Waiting lines in Franklin County, Ohio in 2004 likely deterred thousands from voting. Statistical techniques can illuminate and alleviate the problems.

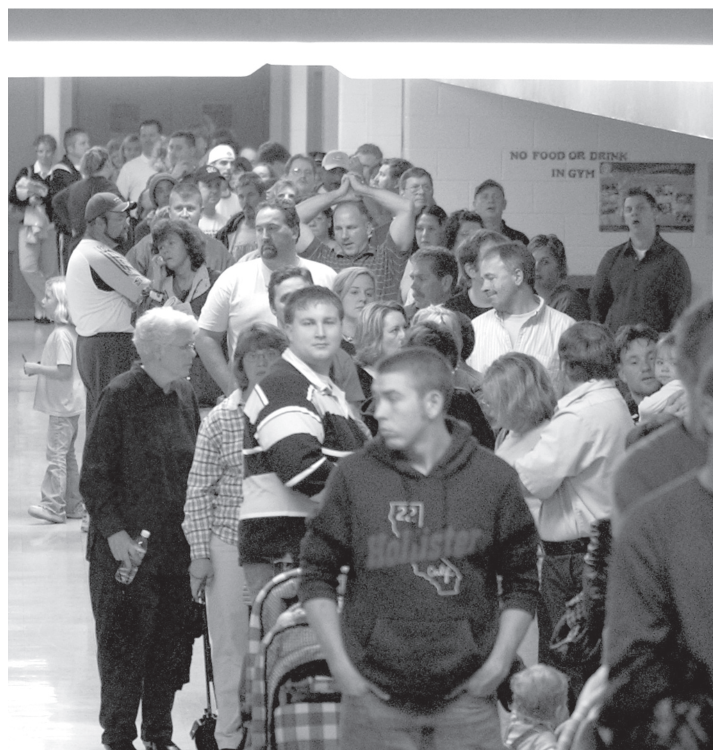
Mitigating Voter Waiting Times

Theodore Allen and Mikhail Bernshteyn

Many around the world are aware that lines in Franklin County, Ohio, likely deterred thousands of would-be voters in 2004. Moreover, a report commissioned by the Democratic National Committee showed that African Americans who voted waited much longer than others. Therefore, it seems likely that a disproportionate number of African Americans were deterred from voting.

What caused the long lines in some voting precincts and no lines in others? How can such lines be avoided in the future? Clearly, purchasing more voting equipment will help, but how much more equipment? And how should it be allocated?

In many places in the United States, the allocation of voting machines to precincts is done on the county level. Typically, county administrators have access to thousands of machines costing several thousand dollars each. Counties also pay set-up and operation costs of the machines that are comparable to the purchasing costs. In some cases, county administrators allocate the machines based on a precise formula or algorithm. In other cases, they use experience and expert judgment. In any case, they allocate the machines with the goal of avoiding long waiting times.



Voters wait in line to cast their ballots at Franklin High School in Franklin, Ohio, a half hour before the polls closed Tuesday, Nov. 2, 2004. According to Carl Bray, the lead voting official for the precinct, the estimated wait time in the line was going to be about three hours for the last person in line at closing. **(AP Photo/Middletown Journal, Pat Auckerman)**

The 2004 election put voting systems to the test because an unprecedented number of people voted. This situation placed added scrutiny on the methods for voting machine allocation and motivated the scientific study of voting machine allocation written about here. The goals of the study include establishing theoretical models relevant to relating machine allocations to waiting times; studying historical practices and their consequences using statistical theory with Franklin County, Ohio, as a case study; proposing a new method for machine allocation; and using theory and historical data to clarify potential advantages of the new method.

Perhaps the most common approaches for allocating machines are based on the ratios of either the number of active voters to machine or the number of registered voters to machine. We argue that, compared with our proposed method, all such ratio-based approaches will result in either longer-than-needed waits, substantial unnecessary expenditures in equipment and personnel, or both. These comparisons use data from the 2004 presidential election results in Franklin County, Ohio.

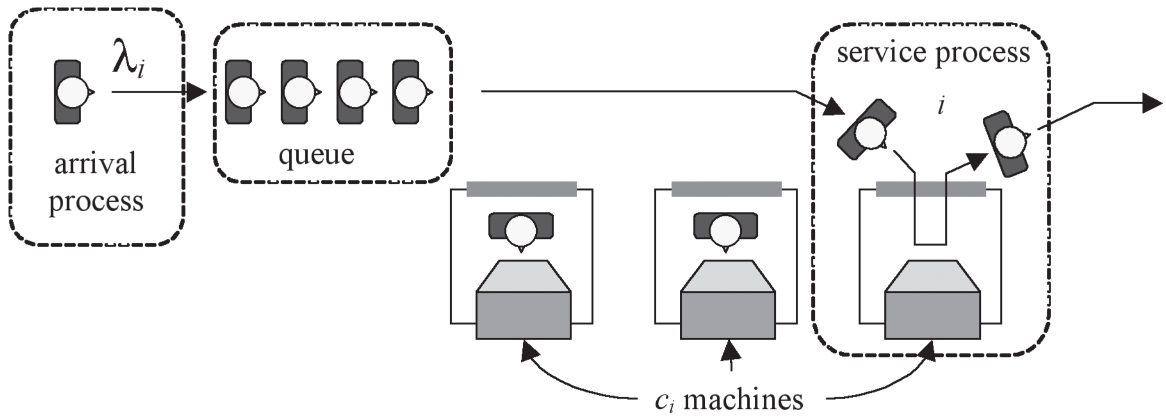


Figure 1. **Illustration of voting in a precinct, i, with ci = 3 voting machines**

Elementary Queuing Theory

In many places in the United States, citizens arrive at their polling place at variable times, wait or "queue" in lines, and then cast their votes using machines. Figure 1 illustrates this process of voting. In some sense, the voting machines offer the 'service' of allowing citizens to express their political preferences. Figure 1 shows people waiting in line, then immediately entering voting booths for voting. This is appropriate for counties in which the bottleneck is related to the service provided by the machine, not the poll workers. If there was no line after the poll workers and before the machines, it could be appropriate to include the time it takes for the poll workers to perform their tasks in the service times.

The phrase "queuing theory" refers to an area of practical academic research related to all operating systems in which there are arrivals, waiting in lines (queuing), and service. Queuing theory can, in many cases of interest, predict the average times people wait and the dependencies of those times on the properties of the arrival and service processes. In the context of voting, queuing theory predictions can be useful for the allocation of machines to precincts, deciding whether additional resources are needed, and predicting problems.

At the county or state levels, many precincts must be considered, each with its own properties. Here, the letter "i" refers to the specific precinct being considered. Each precinct has its predicted "turnout," which is the number of citizens who attempt to vote. The turnout can be converted to a rate, λ_i, at which citizens arrive at the voting system. For example, if 650 citizens arrive at precinct i over 13 hours, the average arrival rate is $\lambda_i = 50$ per hour. Similarly, each precinct has its associated average service rate, μ_i. This is the average speed that voters require to cast their ballot once they finish waiting and are given access to the voting machine. For example, if the average voter requires 3 minutes to vote in precinct i, then $\mu_i = 20$ services per hour.

In the voting context, forecasts of λ_i might depend on the number of active voters in a precinct and the historical turnout percentages associated with that precinct. Similarly, the service rate, μ_i, depends on the number of issues on the ballot in that precinct and the word length, importance, and clarity of the writing.

In queuing theory jargon, "balking" refers to cases in which people choose not to seek service because they perceive waiting times as unacceptable before entering the lines. "Reneging" is the practice of foregoing service after entering the lines. Both balking and reneging are important in voting systems because they correspond to reduced voter turnout, or "deterred" votes.

M/M/c Approximation

For some arrival and services processes, formulas exist that predict the average waiting times and other properties of the system as a function of the parameters λ_i, μ_i, and c_i. For cases in which formulas do not exist, statistical simulations can estimate system properties. Because formulas are computationally more efficient than simulations and generally provide more insight, it can be useful to trade off believability—which simulations generally offer—in exchange for the computational efficiency and intuition afforded by formulas.

The phrase "Poisson arrivals" refers to the assumption that potential voters arrived at the poll place such that the probability of an arrival was constant throughout the day. In the voting context, the assumption of Poisson arrivals is supported by the fact that few voters coordinate their voting with other voters (with a few exceptions, such as married couples). However, this assumption ignores the likely possibility that average or expected arrival rates were variable, peaking at certain predictable times of the day (e.g., early in the morning and late in the afternoon).

The phrase "exponential service" refers to the assumption that voters took a highly variable amount of time to vote as predicted by the so-called exponential probability distribution. In the voting context, this assumption is generally conservative, resulting in longer-than-actual predicted average waiting times because real voters are generally more consistent in their voting times than the exponential distri-

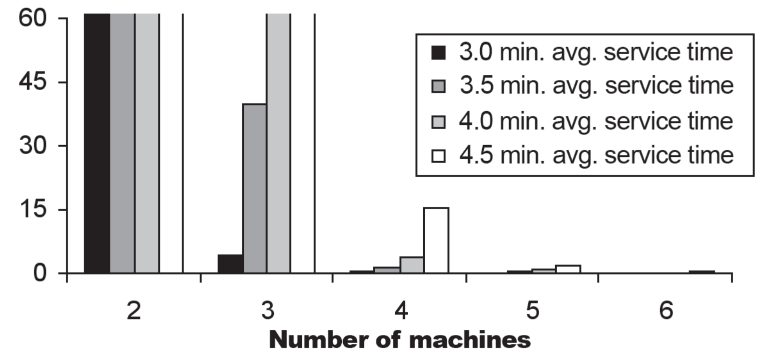


Figure 2. **Predicted average waiting times in minutes for λ_i = 50 people per hour**

bution would predict. This greater consistency of real voters could be enhanced by the enforcement of laws limiting the amount of time to vote allowed to citizens.

The phrase "steady state" refers to the assumption that the properties of the system no longer depend on the initial conditions (e.g., how many people were in line at the beginning of the day). This assumption is relevant for situations in which systems operate over long periods and is only a rough approximation in the context of voting over a single 13-hour or 18-hour election day.

The notation "*M/M/c*" conventionally refers to the combined assumption of Poisson arrivals, exponential servers, and c machines in service. A formula predicting the average waiting times for *M/M/c* queues in steady state is:

$$\text{Avg. Waiting Time} = \frac{(c_i\rho_i)^c}{c_i!}\left((1-\rho_i)\sum_{n=0}^{c_i-1}\frac{(c_i\rho_i)^n}{n!}+\frac{(c_i\rho_i)^{c_i}}{c_i!}\right)^{-1}\left(\frac{1}{1-\rho_i}\right)\left(\frac{1}{c_i\mu_i}\right)\text{ if } \rho_i<1 \tag{1}$$

where $\rho_i = \lambda_i \div (\mu_i \times c_i)$, λ_i is the arrival rate and μ_i is the service rate. It can be shown that the *M/M/c* queue is simple enough that the quartiles of the waiting time distribution are related in a simple way to the average wait. Therefore, by controlling average waits, one is controlling unusually long and unusually short waits also.

The above equation only applies to real situations when $\rho_i < 1$, and then only approximately. Clearly, in real voting situations, the infinite average waits predicted when $\rho_i \sim 1$ are impossible because even if all registered voters voted on the same machine, it could take only a finite number of hours. While this does show an important limitation of the steady state assumption, it is also true that situations in which $\rho_i > 1$ should be avoided because long average waits would result. Despite the limitations associated with all the assumptions of an *M/M/c* queuing system in steady state, this equation can provide useful approximate insights into the behavior of real polling places.

Figures 2 and 3 show predicted average waiting times for different service rates and numbers of machines for different average arrival rates (λ_i). The service rate variability is meant to approximate conditions in which some precincts might have 10 extra ballot issues requesting their attention. The assumption is these extra issues require a combined average of 1.5 minutes.

Figures 2 and 3 establish the following:

- Small changes in the average time per vote can cause large changes in the average waiting times. For example, a ballot initiative adding 30 seconds on average to an otherwise three-minute ballot could add one half hour to the precinct wait times with c = 2 machines. Small additions to the ballot can result in a need for an added machine. For example, if there are c_i = 2 machines in a precinct and a 3.5-minute ballot, it is likely that any additional increase in the ballot length effectively could require the equipping of the precinct with an additional machine.
- In properly functioning voting systems, average waits might be quite short (e.g., less than five minutes). This could lead to a false sense of excessive over-capacity, even while subtle changes in arrival and service rates could cause explosive changes in the waiting times.
- Careful modeling of queuing systems taking into account the combined effects of turnout (through λ_i) and ballot length (through μ_i) can be valuable, as system behavior can be counterintuitive.
- Details of how the poll workers manage the lines can greatly impact performance.

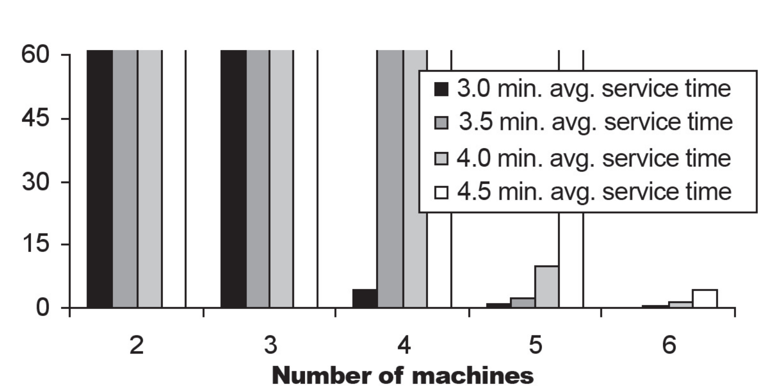


Figure 3. **Predicted average waiting times in minutes for λ_i = 70 people per hour**

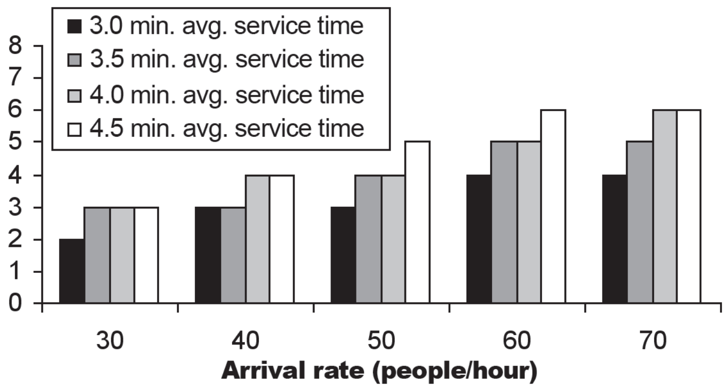


Figure 4. **Minimum numbers of machines (ci) for less than five minutes average waiting time**

- The times in the figures are probably most relevant for properly run polls in which the only bottleneck is voting using the machines. In other words, there is always a line in front of the machines, and leaving the line implies immediate use of machines. Also, appropriate placement of posters showing the issues to be voted on can reduce greatly the waits by reducing service times, as voters are prepared.

The *M/M/c* formula also can be used to determine the minimum number of machines needed to achieve an acceptable average waiting time for properly run polls. Figures 4 and 5 show the minimum numbers of machines to achieve less than or equal to five and 30 minutes of average waiting time, respectively. If 650 voters are predicted to arrive, assuming a 13-hour voting period, one might set $\lambda_i = 50$ per hour. However, it might make sense to focus on average waits associated with peak hours in which arrival rates might increase 20% or more. Note that the assumption $\lambda_i = 50$ per hour does not mean that exactly 50 people come each hour. Poisson arrival process with $\lambda_i = 50$ means that typically 50 people come each hour. The actual number could be as low as 10 or as high as 150 and still follow a Poisson distribution.

Future Work: Not Steady State

So far, we have focussed on the *M/M/c* assumptions and Equation 1 as a preliminary step in voting systems analysis. As additional information becomes available about arrival patterns over the course of election, it likely will become important to investigate alternative assumptions to Poisson arrivals. Timing actual voters prior to elections likely will elicit more relevant service time distributions than the exponential.

In addition, it can be unavoidable for voting systems to run in overload conditions (i.e., $\rho_i > 1.0$), at least for short periods, over an election day. Therefore, departures from steady state assumptions are needed because the waiting times will be large but not infinite. Accurate estimates of waiting times could be important to permit the best possible allocations of machines. All these issues can and should be investigated using statistical simulation. Also, additional queuing theory formulas could be explored for added insights.

Voters per Machine Allocation Methods

Consider the following optimization problem relevant for the selection of the numbers of machines allocated to N precincts, $c_1, c_2, \ldots, c_N$:

$$Minimize\left\{Maximum\left[\frac{\lambda_i}{c_i}\right]\right\}$$
$$\text{subject to: } c_1 + c_2 + \ldots + c_N \leq C. \qquad (2)$$

where λ_i for $i = 1,\ldots, N$ are the forecasted arrival rates for the N precincts. The rates are proportional to forecasts of the precinct turnouts. Also, the particular structure of this formula suggests the solution would be the same if the λ_i were replaced with predicted turnout numbers.

Inspection of Figure 6(a) suggests the majority of precincts in the 2000 presidential election in Franklin County were allocated by solving the above formulation. "Active voters" refers to citizens believed likely to vote in the next election because of their voting in recent elections or from other indicators. In the 2000 election, Franklin County officials apparently used active voters in place of the predicted turnout and solved the formula, resulting in the vertical boundary at approximately 250 active voters per machine. A different approach was used for the 2004 election, as indicated by Figure 6(b). Evidence below suggests officials in 2004 might have used a similar approach, except they applied other estimates of turnout.

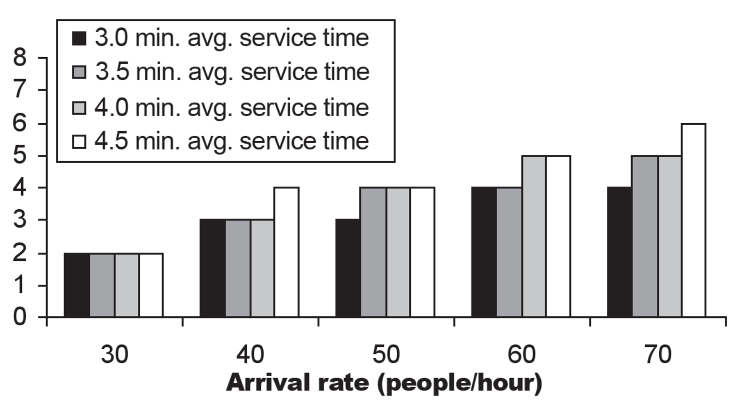


Figure 5. **Minimum numbers of machines (c_i) for less than 30 minutes average waiting time**

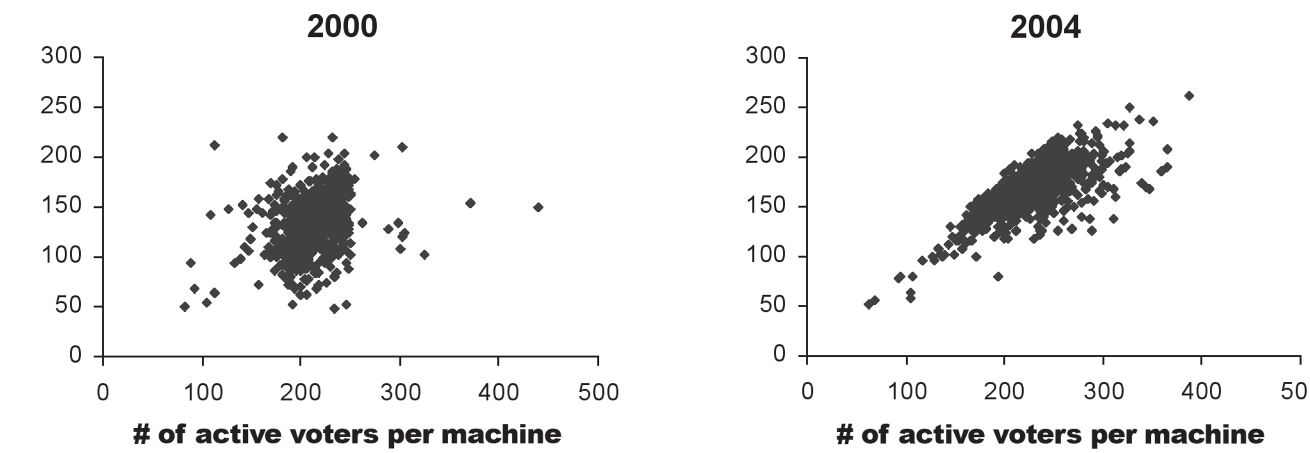


Figure 6. **(a) 2000 and (b) 2004 actual voters per machine vs. active voters per machine**

A simple way to solve Equation 2 is based on the following "greedy algorithm" that can be programmed and used to approximately solve allocation problems involving thousands of machines (i.e., C > 1,000). The greedy algorithm can be expected to provide solutions of reasonable quality, but guarantees of the quality of derived solutions have not been rigorously established.

Greedy Allocation Algorithm

Step 1. Set $c_1 = ... = c_N = 2$. Counter = 2 × *N*.

Step 2. $c_i = c_i + 1$ for the precinct *i* with the worst objective value (i.e., highest $\lambda_i \div c_i$).

Step 3. Counter = Counter + 1.

Step 4. If *Counter* = *C*, stop. The c_i now store numbers of machines roughly proportional to the number of active voters.

Step 5. Go to Step 2.

The obvious weakness of using allocations derived from solving this equation is that approaches based on the equation ignore issues related to service times. As a result, ballot length and other sources of service time variability likely contribute to waiting time variability in elections associated with these allocations. Also, without measuring the voting times and applying queuing theory, it is difficult to predict effectively average waiting times and provision machines.

Deterred Votes in Franklin County

The 2004 presidential election in Franklin County provides a case study for examining applications of queuing theory and the effects of approaches such as the one in Equation 2. That election offered a considerable challenge to election officials for several reasons, including:

- 25% more votes were cast in 2004 than in the 2000 presidential election.
- Numbers of active and registered voters also increased, but at a lower percentage making the actual turnout hard to predict.
- Laws requiring handicap access for future machines caused officials to perceive that any equipment purchased would be used one time only.

Table 1—The First Five Precincts from the Franklin County Board of Elections

	2000					2004							
Precinct	#Active (7/99)	# Reg.	# Mach.	# Ballots Cast	% Gore	# Active (11/04)	# Reg.	# Mach.	# Ballots Cast	# New Voter B. Cast	Hours Late	#+ Ballot Issues	% Kerry
COLS 01-A	972	1096	4	493	62.7%	1018	1412	4	692	91	1.28	10	70.4%
COLS 01-B	1019	1175	5	387	59.6%	1079	1620	3	560	118	2.02	9	69.3%
COLS 01-C	946	1145	4	556	67.0%	1048	1446	4	735	123	0.51	9	67.6%
COLS 02-A	843	976	4	362	73.6%	933	1319	3	502	110	2.16	9	81.8%
COLS 02-B	890	1019	4	534	65.0%	881	1237	3	659	127	1.14	9	70.5%

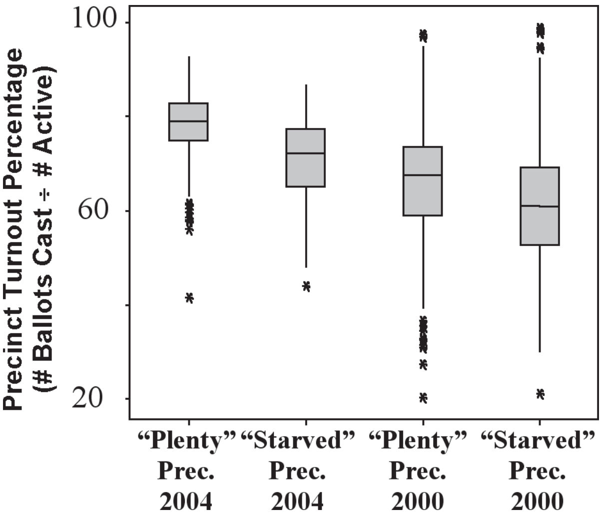


Figure 7. **Box and whisker plot showing turnout percentage variation**

- The state budget situation fostered the perception that one-time voting expenses for more voting machines could not be afforded. Therefore, the total number of machines, C, was constant, despite substantial increases in arrival rates.

As a result of this "perfect storm" of challenges, the average poll closing time of precincts was one "hours late," or around 8:30 p.m. in 2004. All registered voters waiting in line at 7:30 p.m. were permitted to wait and vote. These after-hours waits were reportedly symptomatic of waits throughout the day. It is perhaps obvious, therefore, that some people were deterred by these waits, which included times recorded as longer than five hours. The challenge explored next is to estimate the number of deterred votes.

Table 1 shows data from both the 2000 and 2004 elections, including hours late that the precincts stayed open and the numbers of "new votes" from first-time voters. A simple estimation procedure is based on the first two box plots in Figure 7. In box and whisker plots, horizontal lines show numbers larger than 25%, 50%, and 75% of the data and outliers. The estimation procedure proposed here is related to, but not the same as, the procedure in Elizabeth Liddle's 2005 article, "Votes Lost Due to Under-Provision of Voting Machines in Franklin County, Ohio." The term "plenty" refers to precincts with up to 228 active voters per machine in the 2004 election. Others are "starved" precincts.

It is evident from Figure 7 that the average turnout percentage is substantially higher for the plenty precincts (78.1%) than for the starved precincts (70.6%). It is tempting to conclude all this turnout reduction was caused by the lack of machines in the so-called starved precincts. If 7.5% of the active voters in the starved precincts chose not to vote because of the longer waits, the number of additional or deterred votes would be 21,231, or 4.5%, of the total number of voters. However, further inspection of Figure 7 shows that the same plenty precincts had natively higher turnout percentages in 2000. As no one alleges waiting lines influenced behavior in 2000, the difference in 2000 must have been caused by demographic factors. It seems likely that the natively lower turnout observed in 2000 caused decisionmakers to allocate fewer machines to the starved precincts. This effect is, therefore, confounded with causality running the other way (i.e., from reduced turnout in 2000 to fewer machines in 2004).

Liddle estimated 18,500 as the number of deterred votes using a regression model to compare the actual turnout with one predicted using the plenty precincts. She noted the phenomenon that lower turnout in 2000 correlated to lower turnout in 2004 on the precinct level. Yet, her regression model accounted for only the native precinct turnout variable indirectly by including voter preference. Also, her approach of regressing turnout per machine and then converting to absolute could result in an (unnecessary) error inflation in the final prediction.

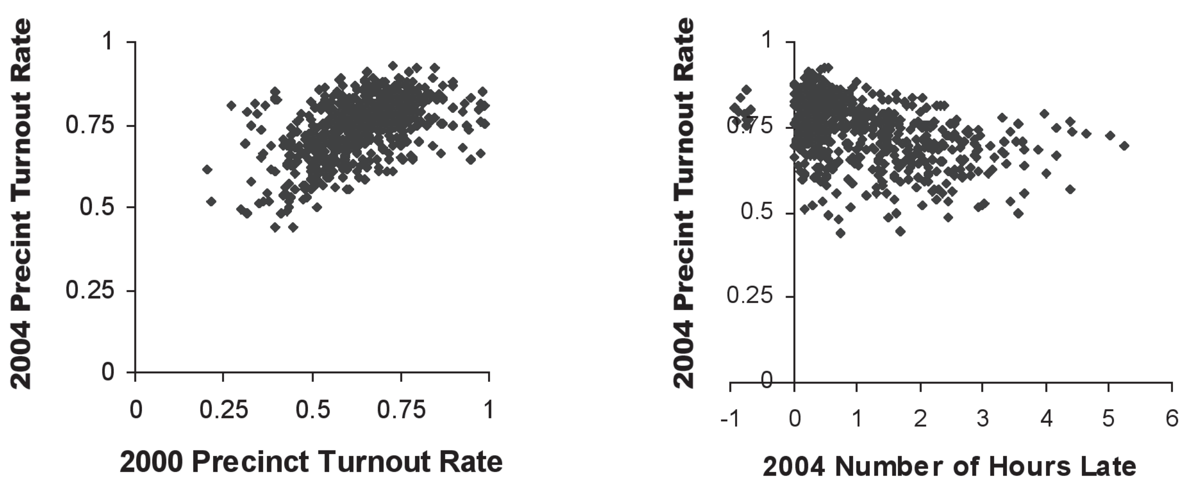


Figure 8. **Plots of the data associated with the regression model**

Table 2—Summary of Deterred Vote Estimates and Assumptions

Method	Method/Assumptions	Estimated % of Votes	Deterred Total #
“plenty” verses “starved” percentage comparison and projection	Average precinct turnout percentages would have been the same for “plenty” and “starved” precincts had the waiting times been small.	4.5%	21,231
“plenty” verses “starved” regression on voters per machine	Native turnout percentage differences can be accounted for by preference for Bush or Kerry.	4.0%	18,500
regression on 2004 turnout percentage as a function of 2000 turnout percentage and hours late	Precinct turnout rate would have been preserved on a percentage basis had there been minimal waits. Also, under usual circumstances, all polls would have closed at 7:30 p.m.	4.9%	23,445
assuming a 15-minute late closing time	Same as previous, except all polls close at 7:45 p.m.	4.0%	18,830

These issues motivate the development of a simpler estimation procedure using precinct turnout rate in 2000 and the extra time polls stayed open to predict turnout rate in 2004. The resulting regression model yielded an adjusted R^2 of prediction equal to 0.41, which might be regarded as acceptable for such a simple model. The prediction model is:

$$\begin{aligned}&(\text{2004 Precinct Turnout Rate})\\&= 0.57 - 0.031 \times (\text{Hours Late})\\&+ 0.32 \times (\text{2000 Precinct Turnout Rate}),\end{aligned} \quad (3)$$

where precinct turnout rate is the ratio of actual voters divided by active voters in that precinct. Note that eight precincts were removed before fitting to corresponding cases in which the number of actual voters was higher than the number of active voters. These omitted precincts corresponded to outliers on the probability plots and likely were caused by late registrants in 2000.

By setting the “Hours Late” in Equation 3, one can estimate the expected number of votes each precinct would have generated in counterfactual situations. For example, if all precincts had been able to close on time (Hours Late = 0), the predicted number of additional votes would have been 23,445. It might seem reasonable to consider a closing time 15 minutes late instead, which generates a deterred vote estimate of 18,830. This is typical of virtually all elections in Franklin County in recent memory. The simplicity and properties of the model in this equation lend it some level of credibility.

Table 2 summarizes the various deterred vote estimates. Standard regression plus or minus error estimates are purposely not included because random error variance for the mean are generally small compared with likely systematic errors due to the large number of data points. Quoting these small variances (all less than ±1,000) would be misleading. The uncertainty in all deterred vote numbers in Table 2 can subjectively be estimated in the thousands of votes.

More on Franklin County, Ohio

The queuing formula in Equation 1 and the prediction model in Equation 3 both offer insight into questions of possible interest:

1. Did the influx of new voters affect the turnout and the number of deterred voters?
2. How much could the addition of 80 machines have reduced the number of deterred votes predicted by the model associated with Equation 3?
3. In hindsight, how much money in additional equipment expenses would have been needed to reduce the number of deterred votes by 90% or more?

The first question can be investigated through adding “# new votes” as a possible independent variable in the regression on turnout percentage. The resulting model is:

$$\begin{aligned}&(\text{2004 precinct turnout percentage})\\&= 0.599 - 0.025 \times (\text{Hours late})\\&+ 0.313 \times (\text{2000 precinct turnout percentage})\\&- 0.00031\ (\text{\# Ballots cast by new voters}),\end{aligned} \quad (4)$$

which has an adjusted R^2 of prediction equal to 0.45. Equation 4 yields predictions of the lost votes that are not qualitatively different from those of Equation 3, beyond what is implied by the subjectively assessed error estimates. The model does, however, lend credence to the assumption that new voters are less likely to vote than other active voters and they can, therefore, reduce the turnout percentage.

The second question is relevant because the county held back roughly 80 machines that conceivably could have been allocated. The following estimate is based on the assumptions that a single machine to any precinct essentially would eliminate the number of deterred votes and the 80 worst precincts could have been identified with sufficient time to program and equip the machines. Note that some precincts had two fewer machines than our approach would suggest (e.g., Columbus precinct 50-C likely needed five machines instead of the allocated three).

Note that sorting the precincts by estimated numbers of deterred voters may identify the truly worst precincts and also result in an average for the sorted precincts far above the true number. This follows because random errors in the analysis would bias estimation, almost certainly resulting in an overestimate for the deterred votes in the selected precincts. As a result, predictions should be used with caution. However, the regression model predicts that the top 80 precincts account for 14,019 deterred votes, but possibly many fewer. Therefore, under the above assumptions, the 80 machines likely could have reduced the number of deterred voters by a substantial fraction, if they had been deployed with perfect hindsight.

Using the above described biased estimation procedure, one estimates that the top 200 precincts account for about 20,000 estimated deterred voters. Adding one machine to each of these precincts would cost about 200 × $5,000 = $1,000,000 in direct expenditures. Additional expenses would be incurred in transportation, storage, and operation. With these costs and assuming perfect allocation, the waiting times and number of deterred voters likely would have been greatly reduced. However, without perfect foreknowledge, costs easily could have been much higher. Also, the calculation assumes space to operate these machines was available in the relevant precincts.

Ballot Length and Waiting Lines

It is our perception that virtually all current machine allocation methods across the United States fail to account for variable service times, including those caused by variable ballot lengths. Figure 9 shows "Hours Late" representing the closing time of individual precincts in Franklin County during the 2004 presidential election versus the number of extra ballot initiatives. The R^2 of prediction for the first-order trend model is 0.28 indicating a substantial fraction of the observed variation in closing times. It is explained by the number of ballot initiatives and, thus, the ballot length. The second model has R^2 of prediction equal to 0.55, and its predictions are given by the fitted model:

$$\begin{aligned}(2004\ \#\ \text{Hours Late}) = &-0.22 + 0.0024 \times (\#\ \text{Active Voters Per Machine}) \\ &- 0.2636 \times (\#\ \text{Extra Ballot Initiatives}) \\ &- 0.0016\ (\#\ \text{Active Voters Per Machine}) \times \\ &- (\#\ \text{Extra Ballot Initiatives}) \end{aligned} \tag{5}$$

This fitted model gives qualitatively similar predictions for the "# Hours Late" that the approximate queuing theory model in Equation 1 gives for the average waiting time. Our conclusion is that the longer ballots in certain precincts caused longer waits and, thus, probably deterred some people from voting. Also, it seems extremely likely that ballot lengths were allowed to cause these waits because little or no provision for them was made in voting machine allocation.

In addition to causing longer waits, there is evidence that longer ballots deterred certain types of voters from voting

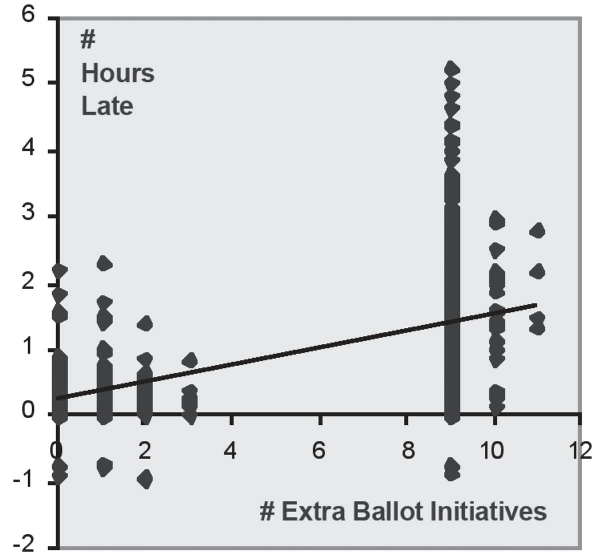


Figure 9. Actual lateness of closing times

more than others. We do not have demographic information about precincts in Franklin County pertinent to the 2004 presidential election. However, the sample correlation between ballot lengths and the percentage of Kerry voters in the precincts in that election was 0.56. This means a high percentage of votes cast in precincts with longer ballots went to Kerry. It might seem reasonable to assume the majority of deterred voters, therefore, would have preferred Kerry, as noted by Liddle. Some or all of the disparities in the waiting times associated with African Americans and other ethnic groups also might ave been explained by variations in ballot lengths.

Note that, in Franklin County, the longer ballots occurred in Columbus precincts because the city added six issues to its ballots. Yet, even if 100% of 20,000 deterred voters would have preferred Kerry and similar dynamics occurred in Cleveland and Toledo, it would almost surely not have affected the election results. Our purpose here is to point out that failure to allocate machines using queuing theory can result in long waits and disproportionate affects to certain types of voters, not to revisit election results from 2004.

Proposed Method

The proposed service and arrivals automatic generation (SAG) allocation method incorporates forecasts of turnout pertinent to the arrival process and time estimates of the voting process relevant to service by the voting machines.

As in previous sections, consider the following optimization problem relevant for the selection of the numbers of machines allocated to N precincts, $c_1, c_2, \ldots, c_N$. The assumed arrival rates for the N precincts are $\lambda_1, \lambda_2, \ldots, \lambda_N$. A simple but potentially reasonable approach for estimating these rates could be to use the number of active voters in the precinct divided by the number of hours the polls are open. Following this approach, we recommend including

Table 3—Comparison of Voters per Machine (VPM) and SAG Allocation Approaches

								Actual		VPM		SAG	
Precinct Common Name	# Active Voters	# Ballots Cast	Hours Late	#+ Ballot Issues	λ_i	μ_i	#Active ÷ # Ballots (C = 18)	# Mach.	Avg. Waiting Time (Min.)	# Mach.	Avg. Waiting Time (Min.)	# Mach.	Avg. Waiting Time (Min.)
JACKSON-C	597	553	0.50	9	34.0	13.8	199.0	3	5.5	3	5.5	3	5.5
COLS 01-A	1018	692	1.47	10	57.9	13.3	254.5	4	∞	5	4.8	5	4.8
COLS 73-D	775	596	1.27	9	44.1	13.8	258.3	3	∞	4	3.2	4	3.2
COLS 45-G	1224	860	1.38	10	69.7	13.3	244.8	5	∞	5	∞	6	3.9
REYNS 4-C	791	603	0.60	0	45.0	20.0	263.7	3	2.3	4	0.4	3	2.3

a safety factor of 1.2 (corresponding to a 20% voter arrival increase) or more to account for peak periods and the possibility of higher than usual voter turnout. This gives rise to the prediction formula:

$$\lambda_i = \frac{\left[\begin{array}{l}(\text{safety factor}) \times (\text{historical turnout fraction}) \\ \times (\#\text{ active voters in precint } i)\end{array}\right]}{(\#\text{ hours precint } i \text{ is scheduled to be open})}$$

$$\text{for } i = 1, \ldots, N, \tag{6}$$

where the "safety factor" could be 1.2 or higher. This equation admittedly does not account for the potentially important effects of variable voter turnout percentages and new voters. Previously, we identified turnout variation and new voters as potentially important in relation to estimations of the numbers of deterred voters. Therefore, Equation 6 can be regarded as a relatively simple, preliminary way to estimate precinct arrival rates.

The proposed method is based on the assumption that average voting times for all precincts, $s_1, \ldots, s_N$, are available and at least roughly proportional to ballot service time for each of the precincts. For example, these might be the number of extra ballot initiatives or the total word lengths of the ballots. Assume precincts are sorted such that $s_1 \leq s_2 \ldots \leq s_N$.

SAG Allocation Method

Step 1. Estimate or collect numbers proportional to ballot length in each precinct, $s_1, \ldots, s_N$.

Step 2. Select $n_1 + n_2$ voters using a random sampling from active voter lists. Measure n_1 voting times for the shortest and measure n_2 voting times for the longest precinct, using a distinct voter for each measurement. By default, use $n_1 = n_2 = 20$ people.

Step 3. Compute estimated average service times for all precincts (μ_i's) using the following formula estimation formula:

$$\mu_i = \{\mu_1^{-1} + [(s_i - s_1)/(s_N - s_1)](\mu_N^{-1} - \mu_1^{-1})\}^{-1} \quad \text{for } i = 1, \ldots, N \tag{7}$$

Step 4. Determine $c_1, c_2, \ldots, c_N$ by solving:

$$\mathit{Minimize}\{\mathit{Maximum}[\text{Average waiting time}(c_i, \lambda_i, \mu_i)]\} \quad \text{subject to: } c_1 + c_2 + \ldots + c_N \leq C, \tag{8}$$

where the average waiting time in Equation 8 could be estimated using Equation 1.

Note that we have used a so-called minimax formulation because it might seem subjectively the fairest, guaranteeing that no precinct is permitted much longer predicted average waits than others. Also note that if a greedy method is used to solve the formulation in Step 4, there might be many iterations in which predicted average waiting times are infinite. For these cases, it might make sense to allocate machines to precincts with the highest $\rho_i = \lambda_i \div (\mu_i \times c_i)$ until all precincts have finite estimate average waits or the procedure terminates.

Numerical Example

Consider an allocation problem based on the 2004 Franklin County experience. In that election, the 74% active voter turnout could have been higher if deterred voters were included. Yet, this turnout is already high compared with past elections. It was 12% higher than in 2000, which was also a historically high turnout. Therefore, in Equation 6, we assume that (safety factor) × (historical turnout) = 0.74. Table 3 shows data from five arbitrarily selected precincts. We very roughly estimate average voting times once there was access to the booths were three minutes for short ballots and 4.5 minutes for long ballots. At present, we have no time data that could be used directly to estimate these service rates, so we are forced to conjecture that results of the SAG method Step 1 would give $\mu_1 = 3 \div 60 = 0.05$ per hour and $\mu_5 = 4.5 \div 60 = 0.075$.

Table 3 shows the values of μ_2, μ_3, and μ_4 calculated using Equation 7 in Step 2. Under these assumptions, it can be checked that there was no possible way to allocate 18 machines such that Equation 1 would yield finite average waiting times (i.e., at least one precinct would need to be overloaded). Note that the precincts predicted by Equation 1 to experience long waits did, in fact, have long waits, as evidenced by the hours late that the polls stayed open. If three additional machines had been made available, however, the theory suggests long waits could have been avoided.

Using the voters per machine allocation method described in Equation 2 and the greedy allocation method, the 19th machine will be allocated to the "REYNS 4-C" precinct and a final allocation in which "COLS 45-G" precinct is in an overload condition. By comparison, Step 4 of the SAG method results in an allocation in which no precinct is in the overload condition, potentially saving multiple hours of waiting time for voters. Also, the voters per machine method allocation provides a machine to the "REYNS 4-C" precinct that does relatively little to improve system performance. Assuming 2,500 machines each cost \$5,000, a 5% allocation of unneeded machines would cost counties in excess of \$600,000 in direct equipment expenditures. This example shows that allocation accounting for the simultaneous effects of turnout and service time variation can dramatically reduce average waiting times and avoid unnecessary capital expenditures simultaneously.

It seems clear that the relative benefits of SAG improve as the diversity of the service times increases. Therefore, the application of SAG or similar approaches is probably most relevant in counties that include both urban and rural precincts, such as Franklin County, Ohio, which includes downtown Columbus. Using SAG, unnecessary expenditures in precincts with short ballots can be avoided and long waits in precincts with extra ballot issues can be mitigated.

Conclusions and Future Work

We conclude that queuing theory is relevant to the allocation of voting machines. Even theories based on simplifying assumptions, such as the *M/M/c* queue, can result in useful recommendations to voting officials. We provide figures such officials might find helpful in making actual decisions about allocations. We also provide evidence that voters were deterred from voting in a real election because of allocations that did not account for the ballot length variability. The proposed SAG method is proposed and illustrated using an example from the 2004 election in Franklin County. The SAG method is demonstrated to offer potential benefits, both through the reduction of average waiting times and the avoidance of unnecessary expenditures on voting equipment.

In general, we consider this work as preliminary (i.e., introducing concepts from management science into the study of voting systems). An incomplete list of additional topics for further study is as follows:

- Because of the sensitive nature of waiting times to service times, it might be important to identify additional best practices for preparing voters before they enter the booth and for managing the lines. Practices that effectively lengthen the service time by making poll worker operations part of the bottleneck on precinct capacity should be avoided.
- Extensions beyond the steady state theory of queues could be helpful for cases in which the available number of machines does not permit all precincts to operate out of overload conditions. Accurate estimates of waiting times could be helpful for equipment purchasing decisions relevant to "worst case" turnout and ballot length–related assumptions.
- Simulation models based on more realistic assumptions than Poisson arrivals with constant rates and exponential service times can be explored. Meta-models constructed based on these simulations could be used in optimal machine allocation to improve computational efficiency.
- More accurate forecast models than Equation 8 could be developed. These models could account for variable conditions around the United States and other countries, the numbers of new voters in each precinct, and demographic factors.
- Commercial software could be developed based on the SAG and improved methods. Such software could help election officials on fixed budgets make actual allocation decisions to reduce unnecessary expenditures and mitigate waiting lines.

Acknowledgments

Fritz Scheuren provided many forms of support and mentorship for this work, including editing help. We deeply thank him. Matthew Damschroder, Karen Cotton, and Michael Hackett of the Franklin County Board of Elections provided data, insight, and many forms of support for this and related project work. Donald Spicer provided valuable insight, helping us interpret data and analysis results. Finally, Steven Hertzberg and Election Science Institute commissioned this work and offered valuable discussions.

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Comment: Queuing To Vote in Franklin County, Ohio, in 2004

Jasjeet S. Sekhon

Theodore Allen and Mikhail Bernshteyn's application of queuing theory to the problem of voting machine allocation is a welcome contribution to the growing field of election administration. Their analysis clarifies many issues and yields substantively important results. The attention they give to the issue of service times—for example, the importance of considering varying ballot lengths when allocating voting machines—is just the kind of actionable advice election administrators need.

The importance of this kind of study was made clear to me at a meeting of the Democratic National Committee's (DNC) Voting Rights Institute in early 2005. There was agreement in the room that the underallocation of voting machines was a serious problem in Franklin County in 2004, particularly for African-American voters. However, it was consternating when it became apparent that the DNC task force, which included political operatives who helped organize the get-out-the-vote effort in Ohio, simply had no idea how voting machines were allocated in Ohio in 2004 or in previous elections, such as in 2000. DNC political organizers were simply outraged ex post that a good job of allocating voting machines was not done. They paid little attention to the issue before election day.

There is one section of the paper, however, about which I do have concerns: the calculation of deterred votes in Franklin County. The authors accurately note that one cannot simply regress voter turnout in 2004 on poll closing times in 2004 in an effort to estimate the number of deterred voters. This is because voting machines in 2004 were allocated in part based on precinct-level turnout in 2000, and baseline characteristics of precincts probably determined the varying turnout rates observed in 2000. Therefore, the authors regress 2004 precinct-level turnout on 2000 precinct-level turnout and 2004 poll closing times.

I wish to discuss three problems that undermine the model: one methodological and two data. First, and most seriously, the same covariates that led to varying baseline turnout in 2000 could have led to differential mobilization changes between 2000 and 2004. This opinion was later confirmed by a rigorous study done by Walter Mebane and Michael Herron and written about in "Ohio 2004 Election: Turnout, Residual Votes, and Votes in Precincts and Wards." There are also statistical issues related to estimating an OLS regression with proportions data, but I will not discuss those here. There are baseline covariates, such as education, that correlate with different likelihoods of being mobilized by political appeals, such as the unusually intense efforts in 2004 (Verba, Schlozman, and Brady, 1995). In other words, when political mobilization increases, it does not do so uniformly across people. Thus, there is no reason for us to expect that, aside from voting times, voter turnout rates should have had a uniform change across precincts. And the factors with which voters' susceptibility to get-out-the-vote efforts varies, such as race and education, also are correlated with the allocation of voting machines.

An example of the problem at hand can be offered by examining the different mobilization tactics that Republican and Democratic get-out-the-vote organizations followed. These differing tactics highlight just how odd it would be if turnout did swing in a uniform fashion. The Republicans relied on volunteers more than the Democrats, who used many volunteers and (lowly) paid organizers. Evidence in *Get Out The Vote! How to Increase Voter Turnout* by *Donald Green and Alan Gerber* shows that volunteers (such as members of church groups calling churchgoers) perform better at getting the vote out than poorly paid organizers. Moreover, the Republican get-out-the-vote organization in Ohio focused, more than the Democratic one, on lapsed voters (i.e., voters with a history of voting, but who have not done so in the past couple of elections). On the other hand, Democrats, more than Republicans, focused on voters who had no history of voting, even if they were registered. Democrats probably registered more new voters than did Republicans in Franklin County (they did so nationally). One consequence of these varying strategies was that Republicans had higher conversion rates (i.e., a larger percentage of contacted lapsed voters were turned into voters than newly registered or previously registered voters without a voting history). My guess is that there were more of the former in white precincts and more of the latter in black precincts. And these racial variables correlate with both waiting times and machine allocations. Democrats were following a strategy that would, relative to the Republican strategy, appear to reduce turnout rates but increase registration rates. Of course, turnout rates were up overall, but these differing strategies play havoc with the assumptions required to interpret the estimates of the regression model as causal estimates. And the fact that the authors estimate a regression with the number of ballots cast by new voters as a covariate does not solve the problem, in part because it does not take into account the more general effects of the different mobilization strategies and kinds of voters at which they were aimed.

Second, Ohio, like all states, underwent a redistricting between 2000 and 2004, and precinct boundaries were redrawn in Franklin County. Franklin County underwent a more extensive redistricting than some other counties because it increased substantially in population between 1990 and 2000. It was the only urban county in Ohio to experience double-digit growth. Because of the redistricting, Mebane and Herron note they were unable to conduct a precinct-level analysis in Franklin County using pre-2004 data. They instead moved up to the ward level in an effort to find consistent geographies. Thus, although precincts may retain the same name or number, one needs to be careful. It would probably be best to simply move up to a stable geography, such as ward. Alternatively, one could restrict analysis to only precincts that have not changed or undergone changes of the kind for which we could reasonably impute the 2000 voting history of the new precinct—such as may be the case for a relatively homogeneous precinct divided into two.

Third, the denominator in the authors' turnout equations—voter registration—is a notoriously noisy measure whose particular biases change from election to election. But in Franklin County between 2000 and 2004, the measure seems to be especially problematic because an unusually large number of inactive voters were still registered in 2004 due to a data issue that occurred in 1999. This unusual state of affairs may be best explained by Franklin County Board of Elections Director Matthew Damschroder:

> In 1999, the Board of Elections changed voter registration systems (database software application) and, in the transition, lost all of its electronic data proving that the board had followed the law and issued the confirmation notices. Therefore, the board chose to reset all inactive voters to active status and began the National Voter Registration Action (NVRA) of 1993 process over. It is hard to determine precisely how many inactive voters in 2004 should have been purged from the rolls in 2001 and 2003, but the bottom line is that the 2004 rolls were bloated due to the abnormally high number of inactive voters. Note here that not all inactive voters were then purged in 2005 (because not all had reached the fullness of their NVRA timetable). Once a voter is purged (canceled) the voter no longer appears on the rolls of the election jurisdiction as a registered voter.

Thus, the population to which we are making an inference is changing between 2000 and 2004 because of both redistricting and because of a changing rule about who should be purged from the voting rolls. Because of problems like this write Michael McDonald and Samuel Popkin in "The Myth of the Vanishing Voter," people often advocate using citizen-voting-age-population, instead of registered voters, to estimate turnout rates. Of course, such measures are difficult to obtain at the precinct level in states where census and political geographies do not line up nicely.

Notwithstanding these concerns, I am hopeful that this article will be widely read, especially by election administrators, and I will be doing my best to make certain it is. I look forward to reading subsequent work on this issue by the authors. As Allen and Bernshteyn note, their current queuing model is preliminary. For example, they acknowledge that the assumption of Poisson arrival times is unrealistic because real arrival times are clustered at certain times of day (such as before and after work) and are overdispersed relative to the Poisson. The authors are on top of this issue and already planning extensions.

A purge was done in 1999 in preparation for the 2000 election. A changing population is always a problem with ecological data of this variety because voters move around. But the situation in Franklin County appears to be even worse than the usual case.

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Authors' Response

We thank Jasjeet Sekhon for his careful attention and constructive criticisms. We agree that the strengths of our paper are the evidence provided that variable ballot lengths played an important role in 2004 election waiting times and our proposed method to mitigate the waiting problems in future elections.

Estimating the number of deterred voters in Franklin County during 2004 is inherently difficult. Yet, this estimation is important partly because it relates closely to methods to forecast turnout in future elections. The chief virtues of our estimation approach, as noted by Sekhon, are that it addresses the issue of differential turnout rates—presumably relating to demographics—and that it is simple.

We propose the development of a defensible turnout forecasting approach that addresses the issues noted by Sekhon (i.e., shifting mobilization, redistricting, and registration issues) is an important topic for future research. The forecasts generated can be applied retroactively to estimate the number deterred in the Franklin County November 2004 election by comparing the forecasted turnout with the actual number who voted. Probably more important is the fact that improved forecasting methods could aid in future decisions about how many machines are needed and how they should be allocated to precincts.