Pricing and mode choice based on nested logit model with trip-chain costs

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Commuters can complete their “home-work-home” trips by three options: subway-only mode, auto-only mode, and park-and-ride mode on a bottleneck-constrained corridor. The purpose of this paper is to enhance the insights into pricing mechanism for subway and parking and corresponding mode choice behavior on the corridor with elastic demand. A nested logit-based stochastic user equilibrium model is proposed to characterize the commuters’ modal choice. Dispersion parameters in the nested logit model reflect the risk or uncertainty of mode choice. It is found by sensitivity analysis that the impacts of subway fare and parking fee on the commute pattern are not always monotonous. Optimal strategies of subway fare and parking fee are discussed, respectively, under four market schemes by assuming that the subway and the parking lot at workplace are operated by either the government or a private owner. A numerical example is presented to illustrate how the pricing policies affect demand implementation, mode choice behavior and benefits of private owners and the whole transportation system.

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1. Introduction

As the result of urban revitalization, many metropolitan areas have witnessed explosive growth of traffic demand. Due to the limited road supply, various traffic demand management strategies have been proposed by researchers, such as congestion toll (de Palma and Lindsey, 2004; Shiftan et al., 2012), parking policy (Inci, 2014; Shiftan and Burd-Eden, 2001), auto restraining (Shiftan and Golani, 2005) and etc., and some of them have been implemented in practice. One of the adopted strategies is to encourage park-and-ride (P&R) travel, namely guiding auto commuters to park at highway bottleneck, and then take the high-capacity public transit to finish the rest of the trip (Lam et al., 2001; Wang et al., 2004). For example, during Beijing’s Twelfth Five-Year Plan period, 26 large-sized P&R facilities are expected to be built in the vicinity of new subway lines. Meanwhile, parking fees are expected to be differentiated according to parking regions, positions, time intervals and forms. On April 1, 2011, the differentiated parking fee policy started to be formally implemented in Beijing by keeping lower charges in P&R parking lots and raising parking fee at the city center. Since then, the volume of passengers on the public transits substantially increases, and the traffic congestion indexes of some major roads within the 5th ring fall significantly.

Parking pricing strategies are important tools for rebalancing the modal split between private car and transit systems in urban areas (D’Acierno et al., 2006). However, it should be recognized that the high price of parking fee raises the travel cost by car, which will certainly intensify the conflicts among government, parking lot owners and commuters. Therefore, the nature of differentiated pricing policy needs to be further discussed and the multimodal trip distribution under the influence of the policy should be reasonably forecasted (Huang et al., 1998; Gkritza et al., 2011; Inturri and Ignaccolo, 2011).

The classical traffic bottleneck model studies the commuting congestion on a highway with a single bottleneck between a residential area and a workplace (Vickrey, 1969). Tabuchi (1993) first studied such a bi-modal competitive system containing transit and highway modes. However, in many large cities, P&R services are provided for auto commuters to choose at highway bottleneck, for example, in Beijing and Hong Kong. Unlike the auto mode, the P&R mode mainly depends on its fare level, parking fee and service quality for attracting commuters. Usually, the parking fee is lower at P&R facilities than that at the city center. Also different from the transit mode, the P&R mode could make use of high velocity of car for attracting commuters, although the trains of railway or subway normally arrive on time no matter how crowded their carriages may be. Obviously, the analysis of this multi-modal system with a P&R option will be significantly different from that of a bi-modal system. Hence, the greatest need perhaps is the development of an
integrated transport pricing system that enables the coordination of parking fee at different positions and transit fares for optimal modal split.

Usually, a commuter complete his/her “home-work-home” trip chain by two single trips that “home to work” and “work to home”. In most academic research, a trip chain is conventionally defined as a sequence of trips that starts at home, involves one or more intermediate stops, and ends back at home (Ye et al., 2007). As Currie and Delbosc (2011) pointed out, the trip chain is an important aspect of travel and has the significant impact on changing travel patterns. McCook et al. (2005) found a 9% increase in chained work trips between 1995 and 2001.

Considering the “home-work-home” trip-chain costs, a nested logit-based stochastic user equilibrium (SUE) model is developed in this paper to investigate subway fare, parking fee and corresponding modal split under different market schemes with elastic demand1. The transportation system studied here is a subway/highway parallel corridor with a P&R option. Commuters first decide to travel by car or subway, and then those by car will choose either parking their cars and riding a train at highway bottleneck or continuing driving to the workplace. The four market schemes are further explored by assuming that the subway and the parking lot at the workplace are operated by either the government or a private owner. Specifically, in the market scheme I, both the subway and the parking lot at the workplace belong to the government; in the market scheme II, the parking lot at the workplace still belongs to the government, but the subway is operated by a private owner; in the market scheme III, contrary to scheme II, the subway belongs to the government whilst the parking lot at the workplace is operated by a private owner; in the market scheme IV, both the subway and the parking lot at the workplace are operated by different private owners. Without loss of generality, it is assumed that, the government hopes to maximize the net social benefit in the whole transportation system, and the private owners want to set the charges for profit maximization.

There are three major contributions of this paper. Firstly, a nested logit SUE model is formulated to depict commuters’ mode choice behavior with the consideration of the “home-work-home” trip-chain costs. Secondly, the sensitivity analysis is conducted to find the impacts of subway fare and parking fee on the travelers’ commute pattern. Thirdly, optimal strategies of subway fare and parking fee are discussed under four different market schemes, respectively. It is proved that the impacts of subway fare and parking fee on the travelers’ commute pattern are not always monotonous. Numerical examples are provided to give new managerial insights into the impacts of pricing policies on the demand implementation, modal choice and benefits of private owners and whole transportation system.

2. Related works

There are lots of studies concerning optimal transit fare, parking fee and their influences on the corresponding travelers’ commute patterns. However, most of them are limited to the public transport or private car mode only. For example, in order to maximize the social welfare, Pedersen (2003) studied the optimal fare policies in a public transport market with capacity constraints. Sharaby and Shiftan (2012) focused on evaluating the impact of fare integration on transit ridership and travel behavior, using the city of Haifa, Israel, as a case study. Ottosson et al. (2013) investigated the sensitivity of on-street parking demand using the automatic transaction data from parking pay stations in Seattle. For more detailed discussions, Interested readers are referred to Li and Hensher (2011) and Inci (in press).

Some studies on the combined trip distribution on a multi-modal corridor have been carried out. Tabuchi (1993) dealt with pricing and modal split in a competitive mass transit/highway system under the deterministic user equilibrium (DUE). Inspired by the seminal work of Tabuchi (1993), Huang et al. (1998) and Huang (2000) extended his study of modal choice by introducing crowding congestion on transit and by admitting the heterogeneity of commuters, respectively. Based on the assumption that the railway line is congestion-free, Wang et al. (2004) investigated the optimal location and pricing of a P&R facility in a linear monocentric city under the DUE. Supposing that P&R services are continuously distributed along a travel corridor, Liu et al. (2009) investigated the commuters’ travel choice behaviors in a competitive railway/highway system under the DUE. Inturri and Ignacccolo (2011) investigated the effects of alternative or joint schemes of road pricing and parking pricing on an idealized urban multimodal traffic corridor under the DUE. Based on the multinomial logit-based SUE, Huang (2002) further analyzed the modal split problem under various pricing regimes. Following the work of Huang (2002), Tian et al. (2005) made an important extension by adding a P&R option at highway bottleneck, yet they still assumed commuters’ multinomial logit-based mode choice behavior. According to Oppenheim (1995), the multinomial logit-based SUE model is more close to the reality than the DUE model. But due to the “independence of irrelevant alternatives” (IIA) assumption of the multinomial logit-based SUE model, i.e., all the alternatives should be irrelevant and independent; it is unsuitable for the case with a P&R option which is a combination of auto and subway. In contrast, a nested logit-based SUE model is more appropriate for mode split prediction when there are correlations among two or more transport modes.

Furthermore, traditional trip distribution models mainly focus on a single trip such as “home to work” or “work to home” trip. They may lead to inappropriate predictions of trip distribution or wrong evaluations of traffic demand management policies because of the separation of the “home-work-home” round journey (Strathman and Dueker, 1995). In fact, there exists a close connection between “to work” and “from work” travelers, especially for those by car. No matter whether his/her car is parked at the work area or a P&R station, the driver has to pick it up at the parking lot and then drives back home. Such the “home-work-home” round travel is one of the most simple and common trip chain (Ye et al., 2007).

3. Nested logit-based SUE model with trip-chain costs

The nested logit-based SUE model will be formulated in detail in this section, which is helpful for understanding further theoretical calculations and analysis.

3.1. Basic description

As shown in Fig. 1, Node H (a residential area or home) and node W (a workplace or central business district (CBD)) are connected by a simplified two-direction corridor with a parallel subway/highway system. Commuters leave their home to work at CBD, and after work, return to home along the corridor everyday. A P&R parking lot (P2) and a transfer station (TS) are located at the highway bottleneck (B). And also, a parking lot (P1) is located at

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1 A preliminary study has been conducted in Lu et al. (2011), in which the numbers of commuters using different travel modes, instead of subway fare or parking fee, are directly optimized. In general, it should be more natural to induce a reasonable modal split solution by adjusting those economic variables, which is what we expect to do in this paper.
the workplace. Denote the distances from H to B and from B to W as $l$ and $l$, respectively, and the highway capacities of two directions as $\bar{s}$ and $\bar{s}$, respectively. For simplification, directions of home to work and back home are represented by $\rightarrow$ and $\leftarrow$, respectively (Zhang et al., 2005).

In such a transportation system as in Fig. 1, commuters have three alternatives for their travel: (i) subway-only mode, i.e., it is both by subway that they go to work and return home; (ii) auto-only mode, i.e., they drive to work on the highway, park their cars at P1, and after work, continue to drive back home; (iii) P&R mode, i.e., the combined use of subway and highway. We assume that there are only trains from H to W, no additional trains between TS and W. Then a train comes from H to TS, additional passengers get on at TS, and the train continues to W. For commuters who choose to travel by P&R mode, they drive from home, park their cars at P2 and then go by train from TS to CBD. When their jobs are finished, they return to TS by subway and then drive back home. Obviously, there is a correlation between auto-only mode and P&R mode because commuters choosing these two modes both drive from home. Let $N$ be the total number of travellers between H and W, $N_1$ and $N_2$ be the numbers of commuters travel without and with a car, respectively, $N_3$ and $N_2$ be the numbers of P&R and auto commuters, respectively. We then have $N_1 + N_2 = N$, and $N_3 + N_1 = N$. For presentation purpose, major notations used throughout this paper are listed in Table 1.

### 3.2. Nested logit-based mode choice

The P&R mode is that the commuters first travel by car to the transfer station and then choose by subway to the workplace. According to our common sense, the P&R mode is a combination of auto-only and subway-only modes, so the auto-only and P&R modes are relevant options for commuters. In this case, a nested logit-based SUE model is more suitable to describe such the mode choice behavior than a multinomial logit model adopted in (Tian et al., 2005). The decision process is shown in Fig. 2. That is, commuters first decide to travel by car or subway, and then those by car will choose either P&R at highway bottleneck or continuing driving to the workplace.

We use the generalized utility function to characterize each mode as follows:

$$V_1 = U_0 - C_1 + \xi_1,$$  \hspace{1cm} (1a)

$$V_2 = U_0 - C_2 + \xi_2 + \xi_2.$$  \hspace{1cm} (1b)

where $U_0$ is a constant term representing the utility received through a working trip; it could be related to individual’s daily income; $C_1$, $C_2$, and $C_3$ are the generalized daily travel costs of

\begin{table}[h]
\centering
\caption{List of notations.}
\begin{tabular}{|l|l|}
\hline
Notation & Definition \\
\hline
W & Workplace or central business district (CBD) \\
H & Residential area or home \\
B & Highway bottleneck \\
P1 & Parking lot at the workplace \\
P2 & P&R parking lot at the highway bottleneck \\
TS & Transfer station \\
L & Distance between H and B \\
l & Distance between B and W \\
\omega & Dispersion parameters in nested logit model \\
\xi & Bottleneck capacity from home to work \\
\xi' & Bottleneck capacity back home \\
N & Total number of travellers between H and W \\
N_1 & Number of commuters travel without a car \\
N_2 & Number of commuters travel with a car \\
N_3 & Number of P&R commuters \\
N_22 & Number of auto commuters \\
C_1 & Generalized daily travel cost of choosing subway-only mode \\
C_21 & Generalized daily travel cost of choosing P&R mode \\
C_22 & Generalized daily travel costs of choosing auto-only mode \\
C_1' & Travel cost from “home to work” trip for subway-only mode \\
C_1'' & Travel cost from “work to home” trip for subway-only mode \\
C_21' & Travel cost from “home to work” trip for P&R mode \\
C_21'' & Travel cost from “work to home” trip for P&R mode \\
C_22' & Cost of “home to work” trip for P&R mode \\
C_22'' & Cost of “work to home” trip for P&R mode \\
\rho & Parking fee at the transfer station \\
\rho_1 & Parking fee at the workplace \\
\rho_2 & Parking fee of “home to work” trip for auto commuters \\
\rho_3 & Parking fee of “work to home” trip for auto commuters \\
\xi(n,x) & Discomfort cost experienced by a subway commuter for “home to work” trip \\
\xi'(n,x) & Discomfort cost experienced by a subway commuter for “work to home” trip \\
\gamma & Subway fare \\
\gamma_1 & Number of commuters selecting this mode \\
\gamma_2 & Traveling distance \\
\gamma_1 & Average velocity of subway \\
\gamma_2 & Average velocity of car \\
\xi_1 & Unit cost of travel time \\
\xi_2 & Unit cost of schedule delay arriving-early \\
\xi_3 & Unit cost of schedule delay leaving-early \\
\xi_4 & Unit cost of schedule delay arriving-late \\
\xi_5 & Unit cost of schedule delay leaving-late \\
\gamma_1 & Round-trip charge for subway-only mode service \\
\gamma_2 & Round-trip charge for P&R mode service \\
\gamma_3 & Round-trip charge for auto-only mode service \\
\hline
\end{tabular}
\end{table}
choosing subway-only, P&R and auto-only modes, respectively; $\xi_1$ and $\xi_2$ are random terms representing the perception errors in specifying the mode utilities. Hence, when commuters decide to travel by car, the conditional utilities of selecting P&R and auto-only modes are

$$V_{1R} = -C_{21} + \xi_1,$$  \hspace{1cm} (2a)

$$V_{2R} = -C_{22} + \xi_2.$$  \hspace{1cm} (2b)

Suppose the random terms $\xi_1$ and $\xi_2$ be identically and independently distributed (IID) Gumbel variables with mean zero and variances $\sigma_0$ and $\sigma_0$, respectively. And then, at equilibrium, the modal split at the aggregate demand level is governed by the two logit formula specified below

$$N_{2i} = N_2 \frac{\exp(-\omega C_{2i})}{\sum_{j=1,2} \exp(-\omega C_{2j})}, \quad i = 1, 2,$$  \hspace{1cm} (3)

$$N_i = N \frac{\exp(-\theta(C_i - U_0))}{\sum_{j=1,2} \exp(-\theta(C_j - U_0))}, \quad i = 1, 2,$$  \hspace{1cm} (4)

where $C_2$ is the expected daily travel cost by car; $\omega = \sigma/\sqrt{6\sigma_0}$ and $\theta = \sigma/\sqrt{6\sigma_0}$. Obviously, the larger variances $\sigma_0$ and $\sigma_0$, the smaller the values of parameters $\omega$ and $\theta$, and more randomly commuters make mode choice; from an individual point of view, this might bring the risk of larger travel cost. In addition, it should be noted that the modal split Eq. (4) is not affected by removing the constant $U_0$.

According to the nested logit model, $C_2$ can be calculated by

$$C_2 = -\frac{1}{\omega} \ln \left( \sum_{j=1,2} \exp(-\omega C_{2j}) \right).$$  \hspace{1cm} (5)

Similarly, the expected daily travel cost between H and W is given by the following expression

$$C = -\frac{1}{\theta} \ln \left( \sum_{j=1,2} \exp(-\theta C_{ij}) \right).$$  \hspace{1cm} (6)

In Eq. (3)–(6), $\theta \leq \omega$ must hold in accordance with the discrete choice theory (Opperhein, 1995).

Generally, the total travel demand $N$ is inversely proportional to the expected travel cost $C_i$, i.e., $N = D(C_i)$, satisfying $dN/dC < 0$. Let $B(N) = D^{-1}(N)$ denote the inverse demand function or the marginal travel cost with the property $dN/dC < 0$. A stochastic user equilibrium between individual travel costs and trip benefit at the expectation level can be formulated as follows:

$$(\ln N_1 + 1)/\omega + C_1 = (\ln N_2 + 1)/\omega + C_2 = (\ln N + 1)/\omega + B(N),$$  \hspace{1cm} (7a)

$$(\ln N_{21} + 1)/\omega + C_{21} = (\ln N_{22} + 1)/\omega + C_{22} = (\ln N + 1)/\omega + B(N),$$  \hspace{1cm} (7b)

with $N_{21} + N_{22} = N_2$ and $N_1 + N_2 = N$. Here, $B(N) = B(N)$ is the marginal travel cost benefit perceived by every commuter, and the other items constitute the marginal daily travel costs perceived by commuters of each mode. According the law of diminishing marginal returns, we have $1/(N(N)) + B \leq 0$ by differentiating the right side of the Eq. (7a) with respect to $N$.

The Eqs. (7a) and (7b) can be derived easily from Eqs. (3)–(6). Indeed, Eq. (4) yields that $\ln N_i = \ln N + (1 - C_i) + \ln(e^{-\omega C_i} + e^{-\sigma C_i}) = \ln N + (1 - C_i) + \sigma C_i$ by Eq. (6), whence $(\ln N_1)/\omega + C_1 = (\ln N)/\omega + C$, which is exactly Eq. (7a) (if the term is added to each side of the equation). The Eq. (7b) also can be obtained in a similar way.

It is easy to prove that the solution $(N_1, N_2, B(N))$ of Eq. (7) satisfies Eqs. (3) and (4) once all parameters appeared so far, the generalized daily travel costs of choosing subway-only, P&R and auto-only modes $C_1, C_2$ and $C_{22}$, and the marginal benefit function $B(N)$ are specified. Moreover, Eq. (7) will be reduced to the case of multinomial logit model when $\omega = 0$ (Tian et al., 2005). Specific components of generalized daily travel costs for the three modes will be discussed in the next section.

3.3. Specific components of generalized daily travel costs

3.3.1. Subway-only mode

The generalized daily travel cost for subway-only mode consists of travel costs resulting from “home to work” and “work to home” trips, $C_1$ and $C_2$, and the constant opportunity cost, $C_{10}$, i.e.,

$$C = C_1 + C_2 + C_{10}.$$  \hspace{1cm} (8)

The travel cost resulting from one-way trip, "home to work" or "work to home", depends on the one-way travel time, the discomfort cost generated by body congestion in carriage, and subway fare, as below

$$C_1 = a(L + b)/v_1 + \tilde{q}(N_1, L) + \tilde{q}(N_1 + N_{21}, L) + f(L + b),$$  \hspace{1cm} (9a)

$$C_2 = a(L + b)/v_1 + \tilde{q}(N_2, L) + \tilde{q}(N_1 + N_{22}, L) + f(L + b),$$  \hspace{1cm} (9b)

where $a$ is the unit cost of travel time, $v_1$ is the average velocity of subway, $\tilde{q}(n, x)$ and $\tilde{q}(n, x)$ represent the discomfort costs experienced by a subway commuter for “home to work” and “work to home” trips, respectively, and $f(x)$ is the subway fare. Generally, $\tilde{q}(n, x)$ and $\tilde{q}(n, x)$ are both positive and increasing functions of the number of commuters selecting this mode, $n$, and the traveling distance, $x$, and $f(x)$ is increasing with the travelling distance.

It should be noted that $\tilde{q}(n, x)$ and $\tilde{q}(n, x)$ may also express the comfort of the travelling by subway. In this case, $\tilde{q}(n, x)$ and $\tilde{q}(n, x)$ attain negative values. However, in some countries, such as Japan, China and Singapore, the carriages of subway get very crowded during the rush hour periods. Functions $\tilde{q}(n, x)$ and $\tilde{q}(n, x)$ more often represent the discomfort costs in the transportation research (Huang, 2002; Liu et al., 2009; Li and Hensher, 2011).

3.3.2. Auto-only mode

If the number of auto users exceeds the bottleneck capacity, a queue develops. In order to avoid or reduce the waiting time in queue, some commuters will leave home earlier or later in the morning which generates schedule delay costs. Let $\bar{p}$ be the unit cost of schedule delay arriving-early, $\bar{p}$ be the unit cost of schedule delay arriving-late. In accordance with bottleneck theory (Vickrey, 1969), at departure-time choice equilibrium $N_{22}$ auto commuters have the same travel cost for the “home to work” trip (Arnott et al., 1990; Huang, 2000), i.e.,

$$\bar{C}_{22} = a(L + b)/v_2 + \tilde{d} N_{22}/\tau + F_2.$$  \hspace{1cm} (10)

where $v_2$ is the average velocity of car, $\tilde{d} = \bar{p} / \bar{p}$, $F_2$ is the fixed cost of driving. Although the cost of driving depends on the distance, for simplicity, we just assume the cost of driving to be a constant in this paper. This will not affect the properties of the model. The second term in Eq. (10) is the comprehensive cost of queuing time and schedule delay. Similarly, at departure-time choice equilibrium, the travel cost for the “work to home” trip is:
is the parking fee at the transfer station. We
and
, if
. Then

is the unit cost of schedule delay
. On the basis of
,
= + ,

\[ \beta \text{ is the unit cost of schedule delay} \]

is the unit cost of schedule delay leaving-early and \( \gamma \) be the unit cost of schedule delay leaving-late. Then the generalized daily travel cost of an auto mode commuter is:

\[ C_{22} = C_{22} + C_{22} + P_b, \]

(12)

where \( P_b \) is the parking fee at the workplace. In accordance with the technical conditions for stability (Oppenheim, 1995), we assume that \( \gamma \geq a \geq \beta \) and \( \beta \geq a \geq \gamma \).

3.3.3. P&R mode

The generalized daily travel cost for P&R mode consists of four parts, i.e.,

\[ C_{21} = C_{21} + C_{21} + C_{20} + P_b, \]

(13)

where \( C_{20} \) is the constant opportunity cost, \( C_{21} \) and \( C_{21} \) are travel costs resulting from “home to work” and “work to home” trips, respectively, and \( P_b \) is the parking fee at the transfer station. We assume that the subway fare and parking fee at the transfer station are collected as a whole. This measure can effectively rule out the parking behavior without transferring and encourage the use of P&R mode. So the round-trip charge \( T_{21} \) of the P&R mode is calculated. In order to compare the charges of the above-mentioned three modes, subway fare and parking fee of round trips \( T_1 \) and \( T_2 \) are also used.

Let \( T_1 = 2f(l + l) \), \( T_2 = 2f(l) + P_b \) and \( T_{22} = P_b \) represent the round-trip charges for subway-only, P&R and auto-only mode services, respectively. It is easy to see from Eq. (7), the modal split depends on each mode’s daily travel cost, \( C_{21} \) and \( C_{22} \), and then the charging level, \( T_1 \), \( T_2 \) and \( T_{22} \). That is to say, since the relation between \( N_1, N_{21} \) and \( N_{22} \) and \( C_{21}, C_{22} \) is defined by Eq. (7), the relation between \( N_1, N_{21} \) and \( N_{22} \) and \( C_{21}, C_{22} \) and \( C_{23} \) is also determined. Proposition 1 gives the properties for changes of the numbers of commuters by three modes, \( N_1, N_{21} \) and \( N_{22} \) with respect to \( T_1, T_2 \) and \( T_{22} \).

Proposition 1. Let \( g(N) = (\ln N + 1)/(\theta + B(N)) \) be the marginal trip benefit perceived by a commuter between H and W, and \( \eta = 1/(\theta - 1) \). If \( g^* = B(N) + 1/(\theta - 1) \) \( \leq 0 \), then we have

(i) \( dN_1/dT_1 \leq 0, dN_{21}/dT_{21} \leq 0, dN_{22}/dT_{22} \leq 0; \)

(ii) \( dN_{21}/dT_1 = dN_{21}/dT_{21} \geq 0, dN_{22}/dT_{21} = dN_{22}/dT_{22} \geq 0; \)

(iii) \( dN_{22}/dT_1 = dN_{22}/dT_{22} \geq 0, \quad \text{if } CE_1(N_1, N_{21}, N_{22}) \geq 0; \)

\[ dN_{22}/dT_1 = dN_{22}/dT_{22} < 0, \quad \text{if } CE_1(N_1, N_{21}, N_{22}) < 0, \]

where \( CE_1(N_1, N_{21}, N_{22}) = -1/(N_{21} + 12(N_{21} + 12N_{22})), \)

\[ CE_2(N_1, N_{21}, N_{22}) = -1/(N_{21} + 12(N_{21} + 12N_{22})) \]

(iv) \( dN_1/dT_1 \leq 0, dN_{21}/dT_{21} \leq 0, \quad \text{but the sign of } dN_{22}/dT_{22} \text{ is determined by modal split and parameter settings. Specifically, if } N_1, N_{21} \text{ and } N_{22} \text{ satisfy } CE_2(N_1, N_{21}, N_{22}) > 0, \text{ then } dN_{22}/dT_{22} > 0; \text{ otherwise, if } CE_2(N_1, N_{21}, N_{22}) < 0, \text{ then } dN_{22}/dT_{22} \leq 0 \).

The proof of Proposition 1 refers to Appendix A. This Proposition provides some managerial insights into the effects of charge level adjustments. Firstly, if the round-trip charge for subway mode, consisting of subway fare for “home to work” and “work to home” trips, is raised, the total travel demand and the number of passengers only by subway certainly decrease, the number of commuters choosing P&R for travel certainly increases, but the change in the number of auto commuters is uncertain according to the conditional expressions in (iii). Furthermore, higher parking fee at the workplace reduces the total travel demand and auto commuters, increases the P&R commuting, but brings the uncertain change in the number of commuters only by subway in terms of the conditions in (iii). Then if the round-trip charge (including the parking fee at transfer station) for P&R mode rises, the number of P&R commuters naturally decrease, commuters by subway alone and auto alone both increase, but the changes in the total travel demand depend on the given condition in (iv). Hence, the traffic management authorities should be careful to use economic control strategies such as fare and parking fee to regulate the flow distribution and modal split in the urban network. Once the control is unsuitable, the outcome might be opposite.

Secondly, it should be noted that the conditional expressions \( CE_1(N_1, N_{21}, N_{22}) \) and \( CE_2(N_1, N_{21}, N_{22}) \) both depend on the parameter combination \((\omega, \theta)\) of the nested logit model. The conclusion is very interesting. When \( \omega = \theta, 1/\eta = 0 \), the model degenerates into the general multinomial logit case, and then we get \( CE_1(N_1, N_{21}, N_{22}) \geq 0 \) and \( CE_2(N_1, N_{21}, N_{22}) \). On the basis of Proposition 1, derivative expressions \( dN_{22}/dT_1 = dN_{22}/dT_2 \geq 0 \) and \( dN_{22}/dT_1 \) always hold. Whereas in the case \( \omega > \theta \), the signs of \( dN_{22}/dT_1 \) and \( dN_{22}/dT_2 \) are not deterministic, depending on the modal split and parameter settings in the conditional expressions. Since the nested logit model is more close to reality than the multinomial logit one adopted in Tian et al. (2005), the corresponding results in Proposition 1 can be used as a reference for designing more reasonable charge policy.

Finally, it is also found that, for any two modes, the cross-charge influence is symmetric. The property is helpful for solving the four optimal pricing strategies in the next section.

Generally, subway and workplace parking lot are operated by different owners. As a result, their charge levels may be optimized for different objectives. In the next section, different optimal strategies of subway charge and parking fee under four market schemes are discussed in detail.

5. Strategies of subway fare and parking fee under four market schemes

First recall the concept of normal form game and the Nash equilibrium. Let \( N = \{1, \ldots, n\} \) be the set of the players, and \( X_i \) is the strategy set of player \( i \) for \( \forall i \in N \). Then \( \phi_i: X_1 \times \ldots \times X_n \rightarrow R \) is the payoff function of player \( i \). A strategy combination \((x_1, \ldots, x_n)\) is a Nash equilibrium if no unilateral deviation in strategy by any single player is profitable for that player, that is
∀ i, x_i ∈ X, x_i ≠ x_{i'}; \phi_i(x_i, x_{i'}) ≥ \phi_i(x_{i'}, x_{i}).

The purpose of this section is to find a Nash equilibrium in the respective game, where the strategy set is always \( (t_1, t_2, t_2) \). When \( n = 1 \), it is a one-player game as in the Section 5.1. The task reduces to find a global maximum of a function. While we are looking for Nash equilibria in two-player games in the remaining three cases, Sections 5.2–5.4.

5.1. Net social benefit maximization

First, we derive the optimal subway fare and parking fee through maximizing the expected net social benefit of the system and solve the corresponding modal split. In this scenario, all facilities are operated and managed by the government or a public operator. The optimization problem is as follows:

\[
\max_{t_1, t_2, t_2} \text{ENB}(N_1, N_2, N_2) \]

\[
= \int_0^N B(w)dw + \text{ESB}(N_1, N_2, N_2) - \text{TTC}(N_1, N_2, N_2) - \text{TOC}(N_1, N_2, N_2),
\]

s. t. (7) and \( N_1 + N_2 + N_2 = N \); \( N_1 ≥ 0 \); \( N_2 ≥ 0 \); \( N_2 ≥ 0 \).

In Eq. (15), the integration term is the deterministic trip benefit of all commuters from travelling; the second term represents the difference between the expected values of random parts of trip benefit and cost (Huang, 2002; Tian et al., 2005); the third term is the total daily travel cost; the last term is the total variable operating cost of the system. Formulations are as below:

\[
\text{ESB}(N_1, N_2, N_2) = N \ln N/\theta - N_1 \ln N_1/\theta - N_2 \ln N_2/\theta
\]

\[
- N_{22} \ln N_{22}/\theta - N_2 \ln N_{22}/\theta.
\]

\[
\text{TTC}(N_1, N_2, N_2) = N_1(C_1 - n_1) + N_2(C_2 - n_2) + N_{22}(C_{22} - n_{22}),
\]

\[
\text{TOC}(N_1, N_2, N_2) = (N_1 + N_2)(C_1 + N_2C_1 + N_2C_2).
\]

where \( c_1, c_2, \) and \( c_{22} \) are the variable operating costs of subway, P&R parking lot and workplace parking lot, respectively. Note that the subway fare and parking fee are eliminated from the total daily travel cost, because it is usually supposed that the government incomes from charges will return to commuters via another form for the net social benefit maximization.

Since the objective function Eq. (15) is to maximize the net social benefit, the same results will be obtained by optimizing either \( N_1, N_2, \) or \( r_1, r_2 \) and \( r_2 \). For simplicity, we select to derive the first-order optimality conditions for objective function Eq. (15) with respect to \( N_1, N_2, \) and \( N_{22} \), respectively, and obtain the optimal pricing strategies as Theorem 1 states.

**Theorem 1.** In order to maximize the expected net social benefit, the round trip charges for subway-only, P&R and auto-only commuters, respectively, should be set as follows:

\[
r_1 = N_1 \left( \frac{\partial q}{\partial N_1} + \frac{\partial q}{\partial N_1} \right)
\]

\[
+ (N_1 + N_2) \left( \frac{\partial q}{\partial (N_1 + N_2)} + \frac{\partial q}{\partial (N_1 + N_2)} \right) + c_1
\]

\[
r_2 = N_2 \left( \frac{\partial q}{\partial N_2} + \frac{\partial q}{\partial N_2} \right) + c_1 + c_2.
\]

\[
\text{Theorem 1.} \text{ indicates that the optimal pricing strategies for maximizing the net social benefit are actually at work under the marginal congestion pricing principle. That is, } r_1 \text{ is the sum of the round-trip body congestion externality cost caused by a marginal subway-only passenger and by a marginal P&R commuter, and the variable operating cost of subway; } r_2 \text{ is the sum of the round-trip body congestion externality cost brought by a marginal P&R commuter and the variable operating cost of the subway and the P&R parking lot; } r_2 \text{ is the sum of the round-trip queue externality cost generated by an additional auto-only commuter and the variable operating cost of the workplace parking lot. If we just consider the one-way trip at the morning or evening peak, all the charges for the three modes will be underestimated. So it is necessary to connect the ‘to’ and ‘from’ travellers’ behavior. Otherwise, the implementation effectiveness of the social-benefit-maximizing pricing policy will be reduced.}

5.2. Subway profit maximization with the public parking lot at the workplace

Different from the case in Section 5.1, the subway and the parking lot at the workplace are assumed to be operated by a private company and the government, respectively. The government still hopes to maximize the expected net social benefit while the subway company wants to set the charges for profit maximization. The problem is formulated as follows:

\[
\max_{t_2} \text{ENB}(N_0, N_2, N_2)
\]

\[
= \int_0^N B(w)dw + \text{ESB}(N_0, N_2, N_2) - \text{TTC}(N_0, N_2, N_2) - \text{TOC}(N_0, N_2, N_2),
\]

s. t. (7) and \( N_0 + N_2 + N_2 = N \); \( N_0 ≥ 0 \); \( N_2 ≥ 0 \); \( N_2 ≥ 0 \).

The first term in objective function Eq. (18) is the total income of subway company, including the fares collected from two stations, where the parking fee at the transfer station is covered in the subway fare. The second term is the sum of variable operating costs of the subway and the P&R parking lot.

\[
\text{Theorem 1.} \text{ indicates that the optimal pricing strategies for maximizing the net social benefit are actually at work under the marginal congestion pricing principle. That is, } r_1 \text{ is the sum of the round-trip body congestion externality cost caused by a marginal subway-only passenger and by a marginal P&R commuter, and the variable operating cost of subway; } r_2 \text{ is the sum of the round-trip body congestion externality cost brought by a marginal P&R commuter and the variable operating cost of the subway and the P&R parking lot; } r_2 \text{ is the sum of the round-trip queue externality cost generated by an additional auto-only commuter and the variable operating cost of the workplace parking lot. If we just consider the one-way trip at the morning or evening peak, all the charges for the three modes will be underestimated. So it is necessary to connect the ‘to’ and ‘from’ travellers’ behavior. Otherwise, the implementation effectiveness of the social-benefit-maximizing pricing policy will be reduced.}

5.4. Discussions about the marginal congestion pricing principle with the stochastic user equilibrium assignment can be found in Yang (1999).
and

\[
\frac{\partial N_1}{\partial \tau_1} \left( c_1 + N_1 + N_{21} \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) \right) - \tau_1
\]

\[
+ \frac{\partial N_{22}}{\partial \tau_2} \left( c_2 + N_{22} \left( \delta \frac{\partial s}{\partial s} + \delta \frac{\partial s}{\partial s} \right) \right) = 0,
\] (19a)

\[
(\tau_1 - c_1) - (\tau_2 - c_1 - c_2) \frac{\partial N_{21}}{\partial \tau_2} = -N_p,
\] (19b)

\[
(\tau_1 - c_1) + (\tau_2 - c_1 - c_2) \frac{\partial N_{21}}{\partial \tau_2} = -N_i
\] (19c)

Solving the Eq. (19), we obtain Theorem 2 as below.

**Theorem 2.** Under the market scheme II (The subway is operated by a private owner and the parking lot at the working place belongs to the government. The objective is to maximize the profit of the owner while the government maximizes the net social benefit.), the optimal round-trip charges for subway-only, P&R and auto-only commuters are, respectively:

\[
\tau_1 = N_{21} \frac{\partial N_{21}}{\partial \tau_1} - N_1 \frac{\partial N_1}{\partial \tau_1} + c_1,
\] (20a)

\[
\tau_{21} = N_1 \frac{\partial N_1}{\partial \tau_1} - N_{21} \frac{\partial N_{21}}{\partial \tau_1} + c_1 + c_2l,
\] (20b)

\[
\tau_{22} = N_{22} \left( \delta \frac{\partial s}{\partial s} + \delta \frac{\partial s}{\partial s} \right) + c_2 - \frac{\Delta \tau_1 \frac{\partial N_1}{\partial \tau_1} + \Delta \tau_{21} \frac{\partial N_{21}}{\partial \tau_{21}}}{\partial \tau_{22}}.
\] (20c)

where

\[
\Delta \tau_{21} = \tau_{21} - (N_1 + N_{21}) \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) - c_1 - c_{21},
\]

\[
\Delta \tau_1 = \tau_1 - N_1 \left( \frac{\partial q (N_1, l)}{\partial \tau_1} + \frac{\partial q (N_1, l)}{\partial \tau_1} \right) - (N_1 + N_{21}) \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) - c_1.
\]

It should be pointed out that the optimal pricing strategies cannot be compared with that in Tian et al. (2005) directly, although the private subway and the public parking lot are both assumed. This is because the parking fee at workplace follows the marginal congestion pricing rule in their study, i.e.,

\[
\tau_{22} = c_{22} + N_{22} \left( \delta \frac{\partial s}{\partial s} + \delta \frac{\partial s}{\partial s} \right), \tau_1 \text{ and } \tau_{21} \text{ are obtained from the first-order conditions by differentiating the profit of subway company with respect to } N_1 \text{ and } N_{21}, \text{ respectively. However, it is more realistic in practice that the subway company influences } N_1 \text{ and } N_{21} \text{ by adjusting } \tau_1 \text{ and } \tau_{21}, \text{ but not by controlling } N_1 \text{ and } N_{21}. \text{ In this Section, the market scheme II really reflects the natural idea by optimizing the charges. Optimal solutions obtained by using the strategy of Tian et al. (2005) could be found in Lu et al. (2011).}

5.3. Parking lot owner’s profit maximization with the public subway

On contrary to the assumption in Section 5.2, the subway is operated by the government and the parking lot belongs to a private owner. Then the government determine \( \tau_1 \) and \( \tau_{21} \) by maximizing the expected net social benefit and the parking lot owner aims to maximize the profit from charging the auto-commuters. The problem is formulated as follows:

\[
\text{maxENB}(N_i, N_{21}, N_{22})
\]

\[
\text{subject to (7) and } N_i + N_{21} + N_{22} = N, N_{21} + N_{22} = N_i, N_i \geq 0, N_{21} \geq 0, N_{22} \geq 0.
\]

The first term in objective function Eq. (22) is the total income of the workplace parking lot, and the second term is the total variable operating cost for its operation. Let \( \delta ENB(N_i, N_{21}, N_{22})/\partial \tau_1 = 0 \) and \( \delta ENB(N_i, N_{21}, N_{22})/\partial \tau_{21} = 0 \), then we obtain

\[
\frac{\partial N_1}{\partial \tau_1} \left( c_1 + N_1 \left( \frac{\partial q (N_1, l)}{\partial \tau_1} + \frac{\partial q (N_1, l)}{\partial \tau_1} \right) \right) + (N_1 + N_{21}) \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) - \tau_1
\]

\[
+ \frac{\partial N_{21}}{\partial \tau_1} \left( c_1 + c_2 + (N_1 + N_{21}) \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) - \tau_1 \right)
\]

\[
+ \frac{\partial N_{21}}{\partial \tau_2} \left( c_2 + N_{22} \left( \delta \frac{\partial s}{\partial s} + \delta \frac{\partial s}{\partial s} \right) \right) - \tau_{22} = 0,
\] (23a)

\[
\frac{\partial N_1}{\partial \tau_1} \left( c_1 + N_1 \left( \frac{\partial q (N_1, l)}{\partial \tau_1} + \frac{\partial q (N_1, l)}{\partial \tau_1} \right) \right) + (N_1 + N_{21}) \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) - \tau_1
\]

\[
+ \frac{\partial N_{21}}{\partial \tau_2} \left( c_1 + c_2 + (N_1 + N_{21}) \left( \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial q (N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} \right) - \tau_1 \right)
\] (23b)
+ \frac{\partial N_{22}}{\partial \tau_{21}} \left( c_{22} + N_{22} \left( \frac{\partial f}{\partial x} + \frac{\partial g}{\partial s} \right) \right) - \tau_{22} = 0, \tag{23b}

N_{22} + (\tau_{22} - c_{22}) \frac{\partial N_{22}}{\partial \tau_{22}} = 0. \tag{23c}

Solving the Eq. (23), we have Theorem 3 below.

**Theorem 3.** Under the market scheme III (The subway belongs to the government and the parking lot at the working place is operated by a private owner. The objective is to maximize the profit of the owner while the government maximizes the net social benefit), the optimal round-trip charges for subway-only, P&R and auto-only commuters are, respectively

\[ r_1 = \frac{\partial f}{\partial N_1}, \quad (N_1 + N_{21} - \frac{\partial f}{\partial N_1}) \left( \frac{\partial f}{\partial N_1} + \frac{\partial g}{\partial s} \right) + c_1 \]

\[ r_{21} = \frac{\partial f}{\partial N_1}, \quad (N_1 + N_{21} - \frac{\partial f}{\partial N_1}) \left( \frac{\partial f}{\partial N_1} + \frac{\partial g}{\partial s} \right) + c_{21} \]

\[ r_{22} = c_{22} - N_{22} \left( \frac{\partial g}{\partial s} \right)^{-1}, \]

where \( \Delta \tau_{22} = \tau_{22} - N_{22} \left( \frac{\partial f}{\partial N_1} \right) \).

5.4. Duopoly price competition

In reality, the subway and the workplace parking lot are usually operated by different private companies, such as in Hong Kong. Both of them expect to maximize their own profits via a duopolistic paradigm (Lu et al., 2010). The game problem is formulated as follows:

\[ \text{maxSP}(N_1, N_{21}, N_{22}) \]

\[ = \{N_{11} + N_{211221} \}, \quad \{N_1 + N_{21} \} \}

\[ \text{maxPP}(N_1, N_{21}, N_{22}) = N_{22} r_{22} - N_{22} c_{22}, \]

s.t. (7) and \( N_{1} + N_{21} + N_{22} = N, \quad N_{21} + N_{22} = N, \quad N_1 \geq 0, \quad N_{21} \geq 0, \quad N_{22} \geq 0. \)

Let \( \partial SP(N_1, N_{21}, N_{22})/\partial \tau_{21} = 0, \quad \partial SP(N_1, N_{21}, N_{22})/\partial \tau_{22} = 0 \) and \( \partial PP(N_1, N_{21}, N_{22})/\partial \tau_{22} = 0, \) then

\[ (r_1 - c_1) \frac{\partial N_1}{\partial \tau_{21}} + (r_{21} - c_{21}) \frac{\partial N_{21}}{\partial \tau_{21}} - N_1, \tag{27b} \]

\[ (r_1 - c_1) \frac{\partial N_1}{\partial \tau_{21}} + (r_{21} - c_{21}) \frac{\partial N_{21}}{\partial \tau_{21}} - N_{21}, \tag{27c} \]

Solving the Eq. (27), we have Theorem 4 below.

**Theorem 4.** Under the market scheme IV (The subway and the parking lot at the working place are operated by two private owners, respectively. The objective is to maximize their profits respectively), the optimal round-trip charges for subway-only, P&R and auto-only commuters are, respectively:

\[ r_1 = \frac{N_{2110} N_{2111} \tau_{21} - N_{10} \partial N_{2110} / \partial \tau_{21}}{- \frac{\partial N_{2110} / \partial \tau_{21}}{2} + \frac{\partial N_{2110} / \partial \tau_{21}}{2}}, + c_1, \tag{28a} \]

\[ r_{21} = \frac{N_{10} \partial N_{2110} / \partial \tau_{21} - N_{10} \partial N_{2110} / \partial \tau_{21}}{- \frac{\partial N_{2110} / \partial \tau_{21}}{2} + \frac{\partial N_{2110} / \partial \tau_{21}}{2}}, + c_1 + c_{21}, \tag{28b} \]

\[ r_{22} = c_{22} - N_{22} \left( \frac{\partial g}{\partial s} \right)^{-1}. \tag{28c} \]

It can be seen from Theorem 4, the expressions of \( r_1 \) and \( r_{21} \) generated from the duopoly game are identical to form that from the scheme II in Section 5.2. And also, \( r_{22} \) here has the same formulation with that from the scheme III in Section 5.3. This phenomenon is not surprising because the subway and the parking lot at the workplace are, respectively, supposed to be private in the cases of schemes II and III. Although the price strategies are identical in form, with specified model parameters, there exist significant distinctions on the values of charging levels, \( r_1, r_{21} \) and \( r_{22} \), under different market schemes. This will be illustrated through an example in the next section.

**Remark 1.** Formally, the cases in Sections 5.2 and 5.3 are also duopolistic games. Two players can only adjust their pricing policies to affect the number of commuters selecting each mode, further to affect their utility functions. We can see that the variables \( N_1, N_{21} \) and \( N_{22} \) are the implicit functions of variables \( r_1, r_{21} \) and \( r_{22} \), which is useful for solving optimal pricing strategies.

6. Numerical example

Up to now, we have formulated four pricing models based on the nested logit-based modal split solutions. Note that the groups of nonlinear equations formulated for each model must be numerically solved. It is then difficult to check the properties of solutions analytically. In this section, we present a numerical example to demonstrate the demand implementation, the mode choice behavior and the benefits of private owners and the whole transportation system generated by four market schemes.

The parameters of the numerical example are: \( \alpha = 20(\$/hour), \quad \beta = (15,30)($/hour), \quad \gamma = (25,10)($/hour), \quad \mu = 3(000,3200)(vehicle/hour), \quad v_1 = 20(km/hour), \quad v_2 = 30(km/hour), \quad l = 30 \text{ km}, \quad l = 5 \text{ km}, \quad \left( \begin{array}{c} T_1, \ T_2 \end{array} \right) = (0.2, 0.15) \text{ (hour)}, \quad F_2 = 10(\$), \quad c_{10} = 0.3(\$/person), \quad c_{20} = 0.12(\$/person), \quad c_{22} = 0.25(\$/person), \quad \theta = 0.1, \quad \omega = 2, \quad c_{10} = 20(\$), \quad c_{20} = 10(\$). \) We adopt the following inverse demand function: \( B(N) = -G(N/N_{max}), \) where \( N_{max} = 10,000. \) This
function implies that the demand is less sensitive to the marginal trip benefit with a larger value of $G$ and thus the final realized demand will be higher. The functions describing the body congestion discomfort cost in carriage for morning and evening trips take the form $\bar{q}(n, x) = \bar{c}(x/v_1) (\bar{D}n^2 + \bar{E}n)$ ($\$) and $\bar{q}(n, x) = \bar{c}(x/v_1) (\bar{D}n^2 + \bar{E}n)$ ($\$), respectively, where $\bar{c} = 1.2 \times 10^{-5}$/$\text{person}$, $\bar{v} = 10^{-5}$ ($\text{person}$), $\bar{D} = 0.05$, $\bar{E} = 0.25$, $\bar{D} = 0.04$, $\bar{E} = 0.2$ (Huang, 2002; Tian et al., 2005). In practical applications, all the function form and parameters should be calibrated.

Fig. 3 shows the change in the charges of subway-only, P&R and auto-only modes under four market schemes with the increase of parameter $G$. It can be seen that, no matter what travel mode, its charge is the least at scheme I (the case of net social benefit maximization). Furthermore, the charging levels in the private subway situation (schemes II and IV) are always higher than the case when the subway are operated by the government (schemes III and I) for any of three travel modes. However, although the workplace parking lot is owned by the government in scheme II (subway company for profit maximization), the subway fare and parking fee generated by this scheme are surprisingly not lower than that by scheme IV (duopoly price game).

Cooing with the order of the charges in Fig. 3, the total travel demand generated by each scheme is decreasing by the order of I, III, IV and II as shown in Fig. 4. This is because the mode split relies on the daily travel costs and then charging levels of three modes. In addition, scheme I and III also produce the most and second-most number of subway users (the sum of commuters by subway-only and P&R modes). But subway users at scheme II are much more than that at scheme IV, which is different from the sequence of total travel demand. This can be explained from Fig. 3. In Fig. 3, compared with scheme IV, fares charged by the subway just get a relative small increase while the parking fee at the workplace rises sharply under scheme II. As a result, some auto users will change to park cars at the transfer station or travel by subway-only, and then the number of commuters using subway goes up.

Since subway users consist of commuters by subway-only and P&R modes, next we investigate the changes in the proportion of commuters by each mode and the proportion of commuters using subway. As shown in Fig. 5(a), under scheme III the proportion of commuters only by subway reaches the top, and the others decrease in the order of scheme I, II and IV, yet the proportion of P&R commuters is sorted as scheme II, III, I and IV in the descending order. Nevertheless, scheme III generates the most subway commuters (see Fig. 5(b)). The results suggest that scheme III (public subway and private parking lot) can effectively encourage commuters to use public transport.

Fig. 6 shows the change in the operating profits of subway and parking lot under different market schemes. Similar to the order of charges in Fig. 3, scheme II is the most profitable, and scheme IV produces the second. Profit under scheme I is the least and scheme III performs between IV and I. The profit generated in the private subway situation (schemes II and IV) is much more than that of scheme I (net social benefit maximization), but the relative loss ratio of expected net social benefits of the system also becomes larger, as shown in Fig. 7. It can be also found that, the relative loss ratio in scheme III is nearly zero. This suggests that scheme III with the public subway and private parking lot can approximately generate the maximum expected net benefit of the system while encouraging the transit travel.

Since scheme III has relative advantages compared with the other schemes, the following discussion is based on its pricing strategy. Figs. 8 and 9 show the effects of the risk (or uncertainty) perception parameter, $\omega$, on the daily travel costs of three modes, the expected net social benefit of the system and the profits of subway and parking lot owners, respectively. It can be seen that, as the $\omega$-value increases, the daily travel cost of each mode decreases.
due to a lower travel risk (or uncertainty), but the expected net social benefit of the system, as well as the profits of subway and workplace parking lot, decreases. This is a so-called “paradox” phenomenon: Individual rationality is difficult to achieve the effect of collective rationality due to the externality. Furthermore, recall that the model will degenerate into a multinomial logit one when $\omega = \theta = 0.1$, the results in Figs. 8 and 9 also indicate that the daily travel cost, the expected net benefit of the system and duopoly's profit will be overestimated if the multinomial logit model is used for modeling the multimodal corridor studied in this paper.

7. Future developments

It should be mentioned that, the Proposition, theorems and numerical results obtained in the above sections critically rely on some simplified but very important assumptions about travelers' commuting and network structure. With the more general framework, there might be many open and potentially valuable extensions for further studies.

First of all, auto-only drivers may have an opportunity to be free from queuing if paying congestion tolls at bottleneck besides making the P&R choice. Considering this, the auto-only commuters could be classified into two groups: ones with tolling and ones without tolling. How to decide the optimal congestion toll level and to redistribute the toll revenue from congestion is a valuable problem (de Palma and Lindsey, 2004; Mirabel and Reymond, 2003004).

Fig. 5. Proportions of commuters using subway and P&R under different market schemes.

Fig. 6. Operating profits of subway and parking lot owners under different market schemes.

Fig. 7. Relative loss ratios of expected net benefits of the system under different market schemes.
8. Conclusions

This paper proposes a nested logit-based SUE model to describe the travel behavior among three modes: subway-only, P&R and auto-only under the background of "home-work-home" trip chain. Through the sensitivity analysis, we find that the effects of the change in subway fare and parking fee on the modal split are not always monotonous, which are different from the results by a general multinomial logit model. Furthermore, four market pricing models, i.e., expected net social benefit maximization, subway's profit maximization with public parking lot, parking lot's profit maximization with public subway and duopoly price competition, are discussed. Since the nested logit model used in this paper is more close to reality than the multinomial logit one, the corresponding results in Section 5 can be used as a reference for designing more reasonable charge policy in social reality. Although the calculating method is not complex, the emphasis is to study the optimal pricing policy of each player under different mechanisms.

Interesting numerical results are also obtained in this paper. It is shown that as long as the subway is operated by the government, travel by public transport can be effectively encouraged and the net social benefit of the system can be almost maximized, whatever the workplace parking lot belongs to the government or a private owner. Once the subway is private, the transit attraction and net social benefit will decrease sharply although the profits of the subway and the parking lot at working place will increase substantially. Furthermore, it is found that both the daily travel cost of each mode and the net social benefit decrease when the travel risk (uncertainty) of commuters is reduced.

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Appendix A. Proof of Proposition 1

Proof. The nested logit equilibrium Eq. (7) can be written as

\[
\ln N_1/\omega + \ln N_2/\omega + \ln N_3/\omega = \ln N_1/\omega + \ln N_2/\omega + \ln N_3/\omega + B(N),
\]

Substituting \( f(L + l) = \tau_2/2, f(l) = (r_2 - r_1)/2, \) and \( B_1 = r_2 \) into Eqs. (9), (14) and (12), respectively, then combining with Eqs. (7–8), (10–11), and (13), we obtain

\[
\frac{\ln N_2}{\eta} + \frac{\ln N_3}{\omega} + a\left(\frac{L}{v_2 + l_1} + \frac{l_1}{v_2 + l_1}\right) + \frac{\ln N_2}{\omega} + \frac{\ln N_3}{\omega} + B(N) = \ln N_1/\omega + \ln N_2/\omega + \ln N_3/\omega + B(N),
\]

Secondly, all the commuters are assumed to have the same preferred work starting and ending time in this paper. This assumption could be relaxed with staggered working hours or flexible schedules (Zhang et al., 2005). The actual scheduling of travel may well depend on some important factors, such as the work duration and the utility variation of different work times. Each individual's work utility could be calculated according to his/her work start time and end time with a predetermined timing utility function (Li et al., 2014).

Finally, this paper only considers single bottleneck and single P&R location. In fact, there might be many P&R facilities on a travel corridor (Liu et al., 2009; Liu et al., 2010). Commuters may consider the use of P&R facilities available anywhere throughout the corridor. A continuum equilibrium model with a nested logit-based SUE could be proposed to characterize commuters' modal choice. It is also very interesting to describe commuters' P&R behavior in more complex transportation systems (Lu et al., 2013). Model calibration is another important work direction, which needs appropriate questionnaire design.

\[
\frac{\ln N_2}{\eta} + \frac{\ln N_3}{\omega} + a\left(\frac{L}{v_2 + l_1} + \frac{l_1}{v_2 + l_1}\right) + \frac{\ln N_2}{\omega} + \frac{\ln N_3}{\omega} + B(N) = \ln N_1/\omega + \ln N_2/\omega + \ln N_3/\omega + B(N),
\]
\[
\frac{\ln N_1}{\theta} + 2\alpha \frac{L + I}{V_1} + [\tilde{q}(N_1, L) + \tilde{q}(N_1 + N_{21}, l) + \tilde{q}(N_1, L)]
+ \tau_{21} + C_{20} = \ln N - B(N),
\]

\[
\frac{\ln N_{21}}{\omega} + \alpha \left( \frac{2L}{V_2} + \tilde{T} + \tilde{T} \right) + [\tilde{q}(N_1 + N_{21}, l) + \tilde{q}(N_1 + N_{21}, l)]
+ \tau_{21} + C_{20} = \ln N_{22} + 2\alpha \frac{L}{V_2} + N_{22} \left( \frac{\delta}{\tilde{s}} + \frac{\delta}{\tilde{s}} \right) + \tau_{22}.
\]

Letting
\[
A_1 = 1/(\xi_0) + B',
\]
\[
A_2 = \frac{\partial \tilde{q}(N_1 + N_{21}, l)}{\partial (N_1 + N_{21})} + \frac{\partial \tilde{q}(N_1 + N_{21}, l)}{\partial (N_1 + N_{21})},
\]
\[
A_3 = \frac{\delta}{\tilde{s}} + \frac{\delta}{\tilde{s}},
\]
\[
A_4 = \frac{\partial \tilde{q}(N_1, L)}{\partial N_1} + \frac{\partial \tilde{q}(N_1, L)}{\partial N_1},
\]

we get
\[
A_1 \leq 0, A_2 \geq 0, A_3 > 0, A_4 \geq 0.
\]

According to the partial derivative rule of implicit function equations, we obtain
\[
\frac{\partial A_1}{\partial \xi_1} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \leq 0,
\]
\[
\frac{\partial A_2}{\partial \xi_1} = \frac{A_1 - A_2 - 1/(\xi N_2) + 1/(\xi N_{21} \omega) + A_3}{-A_0} \leq 0,
\]
\[
\frac{\partial A_3}{\partial \xi_1} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \leq 0,
\]
\[
\frac{\partial A_4}{\partial \xi_1} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \leq 0.
\]
\[
\frac{\partial A_1}{\partial \xi_2} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \geq 0,
\]
\[
\frac{\partial A_2}{\partial \xi_2} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \geq 0,
\]
\[
\frac{\partial A_3}{\partial \xi_2} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \geq 0,
\]
\[
\frac{\partial A_4}{\partial \xi_2} = \frac{A_1 - A_2 - 1/(\xi N_2) - 1/(\xi N_{22} \omega) + A_3}{-A_0} \geq 0.
\]

References


