A Comment on Simulating LRD Traffic with Pareto ON/OFF Sources

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II. BACKGROUND

Self-Similarity is a scaling phenomena - the shape of a feature repeats itself independent on the scale of observation. In a network context, a time series $X$ of packet arrivals (or bytes) counted in intervals of fixed length $\tau$, this means independence on $\tau$ - the resulting time series resembles itself for each intercall length. More formally, let $X = \{X(t), t \geq 1\}$ be a stationary process and let

$$X^{(m)}(k) = 1/m \sum_{t=(k-1)m+1}^{km} X(t), \quad k = 1, 2, 3 \ldots$$

be the level $m$ aggregated sequence by dividing $X$ in non-overlapping blocks of size $k$. The time series exhibit strict SS if it matches condition

$$X \approx m^{(1-H)} X^{(m)}$$

However, this proved to be too strict - actual network traffic is asymptotic self-similar, i.e. with $m \rightarrow \infty$. The degree of SS is expressed by the Hurst parameter, $0.5 \leq H \leq 1$, and increases towards one. Clearly, the Auto-Correlation-Function (ACF) of SS processes resembles itself for each scale of observation. Consequently, network traffic is called asymptotic second-order self-similar.

Moreover, the ACF of network traffic exhibits a significant feature, it decays only very slowly to zero. Mathematically, this means strong dependence between samples (random variables) over large times - so-called LRD (Joseph Effect). Formally, the ACF reads as $C_X(k) \sim C_\gamma k^{(H-1)/2}$ with a finite constant $C_\gamma$ and the Hurst $H$ parameter like above. In the rest of the paper we use SS to refer to network traffic with SS and LRD effects.

Willinger et al. showed in [1] that SS network traffic can be generated using two state Markov ON/OFF source models. In ON state, a source emits packets on peak rate and no packets in OFF state. State sojourn times are drawn from two individually parametrised Pareto distributions with a CDF $P(x) = 1 - \left(\frac{1}{x}\right)^\alpha$. Likely the major finding of their work is the relation $H = \frac{\alpha-1}{\alpha}$. Informally, SS and LRD makes traffic bursty over different time scales. However, mainly due to its asymptotic character and because of limited time scales of interest - real buffer sizes are inherently limited - the actual impact of SS and LRD is still an open issue [2]. Henceforth, the significance of H as a traffic parameter still remains open.

III. EVALUATION

As we are only interested in the arrival process, a simple scenario of two source nodes, one router and one destination suffices. Pareto ON/OFF (POO) sources are uniformly distributed on source nodes. The arrival process, number of bytes, is sampled at the router over intervals $\tau = 100 ms$.

To simulate a realistic scenario, flow arrivals follow a Poisson process with mean $\mu$. Holding times are exponentially distributed with mean $\mu h$. Characteristics of POO sources are configured by the shape parameter $\alpha$, average ON and OFF time $t_{on}$ and $t_{off}$ and peak rate $\rho$. Default settings are listed in Tab. I.

Simulation duration is 140000 $s$ where the first 20000 $s$ are discarded to evaluate the system in equilibrium. As experimental framework we use the NS-2 network simulator.

Not to confuse with Probe Based Admission Control. The latter is a special kind of MBAC

1 http://www.isi.edu/nsnam/ns/

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As mentioned in Sec. II, the degree of SS is expressed in \( H \). Thus, to evaluate SS in the arrival process we have to determine \( H \). Using SELFIS\(^3\), we choose four Hurst estimators: Abry-Veitch based on wavelets \( H_{av} \), Periodogram \( H_p \) and the related Whittle \( H_w \) estimator, both computing in the frequency domain and the R/S statistic \( H \), the initial method developed by the engineer Hurst.

### A. Robustness to Flow Arrival Patterns

In the first simulations we evaluate the robustness of the SS phenomena using multiplexed POO sources for different flow arrival patterns. We choose the default setup, see Tab. I, and varied \( \mu_n \). Results are presented in Tab. II.

This setup examines the dependency of \( H \) on mean traffic intensity. The mean number of flows in the system is \( p = \frac{\mu_n}{\mu_0} \). A source sends with mean rate \( \nu = \frac{\mu_n}{\mu_0} \) and according default settings for \( t_{on} \) and \( t_{off} \), in this case \( \nu = \frac{2}{\alpha} \). Intuitively, one would expect that with a growing number of active sources, the Hurst parameter increases too. However, except for \( H_{av} \), for all other estimators the value of \( H \) and thus the degree of SS decreases with an increasing mean number of sources.

Regarding the estimated value of \( H \) and the analytical relation \( H = \frac{3-\alpha}{2} \), it seems that only the AV-Estimator and the Whittle-Estimator approach the theoretical value of \( H = \frac{3-\alpha}{2} = 0.9 \), while the R/S-Estimator seems to underestimate the degree with a large error.

However, despite of variation in estimated values, for this limited set of simulations, SS and LRD effects seem to be insensible to different values of \( \mu_n \), the issue under investigation in this paper.

### B. Robustness to Burst and Idle Times

Statistical features of the traffic aggregate are cumulative outcomes of individual sources properties. For POO sources, these properties are determined by the triple \( t_{on}, t_{off} \) and \( \alpha \). With the next set of experiments we evaluate SS effects for different source individuals. As before, we apply the default settings, see Tab. I, but diversify \( t_{on} \) and \( t_{off} \). Results are presented in Tab. III.

As a flow is a set of packet bursts (comprising one to \( n \) packets) with a maximum inter-burst time, these experiments can be considered as an increase of investigation granularity. A source can be in ON state with probability \( p_{on} \) and in OFF state with probability \( 1 - p_{on} \) where \( p_{on} = \frac{t_{on}}{t_{on} + t_{off}} \).

For the first experiment with a very short burst period \( t_{on} \) and a still shorter idle time \( t_{off} \), the degree of SS clearly departs from the theoretical level. This suggests some sensibility of the SS degree to high frequent state switching.

The two remaining experiments are carried out to show the sensibility to somewhat extreme burst to idle time ratios. Clearly, the first source with and burst time \( t_{on} \) is much burstier as the last one. Interestingly, although the great difference in \( p_{on} \), the values for \( H \) are relatively close and in contrary to the first set of experiments, the value of \( H \) increases with traffic intensity.

Finally, it is worth to notice that the first three estimators compute rather consistent estimates and only the R/S-Estimator departs significantly.

### C. Diversifying \( \alpha \)

As introduced in [1], the degree of SS is purely a function of \( \alpha \) formulated by the functional \( H = \frac{3-\alpha}{2} \). The results presented corroborate this result, especially as we have no measure of the accuracy of the applied estimators. To further foster this assumption, we finally examine the simulation default setup for different \( \alpha \) levels. Results are presented in Tab. IV.

The results are somewhat surprising. As \( \alpha \) is the crucial parameter, for non of the settings the theoretical values is approached. The reason can be either inaccuracy of the estimators, simulator implementation of POO sources or finally the experimental setup itself as each sampling setup is inherently subject to statistical noise.

### IV. Conclusion

In conclusion we can say, that using POO traffic with Poisson arrival flow arrival processes and exponential distributed holding times generates SS traffic in a robust manner regarding parameter settings. Stepping ahead, another conclusion is, that this limited number of simulations does not allow any statement about the gain of Hurst parameter monitoring for congestion control mechanisms, as for example MBAC. No clear pattern was identifiable and some results were even contradictory to theoretical foundations. Henceforth, the only conclusion in this context is: Further rigorous investigation is necessary.

### Acknowledgements

This work was supported by the Portuguese Ministry of Science and High Education (MCES), and by European Union FEDER under program POSI (project QoSMap).

### References


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\(^3\)http://www.cs.ucr.edu/~tkarag/Selfis/Selfis.html

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### Table I

Simulation standard settings

### Table II

Robustness to different flow arrival patterns

### Table III

Robustness to burst and idle times

### Table IV

Diversifying \( \alpha \)