductance is pronounced and must be taken into consideration [6,9]. To extract the line inductance, a third party tool FastHenry [12] is adopted.

For the transmission line under test, the unit length inductance equals 1.54 pH/um.

In Table 2, the experimental results are summarized in the same format as in Table 1. By checking the experimental data carefully, it is observed that the proposed DDJ analysis method can...
also accomplish high accuracy in the 2nd transmission line model.

Fig. 9 plots the data dependent jitter vs. the bit width $T_b$ when the capacitive load $C_L$ equals 300 fF. Similarly, as the bit width $T_b$ increases (bit rate decreases), the data dependent jitter is reduced. This reduction is due to the fact that as the bit width $T_b$ increases, the amplitude of the pulse response signal reduces rapidly after $T_b$ seconds, and therefore its impact on later bit transitions becomes weak. As a result, the inter-symbol interference diminishes.

As Eq. (27) shows, the signal degradation rate $R_{3/2}$ is also a function of $T_b$. Fig. 10 plots the $R_{3/2}$ vs. the bit width $T_b$ when the capacitive load $C_L$ equals 500 fF. In the figure, $T_b$ sweeps from 100 to 350 ps, thus the signal bit rate in the experiments ranges from 10 Gbps to about 2.8 Gbps. As shown, when the bit width is larger than 190 ps, or when the bit rate is slower than 5.2 Gbps, the signal degradation rate drops below 5%. Thus for this specific transmission line with 500 fF capacitive load, as long as the transmission bit width is longer than 190 ps, the data dependent jitter is mainly affected by two bits backward.

By performing a similar analysis, the minimum bit width under different capacitive load $C_L$ is calculated. The results are summarized in the 3rd column in Table 3. These values are consistent with those listed in Table 2. In Table 3, the 4th column lists the actual minimum bit widths employed in the experiments.

Note, for signal transmission, if the bit width $T_b$ is less than the minimum values listed in column 4, the bit impact will be out of two-bit range. Other bits backward will add considerable impact on DDJ. However, in reality it is unusual to push the bit rate too high, otherwise the transmission signal’s spectrum will significantly overflow the channel’s bandwidth, which will cause critical BER (bit error rate) loss.

### 6. DDJ minimization

In this section, we will explore to minimize the data dependent jitter by optimizing the interconnection line physical size under interconnection RC model.

Fig. 11a shows a 3-dimensional view of an interconnect line on chip. The line length is $h$, the width $w$ and the thickness $t$. The end-to-end resistance $R_{\text{line}}$ is given by the formula

$$ R_{\text{line}} = R_s \frac{h}{w}, \tag{36} $$

Table 2

<table>
<thead>
<tr>
<th>$C_L$ (fF)</th>
<th>$T_b$ (ps)</th>
<th>$t_{c1}$ (ps)</th>
<th>$t_{c2}$ (ps)</th>
<th>$J_C$ (ps) (Cadence)</th>
<th>$J_M$ (ps) (Matlab)</th>
<th>$J_T$ (ps)</th>
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<td>73.41</td>
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</table>

![Fig. 9. Predicted and measured DDJ.](image)

![Fig. 10. Ratio $R_{3/2}$ vs. $T_b$ when $C_L=500$ fF.](image)

![Table 2](image)

![Table 3](image)
where $R_s$ is the line sheet resistance and $(h/w)$ the number of squares with dimensions $(w^n)$. The unit length resistance $R$ thus equals

$$R = \frac{R_s}{w} \, \Omega/cm, \quad (37)$$

This equation shows that, when the sheet resistance $R_s$ is fixed, the unit length resistance $R$ is inversely proportional to $w$. Therefore, by changing the line width $w$, different line resistance will be obtained.

Fig. 11b illustrate the cross-sectional view of the electrical field associated with the interconnect line. As can be seen, there are two electrical field components, one is between the interconnect line’s bottom area and the substrate, and the other one is the fringing electrical field from the line edges and sides. The unit length capacitance $C$ accounting for both electrical field components equals [13]

$$C = \varepsilon_{ox} \left[ 1.15 \left( \frac{w}{T_{ox}} \right) + 2.8 \left( \frac{t}{T_{ox}} \right)^{0.222} \right]. \quad (38)$$

where $t$ is the interconnect line thickness, $\varepsilon_{ox}$ the permittivity of the insulating oxide and $T_{ox}$ the insulating oxide thickness between the line and the substrate.

In the first order interconnect transfer function, it is known that the transmission line time constant is

$$\tau = b_1 = R_s h C + R_s C_l + R h t^2 / 2 + R h C_l \quad (39)$$

Substituting the unit length resistance $R$ and capacitance $C$ formula into Eq. (39) with manipulations, it is obtained

$$\tau = R_s \left[ \frac{1.15 \varepsilon_{ox} h}{T_{ox}} w + \frac{1}{w} \left( R_s h C_l + R_s \varepsilon_{ox} 2.8 \left( \frac{t}{T_{ox}} \right)^{0.222} h^2 / 2 \right) \right]$$

$$+ R_s C_l + 1.15 \frac{R_s \varepsilon_{ox}}{T_{ox}} h t^2 / 2 + 2.8 R_s \varepsilon_{ox} \left( \frac{t}{T_{ox}} \right)^{0.222} h \quad (40)$$

In the above equation, the oxide thickness $T_{ox}$, the line thickness $t$, the oxide permittivity $\varepsilon_{ox}$ and the sheet resistance $R_s$ are constant when a semiconductor process is selected. $R_s$ and $C_l$ are source resistance and load capacitance and $h$ the distance between the transmitter node and the receiver node. These parameters are not varying in a specific case study. Consequently, the only variable is the line width $w$. In other words, the time constant $\tau$ is a function of $w$, $\tau = f(w)$. In Eq. (40), the 1st term is linearly proportional to $w$, while the second term is inversely proportional to $w$. Since $\tau$ is inversely proportional to the channel bandwidth BW, by optimizing the line width $w$, we can maximize BW so as to reduce the data dependent jitter.

To find out the optimum line width, we apply the derivative condition

$$\frac{d\tau}{dw} = \frac{1.15 \varepsilon_{ox} h}{T_{ox}} - \frac{1}{w^2} \left( R_s h C_l + R_s \varepsilon_{ox} 2.8 \left( \frac{t}{T_{ox}} \right)^{0.222} h^2 / 2 \right) = 0 \quad (41)$$

The solution is

$$w_m = \sqrt{\frac{T_{ox}(R_s C_l + 2.8 R_s \varepsilon_{ox} (t/T_{ox})^{0.222} h)}{1.15 R_s \varepsilon_{ox}}} \quad (42)$$

By setting the interconnect line width to $w_m$, the minimum $\tau$ can be obtained, which will maximize the line bandwidth to reduce the DDJ. To validate this finding, a series experiments are conducted. Their results are listed in Table 4.

In the experiments, the parameters of the transmission line are: $h=2 \, \mu m$, $R_s=18 \, \Omega$, $R_o=100 \, \Omega$, $C_l=1 \, pF$, $\varepsilon_{ox}=3.9 \times 8.854 \times 10^{-14} \, F/cm$, $T_{ox}=1 \, \mu m$ and $t=1 \, \mu m$. Plugging these values into Eq. (42), the optimum line width $w_m=2.23 \, \mu m$ is calculated.

In Table 4, the first column shows different line widths in the test. The next two correspond to the unit length resistance and capacitance. The fourth column is the resultant channel bandwidth BW. Column five are bit widths in the test, which are set to 700, 750 and 800 ps, respectively. The last two columns are for data dependent jitter, one is obtained from the Cadence simulations, and the other one is calculated using our proposed model. Both data sets demonstrate good agreements. From the table, it can be seen that, for each bit rate setting, when the interconnect line width $w$ equals $2.23 \, \mu m$, the minimum DDJ is always accomplished.

Fig. 12a is the curve depicting the line width vs. the channel bandwidth, while Fig. 12b shows the line width vs. DDJ. In both the figures, $T_b=750 \, ps$. It is clear to see, when $w=2.23 \, \mu m$, the channel bandwidth BW is maximized and the data dependent jitter is minimized. These are consistent with the experimental data in Table 4.

7. Conclusions and future work

In this paper, we propose a simple yet effective method to analyze the data dependent jitter in the interconnect line. A mathematical model has been developed to show DDJ is dominated by the interference within the two-bit range. Using patterns “001” and “101” as the experimental objects, this method extends the data patterns into periodic signals, and then utilizes the Fourier analysis to investigate the data dependent delay deviations. Furthermore, it explores to minimize the DDJ by
optimizing the interconnection line physical size. In the next step research, we will investigate the DDJ minimization in the high order transmission line models and study the crosstalk caused DDJ among different interconnect lines.

References


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<th>R (m2/um)</th>
<th>C (fF/um)</th>
<th>BW (GHz)</th>
<th>T_b (ps)</th>
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Fig. 12. (a) Bandwidth vs. line width and (b) DDJ vs. line width.

Table 4
DDJ under different line widths.