Diffusion of Innovations and Policy Decision-Making

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This article presents a general mathematical model of the diffusion of innovations, which incorporates mass media and interpersonal influence. The model is applied to three classic diffusion data sets: (a) use of hybrid corn, (b) knowledge of Eisenhower's stroke, and (c) doctors' prescription of a new drug. Nonlinear regression is used to estimate the mathematical model. The results show that diffusion of hybrid corn occurred via interpersonal influence, whereas the diffusion of knowledge of Eisenhower's stroke occurred via the mass media. For the diffusion of the new drug, the model shows that doctors who subscribed to few medical journals learned about the drug primarily through interpersonal influence, while doctors who subscribed to many medical journals learned about the drug through both mass media and interpersonal channels. Policy decision-makers can use diffusion models to (a) evaluate the effectiveness of media versus interpersonal campaigns, (b) make comparisons between subgroups, and (c) evaluate the effect of a policy.

Diffusion of innovations is the process by which a new idea or product is communicated through certain channels over time among members of a social system (Rogers, 1983). Most diffusion studies analyze the spread of new technology or ideas to all (or almost all) members of a social system. Media campaigns are often conducted to promote the adoption of new technology or ideas. These campaigns often rely on mass media and/or interpersonal influence to persuade people to adopt the innovation. The efficacy of the campaign strongly influences how widespread adoption of the technology or idea becomes, which in turn has strong policy implications for society.

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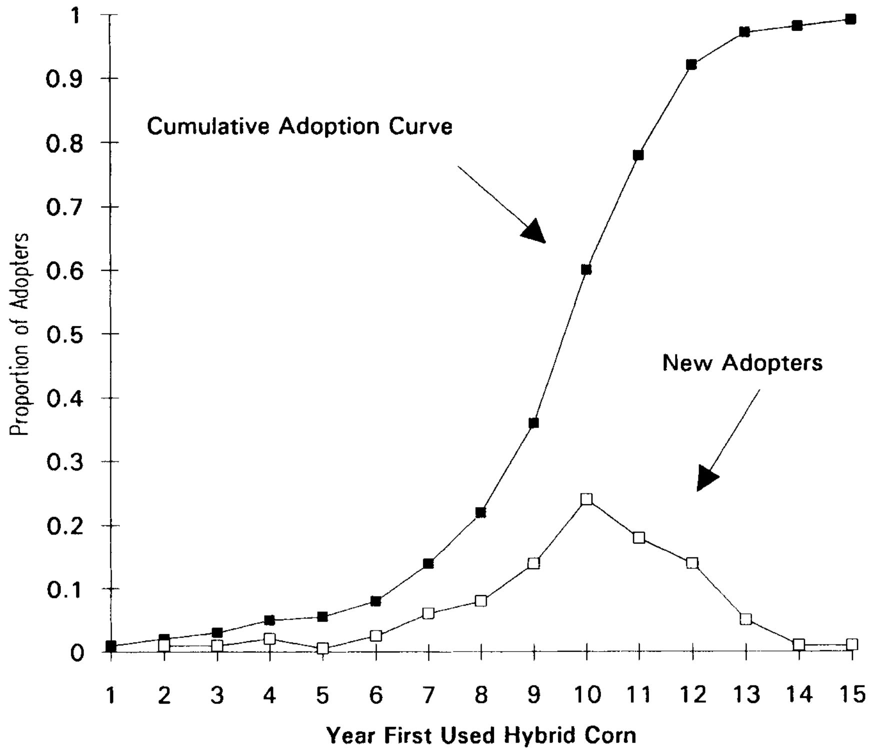

Figure 1. Diffusion of hybrid corn among farmers in two Iowa communities.

This article provides a review of mathematical models of diffusion, and a discussion of how mathematical models can be applied to policy analysis. One specific mathematical model measures mass media and interpersonal influence *during* the diffusion process, which can help guide policy in public- and private-sector communication campaigns.

Diffusion research is inherently policy-oriented. The diffusion paradigm emerged in rural sociology to promote agricultural research results to farmers. Diffusion research was conducted to evaluate and improve Agricultural Extension Services so that research on agricultural innovations could be more rapidly communicated to farmers, and thereby improve the productivity of U.S. farming.

The most well-studied agricultural innovation was hybrid corn, which was originally developed in agricultural research laboratories in the late 1920s. The diffusion of hybrid corn to farmers in two Iowa communities is graphed in Figure 1. This general diffusion model is an *S*-shaped curve referred to as an ogive, a cumulative distribution function, or a logistic curve.

The curve corresponds to a process in which a few members initially adopt an innovation, and then over time more members adopt it until a saturation level is reached. The curve is *S*-shaped because adoption is initially slow and then it "takes off," reaching a maximum rate of diffusion when about half the population has adopted it. After the halfway point, there are fewer potential adopters left in the population so the rate of adoption slows down and the curve levels off.

A second curve we can examine is the new adopters curve, which represents the number of new adopters per time period (also in Figure 1). The new adopters curve is the first derivative of the cumulative function, and shows how many farmers adopted hybrid corn at each point in time.

Mathematical models of diffusion were created to measure how fast or slow the agricultural innovations spread.[1] Early research on diffusion measured how fast hybrid corn was adopted by farmers. The availability and attributes of hybrid corn were communicated to farmers by mass media, salespersons, farm journals, and agricultural extension workers. Some farmers were persuaded by these influences and were early adopters of hybrid corn. The early adopters then persuaded other farmers to adopt hybrid corn (Ryan & Gross, 1943, 1950).

Researchers conducted studies to determine when farmers in a county or state first started using hybrid corn. The initial use of hybrid corn was defined as the farmer's time of adoption. In some areas diffusion occurred rapidly. That is, only a few years elapsed between the time early adopters adopted and the time all farmers in an area adopted. The speed of diffusion of hybrid corn can be more accurately measured by a rate parameter. A *rate parameter* is a specific number that signifies the speed of diffusion of an innovation for a specific population. Rate parameters are calculated with reference to particular mathematical models of diffusion.

Mathematical Models of Diffusion

The diffusion of innovations can be divided into three processes: internal, external, and mixed influence. *Internal influence* is diffusion by interpersonal communication through word of mouth, such as diffusion of rumors, gossip, jokes, and so forth.

External influence is diffusion by any information source external to the interpersonal contact among individuals. For example, mass media broadcasts of important news is external influence diffusion since individuals receive the message independent of their interpersonal contact.

[1] Mathematical models of diffusion were first developed in epidemiology to describe the spread of a disease (Bailey, 1957, 1975; Monin, Benayoun, & Sert, 1976). The models were later applied to the study of the spread of rumors (Daley & Kendall, 1965; Rapoport, 1954; Rapoport & Yuan, 1989), news (Deutschmann & Danielson, 1960), information (Funkhouser, 1970; Funkhouser & McCombs, 1972), innovations (Griliches, 1957), and gossip (Dodd, 1955).

Table 1: Mathematical Models of the Diffusion of Innovations

	Internal	External	Mixed
Cumulative function	$\dfrac{Ne^{Nat}}{(N-1+e^{Nat})}$	$N(1-e^{-at})$	$\dfrac{N-\dfrac{a(N-N_0)}{a+bN_0}e^{[-(a+bN)t]}}{1+\dfrac{b(N-N_0)}{(a+bN_0)}e^{[-(a+bN)t]}}$
Derivative	$a*y(t)[N-y(t)]$	$a*[N-y(t)]$	$[a+b(y(t))][N-y(t)]$
Diffusion of:	Adoption	Awareness	Adoption and awareness
Type of communication	Interpersonal	Mass media	Interpersonal and mass media

Note: N_0 is the number of initial adopters (adopters at t_0); N is the population size; a and b are model parameters.

Although mass media represent the most common external influence, other information sources, such as journals, pamphlets, and fliers, also fall in this category.

Mixed-influence models incorporate both interpersonal and mass media communication simultaneously. Many innovations diffuse via mixed influences since both an external source such as mass media advertising as well as interpersonal sources such as friends and neighbors influence adoption decisions.

The three mathematical functions that represent these processes are presented in Table 1. In the past, appropriate diffusion mathematical modeling required that researchers use the right model for the right process. Generally, researchers compared the internal or external model to diffusion data and determined how well the mathematical model fit the data. Rate parameters were calculated by collecting data on time of adoption and then fitting the data to one of these mathematical functions.

The mathematical model used to represent the diffusion of hybrid corn was

$$y(t) = \frac{Ne^{Nat}}{(N-1+e^{Nat})}, \qquad (1)$$

where $y(t)$ is the cumulative proportion of adopters, N is the population size, a is the rate of diffusion, and t is the time period. The symbol e is the base of the natural logarithm, which is approximately 2.72.

When all members have adopted an innovation, the cumulative proportion, $y(t)$, is equal to N, the size of the population. The rate parameter, a, measures how fast diffusion occurs. Higher values of a indicate faster diffusion. Using this model, if a were .01 and the population size were 100, then diffusion to all members would occur in 10 time periods. If a were .02, then diffusion to all members would occur in 5 time periods.

To fit the mathematical model of diffusion to data requires the first derivative of the function. The first derivative measures the number of new adopters per time, and it measures the rate of change for a function. The first derivative of Equation 1 is

$$\frac{dy}{dt} = [a * y(t)][N - y(t)], \tag{2}$$

where $y(t)$ is the proportion of adopters at time t, N is the population size, a is the rate of diffusion, and $1 - y(t)$ is the proportion who have not adopted at time t. The number of new adopters per time period is dependent on the rate of diffusion, a, and the number who have adopted up to that point, $y(t)$.

The internal influence mathematical model in Equation 1 was used to model the diffusion of numerous innovations (Hamblin, Jacobsen, & Miller, 1973). However, this model provided only one rate parameter estimate. The mixed model allows the estimation of mass media and interpersonal influence processes simultaneously (Bass, 1969; Caldeira, 1985; Lave & March, 1975; Mahajan & Peterson, 1985). The mixed-influence model is

$$y(t) = \frac{N - \dfrac{a(N - N_0)}{a + bN_0} e^{[-(a+bN)(t-t_0)]}}{1 + \dfrac{b(N - N_0)}{(a + bN_0)} e^{[-(a+bN)(t-t_0)]}}, \tag{3}$$

where $y(t)$ is the cumulative proportion of adopters, N is the number of potential adopters, a is the mass media influence parameter, b is the interpersonal influence parameter, N_0 is the number of initial adopters, and t is the time period (Mahajan & Peterson, 1985).[2]

In the discussion of hybrid corn, recall that farmers were persuaded to adopt hybrid corn from external sources such as mass media, salespersons, farm journals, and through communication with other farmers. The mixed model allows us to estimate the diffusion of hybrid corn and compare the relative strengths of mass media and interpersonal influences.

The first derivative of the mixed-influence model is

[2] Diffusion models other than the deterministic ones used in this paper include: Gompertz (Dixon, 1980), stochastic (Bartholomew, 1982), polynomial (Sharif & Ramanathan, 1982), Bayesian (Oliver, 1987), and event history (Strang, 1990, 1991).

$$\frac{dy}{dt} = [a + b(y(t))][N - y(t)], \tag{4}$$

where $y(t)$ is the proportion of adopters, N is the number of potential adopters, a is the mass media influence parameter, b is the interpersonal influence parameter, and t is the time period. The interpersonal influence parameter, b, is multiplied by the number of existing adopters and this product is added to the mass media parameter, a. The product then creates a rate of adoption that is multiplied by $[N - y(t)]$, the number of potential adopters remaining, thereby obtaining the number of adopters at each time.

In sum, the mathematical model can be fit to data to estimate the speed of diffusion. Researchers can measure the rate of diffusion for an innovation and compare the rate of diffusion for various innovations that are similar. For example, the rate of diffusion of hybrid corn can be compared to that of sorghum, fertilizers, pesticides, and so forth.

Determining the rate of diffusion is useful for understanding how a prior innovation diffused, and useful as information to make predictions about how fast similar innovations are likely to diffuse. This is accomplished by using the mathematical model with an estimated rate parameter from a prior study to make a projection about a new innovation's diffusion. Such a technique is called forecasting and is useful for understanding market growth and to plan for future levels of consumer demand. During the diffusion process, researchers and policymakers can continually monitor the adoption pattern to refine the model's prediction.

However, a single rate parameter is not as useful to policy analysts who need to make decisions about campaign design. The mixed model, which provides mass media and interpersonal influence estimates, permits policy analysts to determine the relative effectiveness of mass media and interpersonal communication for promoting the diffusion of an innovation.

For example, suppose a national campaign to promote family planning was implemented in some country. Researchers can apply the mixed model of diffusion to family planning acceptance data. The mixed model provides estimates for the strength of mass media and interpersonal influences on adoption of family planning. The strength (magnitude) of these estimates can be compared, to determine whether mass media or interpersonal communication strategies were more or less effective in promoting family planning. Policymakers can then make campaign strategy adjustments based on this empirical evidence.

Finally, the strength of the mass media and interpersonal effects can be compared across regions or between subgroups to determine where mass media or interpersonal influences are strongest. For example, Griliches (1957) compared the rate of hybrid corn diffusion for different states in the U.S.; he concluded that, due to its profitability, hybrid corn diffused most rapidly in Iowa and Indiana.

In the family planning campaign example, diffusion data for various regions within a country can be compared to determine whether individuals are adopting family planning on the basis of mass media or interpersonal influence. More importantly, subgroups can be compared to determine whether mass media or interpersonal influence is more effective with one group than another. One might hypothesize that educated individuals are more likely to adopt family planning on the basis of mass media influences whereas noneducated individuals are more likely to adopt it through interpersonal influence.

The present mathematical modeling is performed by reanalyzing three diffusion data sets and conducting nonlinear regression with the mixed model to compare mass media and interpersonal influence. The equation analyzed was the mixed-model differential equation, using successive differences in the number of adopters as the dependent variable. This approximates the solution to the differential equation. The objective is to give examples of how researchers can analyze diffusion data to obtain separate estimates for mass media and interpersonal influence, and how to compare estimates between subgroups.

Three Nonlinear Regression Models

This section reanalyzes three diffusion data sets using nonlinear regression (Gallant, 1987). Nonlinear regression fits theoretical curves to empirical data much like linear regression. However, nonlinear regression models may be unstable, that is, the solution to the nonlinear regression may be dependent on the starting values.

Nonlinear regression allows us to fit the mixed-model diffusion curve to data and obtain estimates for mass media and interpersonal influence. The three data sets reanalyzed are taken from (a) Ryan and Gross's 1943 study on hybrid corn adoption, (b) Deutschmann and Danielson's 1960 study of news awareness, and (c) Coleman, Katz, and Menzel's 1966 study of doctors' use of a new drug.

The Ryan and Gross (1943) study of the diffusion of hybrid corn was a classic diffusion study that launched the diffusion paradigm. Ryan and Gross wanted to determine why some farmers adopted hybrid corn early while others delayed their acceptance of the new seed. The authors discovered that early adopters were more socially active, and that later adopters relied on earlier adopters as sources of influence. A principal focus of the study was to determine sources of influence on farmers' adoption decisions, whether they were influenced by farm journals, radio broadcasts, salespersons, or communication with other farmers. Ryan and Gross concluded that the mass media did not influence adoption, but rather that interpersonal communication with other farmers was the principal source of influence on adoption decisions.

Nonlinear regression using the mixed model (Equation 3) on the Ryan

Table 2: Mass Media and Interpersonal Influence Estimates from Nonlinear Regression for Three Studies

	Mass media	Inter-personal	*F*	*df*	R^2	*N*	*T*
Hybrid corn	−.04 (.0195)	.94** (.1122)	44.51**	2,12	.88	14	15 yrs.
Eisenhower's stroke	.49** (.1226)	−.44 (.2193)	23.21**	2,13	.78	15	34 hrs.
Tetracycline Few journals	−.05 (.0829)	.03** (.0085)	35.82**	2,17	.79	19	18 mos.
Many journals	.02* (.0066)	.0052** (.0002)	2424.28**	2,77	.98	79	18 mos.

Note: Parameter estimates are not standardized. *N* = total number of units in population; *T* = duration of diffusion. Standard errors for coefficients appear in parentheses.

*$p < .01$. **$p < .001$.

and Gross data produces estimates for mass media and interpersonal influence. The estimates presented in Table 2 show that interpersonal influence contributed significantly to the pattern of adoption of hybrid corn among the farmers. In other words, it is more likely that farmers adopted hybrid corn due to interpersonal persuasion by other farmers. To be sure, mass media, salespersons, and extension service media were important influences in making farmers aware of hybrid corn, and were important in persuading early adopters. But the majority of farmers adopted due to interpersonal influence. Statistical tests performed on the modeling of this and the other two data sets that follow support these conclusions.[3]

The process of interpersonal influence occurs as early adopters persuade later adopters of the advantages of hybrid corn. "In a sense the early acceptors provided a community laboratory from which neighbors could gain some vicarious experience with the new seed over a period of some years" (Ryan & Gross, 1950, p. 18). The closeness of fit between the data and the predicted diffusion given these parameter estimates is demonstrated by the graph in Figure 2. The predicted diffusion pattern is very similar to the actual diffusion pattern.

[3] Tests comparing the null model (which posits that the mass media and interpersonal influence parameters were equal) with the theoretical model (which posits unequal parameters) were conducted on all three data sets (Borowiak, 1989). The results support the claim (cont.)

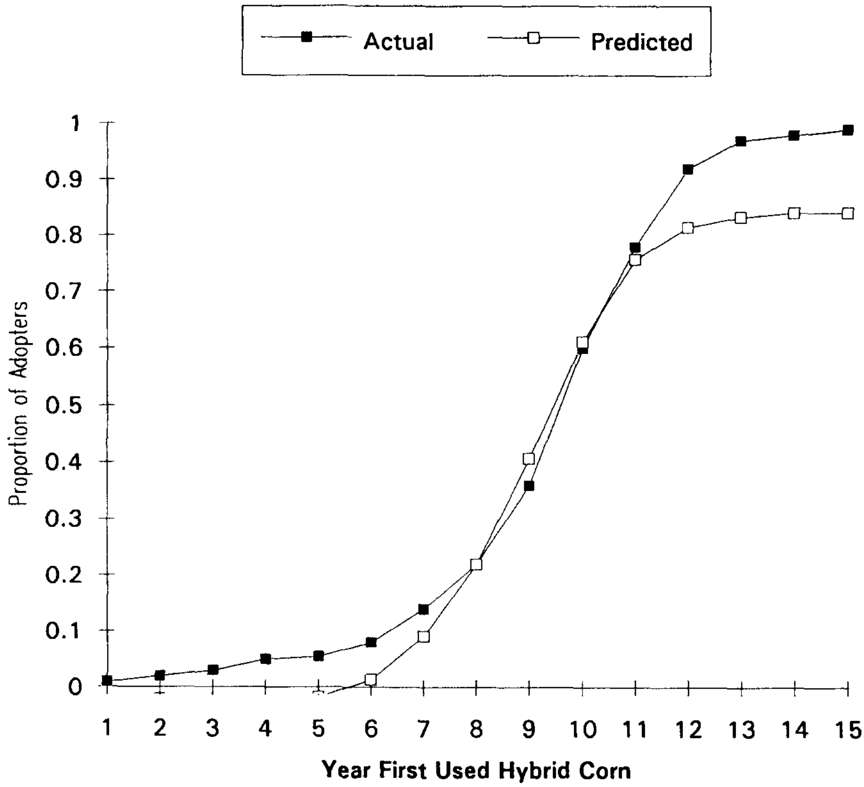

Figure 2. Actual and predicted curves for the diffusion of hybrid corn among farmers in two Iowa communities.

Diffusion of awareness, on the other hand, generally occurs via the mass media. The mixed model applied to diffusion of awareness data should yield a significant estimate for mass media influence and a non-significant estimate for interpersonal influence. Deutschmann and Danielson (1960) provide the data to verify this assertion.

(cont.) that the mass media and interpersonal parameters were statistically different from one another. The test was computed by determining the significance of the generalized likelihood ratio test for nested models (GLRT). The GLRT is

$$F = -2\ln\left(\left[\frac{RSS(null)}{RSS(model)}\right]^{N/2}\right)$$

where *RSS(null)* equals the residual sum of squares for the model with mass media and interpersonal influence parameters constrained to be equal, and *RSS(model)* is the theoretical two-parameter model. The GLRT is distributed as X^2 with one degree of freedom. The GLRT for the null versus theoretical models were as follows: Ryan and Gross, -25.33 ($p <$ .001); Deutschmann and Danielson, -7.01 ($p < .01$); Coleman et al., -4.43 ($p < .05$).

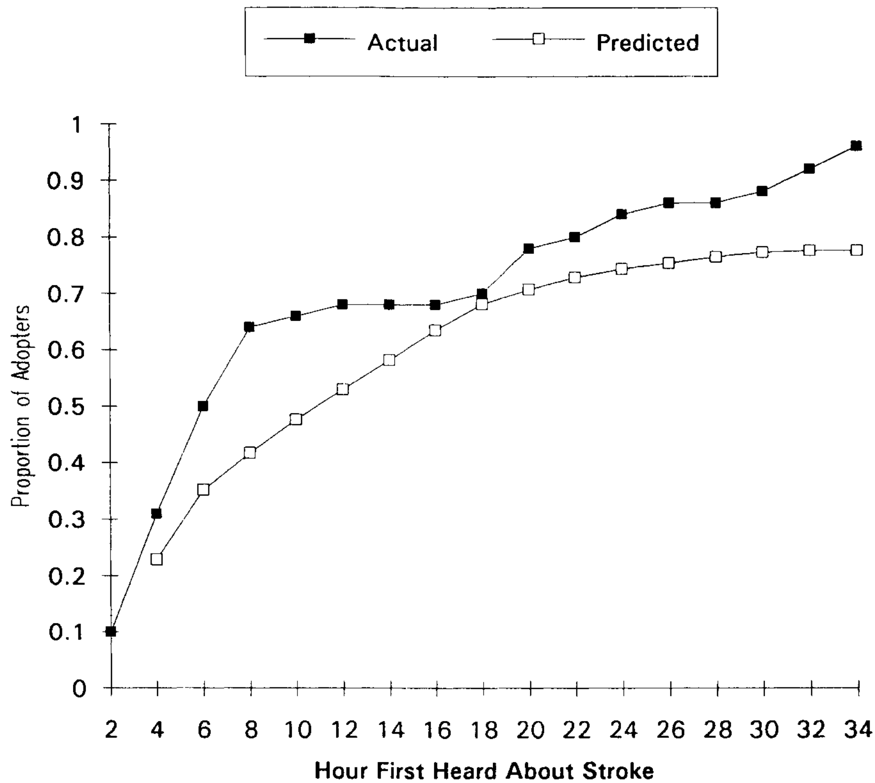

Figure 3. Actual and predicted curves for the diffusion of awareness of Eisenhower's stroke.

Deutschmann and Danielson (1960) studied diffusion of news events such as the public's awareness of Eisenhower's stroke in November 1957. The authors were interested in comparing news sources to determine if people generally get their news from the mass media or from interpersonal communication. Deutschmann and Danielson reported that 82% of the sample heard about Eisenhower's stroke from the mass media.

The mixed model was applied to the Eisenhower stroke data and yielded a significant rate parameter estimate for mass media influence, and a nonsignificant parameter estimate for interpersonal influence. Figure 3 shows how well the mathematical model with these estimates fits the actual diffusion of knowledge of Eisenhower's stroke. As can be seen, most people were aware of the news within a few hours of its occurrence. The significant parameter estimate for mass media influence indicates that mass media were more likely to be the vehicle for this knowledge.

These two examples, diffusion of hybrid corn and diffusion of news, demonstrate how the mixed model can provide significant estimates for interpersonal and mass media influence. However, a more useful application is the comparison of mass media and interpersonal influences on adoption for subgroups of a population or for different regions.

Do all members of a group adopt an innovation through mass media or interpersonal influence? Data collected by Coleman et al. (1966) are reanalyzed to show that the mixed model provides different significant estimates for different subgroups. The analysis attempts to show that some people adopt from mass media influence while others adopt through interpersonal influence.

Coleman et al. (1966) studied the diffusion of tetracycline among doctors in four Illinois communities. Coleman et al. wanted to determine influences on doctors' decisions to adopt new drugs, and sought to understand the role of interpersonal influence on adoption decisions. Time of first use of tetracycline was measured by examining the prescription records at local pharmacies to determine when a doctor first prescribed tetracycline.

Coleman et al. argued that doctors who were interconnected into the social network diffused the innovation among themselves in a chain-reaction process. The present research suggests another model which differentiates media versus interpersonal influence. Some evidence suggests that doctors get information about new drugs via medical journals (Winick, 1961). For doctors, media sources of information for new products are journals and drug company publications. In the present analysis, therefore, the mass media influence was represented by subscriptions to medical journals.

The sample of doctors was divided into two groups: those who received more than four medical journals and those who received less than four. The mixed model was applied to both groups and parameter estimates for mass media (medical journals) and interpersonal (other doctors) influences were obtained. The results indicated that doctors who subscribed to few journals were more likely to be influenced by other doctors: The interpersonal influence estimate was significant whereas the media influence estimate was not (Figure 4).

For doctors who subscribe to many journals (four or more), both the mass media and interpersonal influence estimates were significant. This indicates that doctors who subscribed to many journals were influenced by both the media exposure and by other doctors. Again, statistical tests performed on the models support these conclusions.[4]

A cautionary note is sounded to researchers attempting to compare dif-

[4] For comparison of subgroups, a second GLRT is necessary to determine whether the parameters obtained for the two subgroups are statistically different from one another. The formula is similar to that mentioned in Footnote 3, except that a pooled *RSS* is used in the denominator. The GLRT for the subgroup model was -166.22 ($p < .001$).

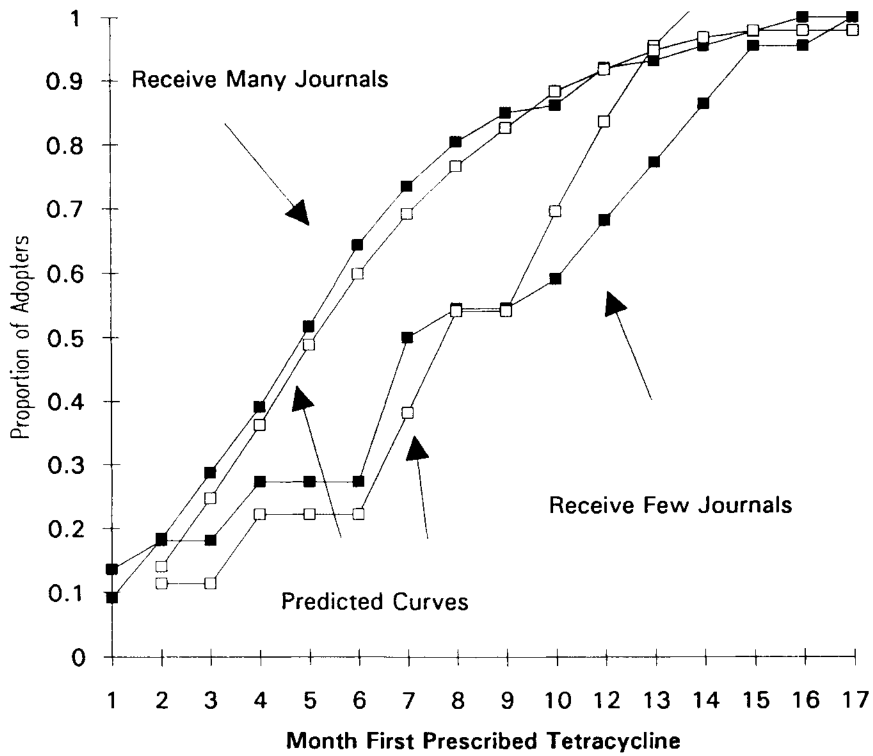

Figure 4. Actual and predicted curves for the diffusion of tetracycline prescriptions among Illinois doctors who receive many and few journals.

fusion between subgroups. Comparison of subgroups may be valid for homogeneous populations such as Illinois doctors. However, comparison between subgroups in a heterogeneous population may be problematic due to social and cultural factors that may make comparisons difficult. In other words, comparing diffusion rates for different geographic areas may in fact be capturing different diffusion rates for subcultures or socioeconomic levels.

Value of the Model

The mixed model provides comparable rate estimates for mass media and interpersonal influence. A study could be conducted that uses the mixed model to plan a mass media and interpersonal campaign. Numerous field studies use levels of treatment to disseminate information. For instance, a family planning campaign might have a control area and two treatment areas, one with a radio program about family planning and one that asks family planning users to recruit nonusers. Empirical results should confirm that the radio treatment area had a higher mass media influence estimate whereas the interpersonal treatment area had a higher interpersonal influence estimate. Both treatment areas can be compared to the control group.

Furthermore, empirical analysis of existing family planning diffusion data might indicate that the mass media influence parameters are nonsignificant. A nonsignificant mass media influence parameter indicates that individuals are not being effectively persuaded to adopt family planning by the mass media. Conversely, greater interpersonal influence estimates indicate that individuals adopt family planning based on interpersonal influence.

A second use of mathematical models for policy is the comparison of diffusion rate parameters for different geographic areas or subgroups. Such analysis, like that conducted by Griliches (1957), will indicate where diffusion occurs most quickly. More importantly, the analysis will indicate which areas had rapid diffusion from the mass media and which had rapid diffusion from interpersonal influence.

By conducting such research, policy analysts can formulate appropriate policies to determine which areas would benefit from a mass media campaign and which would benefit from an interpersonal motivation campaign, and thus accelerate diffusion. For example, policymakers might decide to invest 75% of their resources into a mass media campaign to promote an innovation. After the first year of the campaign, researchers can analyze the diffusion data to determine if individuals are adopting the innovation due to mass media influence. If the mass media are not effective, policymakers can revise the mass media strategy or decide to allocate more resources to an interpersonal communication campaign strategy.

Policies that support an innovation present the opportunity for advertising and mass media campaigns that promote adoption of the innovation. For instance, a government policy that supports family planning allows private companies to advertise and promote their family planning products. Consequently, diffusion of family planning practices may use the mass media influence model to diminish reliance on interpersonal persuasion and thereby promote more rapid diffusion.

The absence of a policy that effectively promotes family planning may deter positive mass media influence and hence slow down diffusion. The absence of a policy increases risk and uncertainty about an innovation, and hence increases the role of interpersonal influence and interpersonal persuasion as forces for adoption.

Mathematical models of diffusion can also be used to evaluate the effect of a policy. Policies designed to promote an innovation's diffusion should, among other objectives, accelerate diffusion. Thus, rate parameters from localities with a favorable policy should be greater than diffusion rate parameters from localities without such a policy. Again, the relative effects of the policy on mass media adoption versus interpersonal adoption can be evaluated.

Since mathematical models indicate the speed of diffusion, mathematical models may be used to predict the time of the critical mass. The *critical mass* is the point in diffusion where the increase in new adopters becomes self-sustaining (cf. Allen, 1988; Markus, 1987). Consequently, policies to promote innovation diffusion should target reaching a critical mass of adopters. Once a critical mass of adopters is reached, policy incentives for adoption become less necessary. If a government provides monetary incentives for adoption, these incentives can be quite costly to the government. If the incentives are targeted to the critical mass, then they may be withdrawn after critical mass is reached, thus saving considerable sums of money.

Prior to the critical mass, potential adopters are uncertain about how many people in the social system will adopt an innovation. The potential adopters' uncertainty increases the risk to adoption. Pre-critical mass adopters are taking a chance by trying something new, often in opposition to traditional societal norms. Consequently, it takes more resources to get people to adopt before the critical mass, to overcome these normative barriers. Thus, knowing when the point of critical mass occurs, and the resources needed to reach it, is a powerful planning tool.

In sum, the mixed mathematical model can specify the rate of diffusion more accurately than studies that report percentage of adoption at certain time periods and thus create a more accurate prediction of diffusion. Policymakers can intervene during the process at appropriate points to make strategy adjustments in a timely and cost-efficient manner, such as at the critical mass. Furthermore, subgroups of a homogeneous population can be compared to make further adjustments to communication campaigns.

By continuing to explore the challenges of diffusing information, policymakers and mathematical modelers can benefit from the application of the mixed model for more effective policy decisions.

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