A New Approach to Cell Loss Analysis for Long-Range Dependent Network Traffic

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SUMMARY A new expression for cell/packet loss probability in an ATM and packet switched queue system with a finite buffer is presented. Cell and packet loss analysis is based on the new concept of a “buffer overflow cluster” and the overflow probability for a queue with an infinite buffer. This approach holds for a wide variety of long-range dependent traffic sources typical of wide-area networks, as well as Internet and other communication traffics. The method is verified by simulations of two long-range dependent traffic scenarios: fractional Gaussian noise and multifractal wavelet model traffic with a beta marginal distribution.

key words: cell loss, buffer overflow, network QoS, traffic prediction, long-range dependent traffic, multifractal wavelet model

1. Introduction

Broadband telecommunications based on asynchronous transfer mode (ATM) switching as well as packet switching in general allow for the provision of services satisfying very stringent quality of service (QoS) requirements while at the same time making efficient use of available resources through statistical multiplexing. Available resources include bandwidth, which must be allocated in consideration of the variability of traffic, and buffer space, which is provided to handle the bursty nature of traffic sources. Bandwidth and buffer space are typically dimensioned so that overflow rarely occurs. Overflow is directly related to the QoS measure of cell loss probability (CLP) or packet loss probability (PLP). This has led to recent research activities related to the computation of CLP/PLP and the tail of the queue length distributions. In the following discussions, only the terminology of CLP is used for both CLP and PLP for the sake of simplicity.

Real measurements in local and wide area networks, World Wide Web networks, variable bit rate video sources, and other communication systems have revealed that traffic demonstrates properties of long-range dependence [6]–[9]. Notably, measured traffic loads exhibit fractal or self-similar behavior in that the traffic appears statistically similar (highly variable) on all time scales. These features appear as burstiness in traffic streams across an extremely wide range of time scales. Long-range dependent (LRD) traffic dramatically influences the performance of system buffers in high-speed networks. Actual traffic measurements and analysis in subsequent papers have shown that the overall cell loss and buffer overflow in LRD traffic models decreases very slowly with increasing buffer size in contrast to classical short-range dependent traffic models such as homogeneous Poisson or Markov models where losses decrease exponentially with increasing buffer size. This implies that long-range dependence affects cell loss and delay and hence must be taken into account carefully when designing networks, providing QoS guarantees, etc.

Although buffer overflow has been studied extensively with regard to a range of traffic models, results for cell loss calculation exist only for a limited number of traffic models, for examples, LRD homogeneous M/G/∞ traffic models [2], [3], and heterogeneous G/G/∞ long-tailed sources [4], [5]. This is because cell loss probabilities in finite queues are often hard or impossible to analyze in an exact analytical way. For other input traffic models, the common approach has been to approximate cell loss probability by buffer overflow probability (BOP), i.e., the probability of exceeding queue levels in an infinite buffer. However, such an approximation is attractive since infinite queues are often amenable to exact analytical solutions. The inaccuracy of this CLP approximation was presented in [1] and will be shown in this paper from the queue performance of real and synthesized traces. Another approach to assess the CLP directly could be more appropriate and accurate as long as it is possible to analyze the CLP mechanism. In this paper, we propose a new approach for determining the CLP based on the analysis of queue performance of real and synthesized traces, employing the new concept of an “overflow cluster” in infinite buffer systems. The proposed method is generally applicable to all types of LRD network traffic and is applied as an example in this study to fractional Gaussian noise (FGN) and the multifractal wavelet model (MWM) as common network traffic models.

This paper is organized as follows. In Sect. 2, the concept of an overflow cluster is defined and used to calculate the BOP and approximate the CLP. Next, the new technique for CLP calculation based on overflow cluster analysis is presented in Sect. 3. The LRD network traffic models, namely, FGN and MWM with a beta distribution marginal are introduced in Sect. 4. The accuracy of the proposed analysis is confirmed through numerical validation by simulation in Sect. 5. Finally, conclusions and future research are
presented in Sect. 6. Additionally, a list of abbreviations is presented in Appendix C; a list of symbols and variables is presented in Appendix D.

2. Overflow Cluster and Cell Loss Probability

The idea of using an overflow cluster to analyze the cell loss performance of network traffic was first introduced by the present authors [16]. Consider a system with an infinite buffer that is accessed by traffic according to a random process, and which is served by a constant transmission link. It is assumed that the time is discrete or slotted. Each time slot represents a time interval with fixed length. At any time slot, a number of bytes \( X_t \) arrive into the buffer, and the buffer is drained at a constant link capacity of \( C \) bytes. It is assumed that the model of input traffic \( X_t \) is given. The arrival process is assumed to be wide-sense stationary, i.e., the mean \( \mu \), the variance \( \sigma^2 \) and the auto-covariance function \( r(k) = \text{cov}(X_t, X_{t+k}) \) of \( X_t \) are independent of time \( t \).

Furthermore, the input traffic process \( X_t \) has the mean \( \mu < C \) such that the system is stable, thus allowing the state of the system to be considered at any arbitrary time instant. This condition guarantees the existence of a stationary work load and queue length processes. Let \( A_t \), where \( t \geq 0 \), be the amount of traffic arriving at the system in the interval \((0, t)\), and let \( Q_t \) be the work remaining in the buffer at time \( t \). In queue terms, \( Q_t \) is also the queue length in the buffer at time \( t \). The amount of unfinished work \( Q_t \) at time \( t \) stored in the buffer and is defined by Lindley’s equation as follows

\[
Q_t = \max\{Q_{t-1} + X_t - C, 0\} \tag{1}
\]

This buffer occupancy can also be given by Reich’s formula as

\[
Q_t = \sup_{u \geq t} (A_t - A_u - C(t-u)) \tag{2}
\]

We then consider the buffer overflow state of the system. In the infinite buffer system, buffer overflow occurs when the queue exceeds a prescribed buffer level \( B \). Thus, the buffer overflow probability is defined as

\[
P(\text{Overflow}) = P(Q_t > B) \tag{3}
\]

It is then possible to introduce the concept of a buffer “overflow cluster,” as described in Fig. 1. A time interval in which the buffer is always non-empty and buffer overflow occurs instantly is defined as an “overflow period.” In other words, the overflow period is a busy period during which the overflow occurs. The overflow period length is defined as the time from when the buffer becomes non-empty to the time it becomes empty again. The part of a queue continuously exceeding the buffer level \( B \) then represents an “overflow cluster” (OC). The interval of time during which an overflow cluster is present is called the “overflow cluster length” (OCL), and maximum amount of queue exceeding the buffer level \( B \) in an OCL is called the “overflow cluster peak” (OCP). During the overflow period the overflow cluster with the highest overflow cluster peak is called the “main overflow cluster” (MOC), having its own length (MOCL) and peak (MOCP). Therefore, there is only one main overflow cluster in every overflow period. Assuming that the input traffic has length of \( N \rightarrow \infty \) time slot units, the BOP can be written in terms of OCL as

\[
P(\text{Overflow}) = \lim_{N \to \infty} \frac{\sum \text{OverflowClusterLength}}{N} \tag{4}
\]

Because cell loss only occurs in systems with a finite buffer size, we consider the queue in a system with buffer size \( B \), loaded by the same input traffic process as a system with an infinite buffer. It is assumed that both systems have the same buffer initial condition. Let the queue length in the infinite buffer system be denoted \( Q_n \), and the queue length in the finite buffer system be \( Q_t^* \). To aid in understanding the concept of the overflow cluster as it applies to cell loss approximation, the simple typical cell loss and buffer overflow performance of the system in Fig. 1 is examined. In the starting interval of the overflow period, the queue occupancies \( Q_t \) and \( Q_t^* \) are the same up to the threshold \( B \), above which overflow occurs. The cell loss from the finite buffer system associated with overflow cluster 1 will be of size \( \text{OverflowClusterPeak1} \) (the difference between the buffer occupancy of the OCP and the buffer level \( B \)). After the queue \( Q_t \) passes the peak of overflow cluster 1, both queues \( Q_t^* \) shrink in parallel, with \( Q_t - Q_t^* = \text{OverflowClusterPeak1} \), until \( Q_t^* \) returns to 0. While \( Q_t^* \) is at 0, \( Q_t \) continues to shrink until \( Q_t \) reaches a minimum value of \( Queue1 \). When \( Q_t \) and \( Q_t^* \) expand again, the difference between them will be \( Q_t - Q_t^* = Queue1 \). If the value of \( \text{OverflowClusterPeak2} \) is sufficiently small \( (\text{OverflowClusterPeak2} \leq Queue1) \), then the cell loss in \( Q_t^* \) will not occur in overflow cluster 2 even if overflow of \( Q_t \) occurs. The volume of cell loss associated with the main overflow cluster will then be

![Fig 1](image-url)
Fig. 2  Cell loss and overflow performance of Bellcore 1994 LBL-TCP-3 WAN traffic. Link utilization is 80%.

MainOverflowClusterPeak - Queue1. Similarly, as we can see in Fig. 1, the cell loss associated with overflow cluster 3 will be OverflowClusterPeak3 - Queue3, and no cell loss will occur in the next overflow cluster. The total volume of cell loss in the overflow period in Fig. 1 is then calculated as follows

\[
\text{CellLoss} = \text{MainOverflowClusterPeak} + (\text{OverflowClusterPeak1} - \text{Queue1}) + (\text{OverflowClusterPeak3} - \text{Queue3})
\]

It is clear that the volume of cell loss in the overflow period is never less than MainOverflowClusterPeak. Note that if the buffer level \( B \) increases, then OverflowClusterPeak1 and OverflowClusterPeak3 decrease. In fact, Queue1 and Queue3 do not change as the buffer level \( B \) is increased and the elements (OverflowClusterPeak1 - Queue1) and (OverflowClusterPeak3 - Queue3) moves closer to zero. This means that the volume of cell loss can be approximated by the MOCP given a large buffer size \( B \).

Although the scenario of cell loss performance for actual network traffic will differ from that described in Fig. 1, it is expected that the cell loss can be derived from the MOCP and an additional element that converges to zero with increasing buffer size \( B \). Therefore, given a sufficiently large buffer, the CLP can be estimated by its lower bound as follows

\[
P(\text{CellLoss}) \approx \lim_{N \to \infty} \frac{\sum \text{MainOverflowClusterPeak}}{\mu N}
\]

Simulations of cell loss and overflow probability for real traffic [33], FGN traffic [12], and MWM with stable marginal distribution traffic [14], [15], [24] are illustrated in Figs. 2, 3, and 4, respectively. The parameters of the FGN traffic were chosen for Ethernet traffic data from August 1989, while parameters of the MWM with stable marginal distribution traffic were chosen from LBL-PKT-4 WAN traffic data (both available in [33]). The simulation results confirm the accuracy of cell loss approximation by Eq. (5). It can be seen that when the buffer is sufficiently large, the error of cell loss approximation is almost zero for synthesized traces, and equal to zero for a real TCP-LBL-3 WAN trace. We also note that the numerical results of CLP and BOP are different; therefore, it is not possible to approximate one from the other.

3. Cell Loss Calculation

Assumed that BOP is given, in this section we propose a new method for cell loss calculation based on the relationship between cell loss and buffer overflow. The results in Sect. 2 showed that the BOP is defined by the sum of all OCLs, while CLP is well approximated by sum of all MOCPs. Therefore, the relationship between the sum of all OCLs and the sum of all MOCPs will allow the CLP to be determined from the BOP.
3.1 Approximation of Total Overflow Cluster Length

In this subsection, we estimate the total overflow cluster length (TOCL) in one overflow period by its MOCL. We suppose that a single overflow period starts at time zero and let \( T_p \) be the time instant at which the MOC reaches the peak. Consider the time instants at which the queue passes the buffer level \( B \): these instants are referred to as start points and end points of overflow clusters. They are denoted by \( T_1^{2k-1} \) and \( T_2^{2k} \), \( k = 1, 2, ..., K \), in the left-hand side of the MOC, and are \( T_{f}^{2l-1} \) and \( T_{e}^{2l} \), \( l = 1, 2, ..., L \), in the right-hand side of the MOC, as described in Fig. 5. Here, \( K \) is the number of overflow clusters on the left of the MOC, and \( L \) is the number of overflow clusters on the right of the MOC. In other words, \( T_1^{2k-1}, T_2^{2k} \) are points in the buffer MOC formation process, and \( T_{f}^{2l-1}, T_{e}^{2l} \) are points in the buffer MOC erosion process. The intervals \( (T_1^{2k-1} - T_2^{2k}), k = 1, 2, ..., K \), and \( (T_{f}^{2l-1} - T_{e}^{2l}), l = 1, 2, ..., L \), are called the underflow interval in the overflow period. From the definitions of \( T_1^{2k-1}, T_2^{2k} \) and \( T_{f}^{2l-1}, T_{e}^{2l} \) and from Fig. 5, the summation of OCLs in one overflow period is calculated by

\[
T_{TOCL} = T_{MOCL} + \sum (T_1^{2k-1} - T_2^{2k}) + \sum (T_{f}^{2l-1} - T_{e}^{2l})
\]  

(6)

where \( T_{TOCL} \) is the TOCL in an overflow period and \( T_{MOCL} \) is the MOCL.

Observations of the behavior of the queue system for synthesized LRD network traffic show that in the overflow period, the average of \( (T_1^{2k} - T_2^{2k-1}), k = 1, 2, ..., K \), and \( (T_{f}^{2l} - T_{e}^{2l-1}), l = 1, 2, ..., L \), i.e., the average length of OCLs except the MOC, is much smaller than the average length of the MOC. It almost equals the average length of the underflow interval. This is because burstiness in LRD traffic streams appears across an extremely wide range of time scales. In the overflow period the burstiness of traffic at a large time scale makes the queue overflow and formulates the MOC. On the other hand, the burstiness of traffic at smaller time scale makes the queue fluctuate around buffer level \( B \) and hence formulates the other OCLs. The ratios of the average length of OCLs except the MOC and the average underflow interval length to MOCL are demonstrated in Figs. 6 and 7 for FGN and MWM traffic models, respectively. While the number of OCLs in one overflow period is small for a wide range of buffer size \( B \) (about 2 – 5 for a FGN traffic model and 4 – 8 for a MWM traffic model as seen in Figs. 8 and 9 of Sect. 3.3) the average of \( (T_1^{2k+1} - T_2^{2k}) + (T_{f}^{2l+1} - T_{e}^{2l}) \) is also small in comparison to the average MOCL. This means that the expected value of the TOCL can be approximated by its lower bound as

\[
E[T_{TOCL}] \approx E[T_{MOCL}]
\]  

(7)

For simplicity denote \( T_1^{2k} \) as \( T_f \) and \( T_2^{2k} \) as \( T_e \). Let \( T_{pf} = T_p - T_f \) and \( T_{pe} = T_e - T_p \) we can estimate \( E[T_{TOCL}] \) as

\[
E[T_{TOCL}] \approx E[T_{pf}] + E[T_{pe}]
\]  

(8)

The accuracy of the TOCL approximation (8) is verified through simulation in Figs. 6 and 7 for FGN and MWM traffics, respectively. The accuracy of the TOCL approximation
is valuated by ratio $\log E[T_{MOCP}] / E[T_{TOCL}]$.

3.2 Relationship between TOCL and MOCP in One Overflow Period

Now we establish the relationship between TOCL and MOCP. To do this, denote MOCP by $B_{MOCP}$ and fix its value, then perform analysis only for those overflow clusters with the same $B_{MOCP}$. Generally, investigating the distribution of the random variable $T_{pf}$, $T_{pe}$ is difficult. However, noting that an overflow period appears with very low probability and applying the well-known principle “rare events occur in the most likely way” in the buffer overflow analysis, we believe that $T_f$, $T_{pf}$, $T_{pe}$ can be determined over a relatively small interval around the typical values. This translates as the supposition that the queue is unlikely to ever reach a given level $B$, if the queue is conditioned to pass $B$ and reach $B + B_{MOCP}$, then we will do so at the time when this is most likely to occur. For details of this argument, refer to [17]–[20]. In a standard analysis $T_f$, $T_{pf}$ and $T_{pe}$ can be defined by their most probable sample values. However, given that the distributions of $T_f$, $T_{pf}$ and $T_{pe}$ are unknown, investigation of the most probable values is impossible. Therefore, we propose an approximation of the expected values of $T_f$, $T_{pf}$ and $T_{pe}$ by the probable, but not necessarily most probable, deterministic values $\bar{T}_f$, $\bar{T}_{pf}$, $\bar{T}_{pe}$ of a typical overflow period. The idea behind this approximation is the estimation of $\bar{T}_f$, $\bar{T}_{pf}$ and $\bar{T}_{pe}$ through sample mean prediction of traffic rates over these intervals. Therefore, we can write the estimation of $E[T_{TOCL}]$ as

$$E[T_{TOCL}] \approx \bar{T}_{pf} + \bar{T}_{pe}$$

(9)

While this approximation of the average TOCL may seem crude and simple, it is acceptable. Using (9), the cell loss calculation is still accurate as it will be shown by the simulation results in Sect. 5.

Let the rate of traffic arriving at the system in arbitrary intervals of length $\bar{T}_f$, $\bar{T}_{pf}$ and $\bar{T}_{pe}$ be $R_f$, $R_{pf}$ and $R_{pe}$, respectively. In a typical overflow period, let the typical rate of traffic arriving at the system in the intervals $(0, \bar{T}_f)$, $(\bar{T}_f, \bar{T}_{pf})$ and $(\bar{T}_{pf}, \bar{T}_{pe})$ be $\bar{R}_f$, $\bar{R}_{pf}$ and $\bar{R}_{pe}$, respectively. Noting that at the points $T_f$ and $\bar{T}_{e}$, the queue equals the buffer level $B$, we have the following relations

$$\bar{R}_f = C + \frac{B}{\bar{T}_f}$$

(10)

$$\bar{R}_{pf} = C + \frac{B_{MOCP}}{\bar{T}_{pf}}$$

(11)

$$\bar{R}_{pe} = C - \frac{B_{MOCP}}{\bar{T}_{pe}}$$

(12)

Fixing the value $B_{MOCP}$ of the MOCP, to investigate $\bar{R}_f$, $\bar{R}_{pf}$ and $\bar{R}_{pe}$ of a typical overflow period we use the future sample mean prediction rule for stochastic processes. Denote the arriving traffic rate in the interval of length $\bar{T}_{pf}$ under the condition that the traffic rate arriving in the preceding interval of length $\bar{T}_f$ is $\bar{R}_f$ as a random variable $R_{pf}^i$, i.e., $R_{pf}^i \equiv \{R_{pf} \mid \bar{R}_f\}$. If $\bar{R}_f$ is given, we will estimate the typical rate $R_{pf}$ by the likely traffic rate at which the queue length increases in the interval $(\bar{T}_f, \bar{T}_{pf})$ to reach level $B + B_{MOCP}$. Because the queue length increases in the interval $(\bar{T}_f, \bar{T}_{pf})$, we consider only the traffic rate $R_{pf}^i > C$. Using a minimized mean square predictor, $\bar{R}_{pf}$ is estimated by the sample mean of the arrival traffic rate in the interval $(\bar{T}_f, \bar{T}_{pf})$ as

$$\bar{R}_{pf} = E[R_{pf}^i \mid R_{pf}^i > C]$$

(13)

Similarly, denote the arriving traffic rate in the interval of length $\bar{T}_{pe}$ under the condition that traffic rate arriving in the preceding interval of length $\bar{T}_{pf}$ is $\bar{R}_{pf}$ as a random variable $R_{pe}$, i.e., $R_{pe}^e \equiv \{R_{pe} \mid \bar{R}_{pf}\}$. If the traffic arrival rate $R_{pf}$ in the interval $(\bar{T}_{pf}, \bar{T}_{pe})$ is given, we estimate $R_{pe}$ by the likely traffic rate at which the queue length decreases in the interval $(\bar{T}_{pf}, \bar{T}_{pe})$ to level $B$. Because the queue length decreases in the interval $(\bar{T}_{pf}, \bar{T}_{pe})$ we consider only the traffic rate $R_{pe}^e < C$. Using a minimized mean square predictor, $\bar{R}_{pe}$ is estimated by the sample mean of the arrival traffic rate in the interval $(\bar{T}_{pf}, \bar{T}_{pe})$ as

$$\bar{R}_{pe} = E[R_{pe}^e \mid R_{pe}^e < C]$$

(14)

When the conditional expected values in Eqs. (13) and (14) are in effect the mean of the singly truncated distribution of the random variables $R_{pf}^i$ and $R_{pe}^e$, they are calculated by

$$\bar{R}_{pf} = \frac{\int_{0}^{\infty} f_{pf}(x)dx}{P(R_{pf} > C)}$$

(15)

$$\bar{R}_{pe} = \frac{\int_{0}^{\infty} f_{pe}(x)dx}{P(R_{pe} < C)}$$

(16)

where $f_{pf}(x)$ and $f_{pe}(x)$ are the density functions of the random variables $R_{pf}$ and $R_{pe}$, respectively. The explicit expressions of $f_{pf}(x)$ and $f_{pe}(x)$ can be obtained if a concrete model of the input traffic process $X_t$ is known. If the input traffic model is FGN, because of the finite-dimensional Gaussian nature of the FGN process, $R_{pf}$ and $R_{pe}$ have normal distribution (Chapter 3, Sect. 2 of [27]), whose mean and variance are given in Appendix A. If the input traffic model is a MWM with beta marginal distribution, the probability density functions $f_{pf}(x)$ and $f_{pe}(x)$ are more complicated and are given by Eq. (56) in Sect. 4.2.

Consequently, given the past value of the traffic rate $\bar{R}_f$ in the interval $(0, \bar{T}_f)$, applying the minimized mean square prediction error linear estimator ([10], Sect. 8.5 of [11]) for predicting the sample mean of the arriving traffic rate $E[R_{TOCL}]$ in the interval $(\bar{T}_f, \bar{T}_{e})$, which is used for approximating $E[T_{TOCL}]$, we obtain the following equation

$$E[R_{TOCL}] = C$$

$$= \mu + \rho_{f;TOCL} \frac{\sigma_{TOCL}}{\sigma_f} (\bar{R}_f - \mu)$$

(17)

where $\sigma_f$ is the standard deviation of the random variable of the arriving traffic rate in the arbitrary interval of length $\bar{T}_f$. 

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$\sigma_{TOCL}$ is the standard deviation of the random variable of the arriving traffic rate in the arbitrary interval of length $T_{TOCL}$, and $p_{TOCL}$ is the correlation between the two random variables of the arriving traffic rate in the two next intervals of length $T_f$ and $T_{TOCL}$.

Given the value of MOCP, we will estimate the average value $E[T_{TOCL}]$. To do this, we will determine the traffic rates $\bar{R}_f$, $\bar{R}_p$ and $\bar{R}_{pe}$ in the intervals $T_f$, $T_p$ and $T_{pe}$ by Eqs. (9) to (17). Note that if the model of input traffic $X_t$ is given, these equations can be completely solved. The concrete LRD traffic models and their parameters will be introduced in the next section. There are various ways to solve these equations. However, in this paper we propose a simple algorithm that is suitable for execution by a computer to find $E[T_{TOCL}]$, as outlined in Appendix B.

In general the average of the TOCL in one overflow period $E[T_{TOCL}]$ is a function of buffer level $B$ and the MOCP value, $B_{MOCP}$, and is expressed as

$$E[T_{TOCL}] = E_{TOCL}(B, B_{MOCP})$$

(18)

3.3 Probability of Overflow Cluster and Main Overflow Cluster Occurrence

Consider input traffic of length $N \rightarrow \infty$ time slots. Let $N_{OC}$ and $N_{MOC}$ be the average number of OCs and MOCs over a time interval of length $N$ time slots. Note that $N_{MOC}$ is also the average number of overflow periods over the same time interval. Let $P_{MOC}(B)$ be the probability of MOC occurrence given buffer size $B$. We expect that the probability of OC occurrence $P_{OC}$ and the probability of MOC occurrence $P_{MOC}$ depends on the buffer level $B$. Thus, they are written as

$$P_{OC}(B) = \lim_{N \rightarrow \infty} \frac{N_{OC}(B)}{N}$$

(19)

$$P_{MOC}(B) = \lim_{N \rightarrow \infty} \frac{N_{MOC}(B)}{N}$$

(20)

Let $n_{OC}(B)$ be a function of $B$ that represents the average number of overflow clusters in one overflow period. It can be expressed as follows

$$n_{OC}(B) = \frac{P_{OC}(B)}{P_{MOC}(B)}$$

(21)

The function $n_{OC}(B)$ is strongly dependent on the input traffic characteristics and weakly dependent on the buffer level $B$. Experimental measurements of buffer overflow performance for real and synthesized traffic indicate that the function $n_{OC}(B)$ changes very slowly with changes in the buffer level $B$, and is almost constant over a wide range of $B$. In particular, the function $n_{OC}(B)$ changes much slower than the function $P_{OC}(B)$. This statement is verified by simulation results for infinite buffer systems such as illustrated in Fig. 8 the FGN traffic model and Fig. 9 for the MWM traffic. The number of overflow clusters with respect to the buffer level $B$ is normalized by the number of overflow clusters at the smallest buffer level over the buffer range considered (the average number of OCs at 0.5 Mbytes for the FGN trace and at 0.1 Mbytes for the MWM trace).

In a system with infinite buffer size, consider the queue length at the time instant the queue changes from the overflow to the non-overflow state. We note that the occurrence of an overflow cluster represents a buffer overflow at the time instant $t-1$, $Q_{t-1} > B$, and the transmission link drains the buffer queue to below level $B$ at the next time slot $t$. Therefore, the probability of overflow cluster occurrence is given by

$$P_{OC}(B) = \int_0^\infty P(Q_{t-1} = B + x, X_t \leq C - x) dx$$

(22)

In general, calculating the joint probability of these two dependent events is often complicated and sometimes impossible. However, we can rewrite the probability of overflow cluster occurrence as follows

$$P_{OC}(B) = \int_0^\infty \frac{dP(Q_{t-1} \leq y)}{dy} \bigg|_{y=B+x} P(X_{oc} \leq C - x) dx$$

(23)
where \( X_{toc} \) is a random variable with a distribution

\[
P(X_{toc} \leq C - x) = P(X_t \leq C - x) \big| Q_{t-1} = B + x
\]

(24)

The random variable \( X_{toc} \) can be simply estimated by a random variable having the same marginal distribution as \( X_t \), i.e., if \( X_t \) has Gaussian or Beta distribution then \( X_{toc} \) also has Gaussian or Beta distribution, ... An expected value of \( X_{toc} \) equals to the link capacity \( C \) and other parameters are the same as \( X_t \). A heuristic motivation is as follows. During the overflow period in general, and during an overflow cluster event in particular, the sample mean of the arriving traffic rate is exactly the link capacity. Thus, it can be assumed that the amount of traffic arriving in the buffer in time slot \( t_{toc} \), at which the queue changes from the overflow to the non-overflow state, acting as a random variable with mean \( C \).

In real communication networks, in order to guarantee rare cell loss and overflow, the buffer usually designed to be large to consume traffic burst data over large time scale. However, the time slot unit is chosen very small so as to allow fine-tuning of traffic modeling and network performance. Therefore, the link capacity \( C \) is usually much smaller than the buffer size \( B \). This leads to that \( P(Q_t > y) \) and \( \frac{dP(Q_t > y)}{dy} \) almost constant over the range \([B, B + C]\) of buffer size, i.e., \( P(Q_t > u) \approx P(Q_t > v) \) and \( \frac{dP(Q_t > y)}{dy} \approx \frac{dP(Q_t > y)}{dy} \) for \( B \leq u, v \leq B + C \). This is true particularly for LRD network traffic, where buffer overflow is reduced only very slightly with increasing buffer size (see Figs. 12, 13 and 16, 17 in Sect. 5). Using this observation, a simpler estimation for the probability of overflow cluster occurrence is given as

\[
P_{OC}(B) \leq \int_0^C P(X_{toc} \leq C - x) dx
\]

(25)

While time slot length is chosen small so as link capacity much smaller than buffer size, the approximation of (25) is quite accurate. Therefore, the effect of the time slot length in accuracy of the approximation is negligible. The expression of (25) shows that the probability of overflow cluster occurrence is largely proportional to the differential of the BOP. Although in this section the random variable \( X_{toc} \) was adopted to estimate the probability of overflow cluster occurrence, it is shown later that this random variable plays only a minor role in the estimation of CLP.

### 3.4 Cell Loss Probability Calculation Using BOP

Using Eq. (5) the CLP for a system is calculated by

\[
P(CellLoss) \approx \int_0^\infty \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} x dx \mu
\]

(26)

where \( \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} \) is understood as the probability of MOC occurrence with MOCP equal to \( x \), i.e., \( B_{MOCP} = x \).

Using the slow-changing feature of the function \( n_{OC}(B) \) we can approximate the CLP in Eq. (26) as

\[
P(CellLoss) \approx \int_0^\infty \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} x dx\frac{n_{OC}(B)}{\mu}
\]

(27)

with \( B_0 \geq B \) a constant depending on the buffer level \( B \).

Note that the average number of OCs in one overflow period \( n_{OB}(B_0) \) is not less than 1. Thus, if the average number of OCs is small, the CLP can be approximated by its upper bound as

\[
P(CellLoss) \approx \int_0^\infty \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} x dx\frac{n_{OC}(B)}{\mu}
\]

(28)

This approximation of CLP is simple, and does not involve the term \( E_{TOCL}(B, x) \) described in Sect. 3.2. However, as the average number of OCs in one overflow period increases with increasing buffer level \( B \) (see Figs. 8 and 9), \( n_{OB}(B_0) \) also increases with increasing buffer level \( B \). Therefore, the accuracy of (28) decreases with increasing buffer level \( B \), as will be evaluated in Sect. 5. Calculation of CLP by (28) is summarized in an algorithm of Fig. 10.

On the other hand, using Eq. (4) we can calculate the BOP

\[
P(Q_t > B) = \int_0^\infty \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} E_{TOCL}(B, x) dx
\]

Using the slow-changing feature of the function \( n_{OC}(B) \) we rewrite Eq. (29) as

\[
P(Q_t > B) \approx \int_0^\infty \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} E_{TOCL}(B, x) dx \frac{n_{OC}(B)}{P(Q_t > B)}
\]

(30)

From Eq. (30) we specify the function \( n_{OC}(B_0) \) by

\[
n_{OC}(B_0) \approx \int_0^\infty \frac{-dP_{MOC}(y)}{dy} \big|_{y=B+x} E_{TOCL}(B, x) dx \frac{P(Q_t > B)}{P(Q_t > B)}
\]

(31)
Finally, substituting Eq. (31) into Eq. (27) we can estimate the CLP by

\[
P(\text{CellLoss}) = \sum_{t=0}^{\infty} d\frac{P(Q_t > B)}{dy} |_{y=B+x} xdxP(Q_t > B)
\]

Given the BOP, the CLP is fully approximated by Eqs. (23), (32) and the calculation algorithm of \(E_{\text{TOCL}}(B,x)\). In Eq. (32) the BOP directly appears in the calculation of the CLP, and as such it can be expected that the CLP is almost proportional to BOP. This conclusion will be confirmed through numerical analysis.

Using Eq. (25) for estimating the probability of overflow cluster occurrence \(P_{OC}(B)\) in Eq. (32), we have another approximation of the CLP as follows

\[
P(\text{CellLoss}) \approx \sum_{t=0}^{\infty} d\frac{P(Q_t > B)}{dy} |_{y=B+x} E_{\text{TOCL}}(B,x)dx
\]

\[
P(\text{CellLoss}) \approx \sum_{t=0}^{\infty} d\frac{P(Q_t > B)}{dy} |_{y=B+x} E_{\text{TOCL}}(B,x)dx
\]

In this equation the integral of the arrival traffic distribution function in the overflow cluster period \(\int_0^C \frac{P(X_{toc} \leq C-x)}{dy} dx\), as used for estimating \(P_{OC}(B)\) in Eq. (25), disappears, suggesting that the cell loss calculation is primarily defined by the overflow probability, and is not affected by how the parameters of the random variable \(X_{toc}\) are chosen. We summarize final result (33) in an algorithm calculation of Fig. 11.

4. Long-Range Dependent Network Traffic

Long-range dependence and self-similarity are established concepts required for modeling broadband data networks. This is especially true when analyzing Internet data. A change in the time scale can be represented by forming the aggregate process \(X_t^{(m)}\), which is obtained by summing \(X_t\) over a non-overlapping block of length \(m\), i.e.,

\[
X_t^{(m)} = X_{tm} + \ldots + X_{(tm+m-1)}
\]

The process \(X\) exhibits LRD if its auto-covariance decays slowly enough to render \(\sum_{k=-\infty}^{\infty} r(k)\) infinite. For details of LRD processes and characteristics, see [11]. In this section we introduce two models of LRD traffic, the FGN and the MWM with a beta marginal distribution. The correlation, variance and conditional density function of these traffic models are discussed. We also introduce the results of overflow probability calculation in queue systems with infinite buffer sizes for LRD traffic models.

4.1 FGN Traffic

The FGN processes represent the traditional LRD traffic model and have received a lot of attention for modeling and analysis of queuing behavior in high-speed networks. Due to the huge link capacity of high-speed networks, hundreds or even thousands of network applications are likely to be served by one multiplexer. By applying the central limit theorem, we can accurately characterize the aggregated process as a Gaussian process. By the nature of Gaussian processes, the FGN is an exactly self-similar process [12], [13], i.e.,

\[
m^{-H}X_t^{(m)} \overset{d}{=} X_t
\]

where \(d\) means equality in the sense of a statistical distribution. The autocorrelation function of FGN with Hurst parameter \(H\), \(0.5 < H < 1\), is

\[
r(k) = \frac{1}{2} \left( (k+1)^2H - 2k^2H + (k-1)^2H \right)
\]

For FGN, the standard deviation of the sample mean of \(m\) observations \(\frac{1}{m}X_t^{(m)}\) is

\[
\sigma_m = m^{H-1/2} \sigma
\]

The correlation between two consecutive sample mean random variables of lengths \(m\) and \(n\) time slots is

\[
\rho_{mn} = \frac{1}{mn^{1/2}} |q+1|^{2H} - q^{2H} - 1
\]

with \(q = \min(n/m, m/n)\).

Employing the notion of critical time scale, the BOP of FGN traffic can be approximated by its lower bound as [12]

\[
P(Q_t > B) \approx \Phi \left( \frac{(C - \mu)tc_{CTS} + B}{\sigma tc_{CTS}} \right)
\]

where \(tc_{CTS} = \frac{BH}{(C - \mu)(1 - H)}\) and \(\Phi(\cdot)\) is the residual distribution function of the standard Gaussian distribution. Various approximations for the function \(\Phi(\cdot)\) can be found in [31].

4.2 MWM Traffic with Beta Marginal Distribution

Recently, a radical approach to understanding and describing the actual dynamics of modern network traffic is based on multifractals. The investigation into the fractal nature of network traffic is based on using wavelet based analysis. The analysis for a particular traffic class of multiplicatively generated multifractals, called conservative cascades, is presented in [21]–[23]. The general MWM based on the multiplicative cascade is proposed in [24] and its generator and
marginal distribution are developed in [14], [15]. The primary goal of the MWM is to model a class of non-negative discrete LRD processes. The flexibility and accuracy of the model and fitting procedure result in a close fit to real data statistics and queuing behavior can be found in [15], [24].

We now consider a process \( X_t \) for \( t = 0, 1, \ldots, 2^J - 1 \) of \( N = 2^J \) data points for some integer \( J \). The MWM iteratively constructs a multiscale representation of process \( X_t \). To start the iteration, let \( U_{0,0} \) be the coarsest solution or root of the process, which equals the total traffic that arrives at the system in \( 2^J \) time units. At higher levels, corresponding to the resolutions of the process \( X_t \) at time scale \( j = 1, ..., J \), we define

\[
U_{jk} = X_k^{(2^{j-k})} \tag{40}
\]

The MWM uses the independent multiplicative innovations \( M_{jk} \) to model the process at a finer scale as

\[
U_{j+1,k} = U_{jk}M_{j+1,k} \tag{41}
\]

\[
U_{j+1,k+1} = U_{jk}(1 - M_{j+1,k}) \tag{42}
\]

For stationary processes at each scale \( j \) the multipliers \( M_{jk} \) are identically distributed to some random variable \( M_j \) where \( M_j \) is symmetric about 1/2 with \( M_j \in [0,1] \). The LRD process resolutions \( U_{jk} \) at a given scale \( j \) are also identically distributed over some random variable \( U_j \)

\[
U_j = U_{0,1}M_2 \ldots M_j = U_{0}G_j \tag{43}
\]

The existence of scaling in network traffic is now well established, and one of the central ideas is that scaling is generally associated with some form of linearity in a log-log diagram of scale energy. In most cases, the scale function is piecewise, i.e., the Hurst parameter is approximately constant at the range of both large-time scales and small-time scales, see [15], [21]–[23]. We assume that the breakpoint of a scaling linear relationship occurs at scale \( J_b \) of network data. The self-similar parameter \( H \) equals \( H_1 \) over large-time scales and equals \( H_2 \) over small-time scales. This corresponds to the fact that parameters of the cascade generator \( M_j, j \in [1,J_b] \) are defined by \( H_1 \) and the parameters of the cascade generator \( M_j, j \in [J_b + 1, J] \) are defined by \( H_2 \). If we let \( J_s = J - J_b \) and \( j_s = j - J_b \) be the scale indices at the range of small scales, with \( j \in [J_b + 1, J] \), we obtain similar results to the first generators in each scale range, as follows

\[
E[M_j^2] = \frac{a_1 b(a_1^{j_s-1} - b^{j_s-1}) E[U_{j_s}^2]}{(a_1^{j_s} - b^{j_s}) E[U_{j_s}]} E[U_{j_s}^2] \tag{44}
\]

\[
E[M_{j+1}^2] = \frac{a_2 b(a_2^{j_s-1} - b^{j_s-1}) E[U_{j_s}^2]}{E[U_{j_s}^2]} \tag{45}
\]

where \( a_1 = 2^{-2H_1} \) and \( a_2 = 2^{-2H_2} \). The second moment of generators \( M_j \) and the products are calculated for \( j \in [2, J_b] \) as

\[
E[M_j^2] = \frac{(a_1^j - b^j) E[M_j^2] - a_1 b(a_1^{-1} - b^{-1})}{(a_1^{-1} - b^{-1}) E[M_j^2] - a_1 b(a_1^{-2} - b^{-2})} \tag{46}
\]

\[
E[G_j^2] = \frac{(a_2^j - b^j) E[M_j^2] - a_2 b(a_2^{-1} - b^{-1})}{a_2 - b} \tag{47}
\]

and for \( j \in [J_b + 1, J] \) as

\[
E[M_j^2] = \frac{(a_2^j - b^j) E[M_{j+1}^2] - a_2 b(a_2^{-1} - b^{-1})}{(a_2^{-1} - b^{-1}) E[M_{j+1}^2] - a_2 b(a_2^{-2} - b^{-2})} \tag{48}
\]

\[
E[G_j^2] = \frac{(a_2^j - b^j) E[M_{j+1}^2] - a_2 b(a_2^{-1} - b^{-1})}{a_2 - b} \tag{49}
\]

At scale \( J_b \) the second moment \( E[U_{J_b}^2] \) can be estimated from the training data, because its time scale is small. But at the largest scale \( 0 \) the second moment \( E[U_{J_b}^2] \) can only be calculated from the approximated \( var[U_0] \) with

\[
var[U_0] \approx 2^{2(J-J_b)H_1} var[U_{J_b}] \tag{50}
\]

In the case of the multifractal LRD process, the \( M_j \) generators of the MWM are completely specified by the two LRD parameters, \( H_1 \) and \( H_2 \), the first and the second moments of the scale resolution random variables \( U_j \) and \( U_{J_b} \).

The natural choice for a cascade generator is a symmetric standard beta distribution because of its flexibility. If the traffic root \( U_{0,0} \) is also chosen to be a beta distribution then at each scale \( j \) the solution \( U_j \) is a beta distribution since the product of the beta random variables is approximated by a beta distribution as explained in [28]. Therefore, sometimes the MWM with a beta traffic root and a beta generator is called a beta MWM.

The distribution of traffic arrival in an interval of length \( m \) time slots \( X_{t,m} \) is estimated by

\[
X_{t,m} = G_j U_{0,0} \tag{51}
\]

where \( j = \log_2(\frac{m}{N}) \), \( G_j \) is a random variable with mean \( \frac{m}{N} \) and \( E[G_j^2] \) is calculated using Eq. (47) if \( 1 \leq j \leq J_b \) and using Eq. (49) if \( J_b < j \leq J \). The standard deviation of the sample mean of \( m \) observations \( \frac{1}{m} X_{t,m} \) for MWM traffic is approximated by

\[
\sigma_m \approx \left( \frac{E[G_j^2] E[U_{0,0}]}{m^2} - \mu^2 \right)^{1/2} \tag{52}
\]
The correlation between two consecutive sample mean random variables of length \( m \) and \( n \) time slots will be

\[
\rho_{m,n} \approx \frac{(m+n)^2 \sigma_m^2 \sigma_n^2 - m^2 \sigma_m^2 - n^2 \sigma_n^2}{2mn \sigma_m \sigma_n} \tag{53}
\]

Denote traffic arrival in an interval of length \( m \) time slots as the random variable \( X^{(m)} \) and traffic arrival in the next interval of length \( n \) time slots as \( X^{(n)} \). Due to the cascade structure of the MWM process, assume that \( X^{(m)} = X^{(m+n)} M_{m,n} \) and \( X^{(n)} = X^{(m+n)} (1 - M_{m,n}) \), where \( X^{(m+n)} = X^{(m)} + X^{(n)} \) is a random variable defined by (51); \( M_{m,n} \) is a cascade generator with mean \( \frac{m}{m+n} \) and second order statistic \( \frac{E[G^2_j]}{E[G_j]^2} \), \( j_1 = \log_2 \left( \frac{n}{m+n} \right) \), \( j_2 = \log_2 \left( \frac{2}{m} \right) \). We consider the conditional probability of \( X^{(n)} \) given that \( X^{(m)} = x_m \) to be

\[
P(X^{(n)} < y \mid X^{(m)} = x_m) = P(X^{(m+n)} < x_m + y \mid X^{(m+n)} M_{m,n} = x_m) \tag{54}
\]

With some manipulation, we have

\[
P(X^{(n)} < y \mid X^{(m)} = x_m) = \frac{\int_{x_m}^{x_m+y} f_0(x) \frac{1}{2} f_{M_{m,n}}(\frac{x}{x_m}) dx}{f_1(x_m)} \tag{55}
\]

where \( f_0(x) \), \( f_M(x) \), and \( f_1(x) \) are the probability density function of \( X^{(m+n)} M_{m,n} \) and \( X^{(m)} \), respectively. The conditional density of the traffic rate \( \frac{1}{n} X^{(n)} \) given that the traffic rate \( \frac{1}{m} X^{(m)} = x_m \) is given by

\[
f \left( y \mid \frac{1}{m} X^{(m)} = x_m \right) = \frac{d}{dy} \frac{dP(X^{(n)} < ny \mid X^{(m)} = mx_m)}{dP(X^{(m)} = x_m)} \tag{56}
\]

The BOP for beta MWM traffic is calculated in this paper using multiscale queuing analysis to approximate the queue tail probability [25], [26], i.e.,

\[
P(Q_t > B) \approx 1 - \prod_{j=0}^{j} P(U_j < (C - \mu) 2^{-j} + B) \tag{57}
\]

To calculate BOP for a beta MWM using Eq. (57) we need to approximate the tail probability of the beta distribution. A number of beta distribution approximations are given in [32]. In this paper, we use a normal approximation for the beta distribution as presented in [29], [30].

5. Numerical Results

The numerical validation of the proposed cell loss calculation method through simulation is presented for the FGN traffic and the beta MWM traffic. The results of estimated BOP are based on well-known Norros’s (39) [12] for FGN traffic and on (57) [25], [26] for MWM traffic. The results of estimated CLP are based on calculation formulas (23), (33) and the calculation algorithm of \( E_{TOCL}(B, B_{MOCP}) \) in Appendix B. The parameters of the simulated FGN traffic were chosen to be identical to the parameters of a real Bellcore Ethernet traces from October and August 1989. The parameters of the simulated beta MWM traffic were set as those for a real DEC-PKT-4 and LBL-PKT-4 WAN traces from 1994. Four traces are available from the Internet traffic archive [33]. The simulation results, shown in Figs. 12, 13 and 16, 17, demonstrate the high accuracy of the proposed cell loss calculation. For both traffic models with different traffic parameters (Hurst parameter, traffic mean, variance) and link utilization the simulated CLP and estimated CLP are very close, and have very similar shapes. Notably, the ratio of the CLP to the BOP (Figs. 14, 15 and 18, 19) shows that the CLP approximation is extremely accurate for a large buffer size, with error less than 0.1 for FGN traffics and less than 0.2 for beta MWM traffics in the logarithm of CLP/BOP. It is also concluded that the present CLP approximation is asymptotic with increasing buffer size as an obvious outcome of our derivation using the concept and analysis of an overflow cluster. The simulation results show that the CLP estimation using Eq. (28) is not accurate and asymptotic, especially for a large buffer size \( B \). This con-
Fig. 14  Cell loss probability and buffer overflow probability ratios of an FGN trace. The traffic mean $\mu = 35.295$ kbytes and standard deviation $\sigma = 19.604$ kbytes. The Hurst parameter $H = 0.8$. Link utilization is 80%.

Fig. 15  Cell loss probability and buffer overflow probability ratios of an FGN trace. The traffic mean $\mu = 144.441$ kbytes and standard deviation $\sigma = 88.030$ kbytes. The Hurst parameter $H = 0.84$. Link utilization is 66.7%.

Fig. 16  Queuing performance of a beta MWM trace. The traffic mean $\mu = 447.543$ bytes and standard deviation $\sigma = 555.746$ bytes. The Hurst parameter $H_1 = 0.867$ over large time scales and $H_2 = 0.729$ over small time scales. Link utilization is 66.7%.

Fig. 17  Queuing performance of a beta MWM trace. The traffic mean $\mu = 62.062$ bytes and standard deviation $\sigma = 229.670$ bytes. The Hurst parameter $H_1 = 0.939$ over large time scales and $H_2 = 0.647$ over small time scales. Link utilization is 33.3%.

Fig. 18  Cell loss probability and buffer overflow probability ratios of a beta MWM trace. The traffic mean $\mu = 447.543$ bytes and standard deviation $\sigma = 555.746$ bytes. The Hurst parameter $H_1 = 0.867$ over large time scales and $H_2 = 0.729$ over small time scales. Link utilization is 66.7%.

Fig. 19  Cell loss probability and buffer overflow probability ratios of a beta MWM trace. The traffic mean $\mu = 62.062$ bytes and standard deviation $\sigma = 229.670$ bytes. The Hurst parameter $H_1 = 0.939$ over large time scales and $H_2 = 0.647$ over small time scales. Link utilization is 33.3%.
firms the importance and accuracy of \(E_{TOCL}(B, B_{MOCP})\) calculation. It is seen that proposed approximation of CLP is much more accurate than traditional approximation of CLP by BOP that confirms the necessity of our CLP estimation technique.

6. Conclusions

A new concept of an overflow cluster was introduced for the analysis of cell/packet loss in ATM and packet switched networks. The new approach is based on well-studied buffer overflow results, and represents an accurate method of CLP/PLP calculation for general aggregated LRD network traffic, including FGN and MWM traffic models, with different traffic parameters and under different network load. Numerical results validated the accuracy and flexibility of the new technique. The accuracy and generality of the proposed method make it widely applicable to analysis of many aspects of network performance including resource allocation, call admission control and QoS guarantees. Analysis of overflow clusters, overflow cluster length, and the main overflow cluster peak also allows us to understand the nature of buffer overflow and cell/packet loss mechanisms. For further study issues, it is important to evaluate the usefulness of the proposed approach for practical applications such as control and management of buffer overflows, cell/packet loss rates, as well as for guaranteeing QoS in network systems.

References


Appendix A: Mean and Variance of Conditional Traffic Rate

Assuming that the value of \( \bar{R}_f \) is known a priori, we predict
the future sample mean \( E[R_{pf}'] \) using the minimized mean square prediction error linear estimator ([10], Sect. 8.5 of [11]) as follows

\[
E[R_{pf}'] = \mu + \rho_{pf}\frac{\sigma_{pf}}{\sigma_f}(\bar{R_f} - \mu)
\] (A-1)

where \( \sigma_f \) is the standard deviation of the random variable of arriving traffic rate in an arbitrary interval of length \( \bar{T_f} \), \( \sigma_{pf} \) is the standard deviation of the random variable of arriving traffic rate in an arbitrary interval of length \( \bar{T}_{pf} \), and \( \rho_{pf} \) is the correlation between the two random variables of arriving traffic rates in next intervals of lengths \( \bar{T_f} \) and \( \bar{T}_{pf} \). The variance of the random variable \( R_{pf}' \) is given by

\[
Var[R_{pf}'] = (1 - \rho_{pf}^2)\sigma_{pf}^2
\] (A-2)

If \( \bar{R}_{pf} \) is given we can predict the future sample mean \( E[R_{pf}'] \) as

\[
E[R_{pf}'] = \mu + \rho_{pf,pe}\frac{\sigma_{pe}}{\sigma_f}(\bar{R_f} - \mu)
\] (A-3)

where \( \sigma_{pe} \) is the standard deviation of the random variable of arriving traffic rate in an interval of length \( \bar{T}_{pe} \), and \( \rho_{pf,pe} \) is the correlation between the two arriving traffic rates in next intervals of lengths \( \bar{T}_{pf} \) and \( \bar{T}_{pe} \). The variance of the random variable \( R_{pe} \) is given by

\[
Var[R_{pe}'] = (1 - \rho_{pf,pe}^2)\sigma_{pe}^2
\] (A-4)

Appendix B: TOCL Calculation Algorithm

In this paper TOCL is calculated using heuristic algorithm outlined in Fig. A-1. There are three loops in this algorithm. The first loop compute \( \bar{T}_{pf} \). Note that when \( \bar{T}_{pf} \) increases to infinity by the step \( \Delta\bar{T}_{pf} \) then \( \bar{R}_{pf} \) decreases from a very large value to zero, \( \bar{R}_{pe} \) decreases from value of \( \bar{R}_f \) to \( \mu \), and \( \bar{R}_{pf}' \) increases from some value near \( \bar{R}_f \) to infinity. Thus, increase the value of \( \bar{T}_{pf} \) we always obtain \( \bar{R}_{pf} \geq \bar{R}_{pf}' \). In the second loop when \( \bar{T}_{pe} \) increases by the step \( \Delta\bar{T}_{pe} \) then \( \bar{R}_{pe} \) increases from zero to \( C \). At the same time, \( \bar{R}_{pe} \) decreases from \( \bar{R}_{pf} \) to \( \mu \), and \( \bar{R}_{pe}' \) gets close to \( \mu \) from some initial positive value. Because \( C > \mu \), the second loop will be always stopped. We discuss the overall loop of calculation \( \bar{R}_f \) as follows. When the initial value of \( \bar{R}_f \) is large, \( \bar{R}_{TOCL} \) computed by Eq. (17) is also large so that it is larger than \( C \). If \( \bar{R}_f \) decreases by the step \( \Delta\bar{R}_f \), equivalently \( \bar{T}_f \) increases, then \( \bar{R}_{TOCL} \) decreases closely to \( \mu < C \). Thus, we can obtain the value of \( \bar{R}_f \) so that \( \bar{R}_{TOCL} \leq C \) and the procedure will end.

Appendix C: List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>ATM</td>
<td>asynchronous transfer mode</td>
</tr>
<tr>
<td>BOP</td>
<td>buffer overflow probability</td>
</tr>
<tr>
<td>CLP</td>
<td>cell loss probability</td>
</tr>
<tr>
<td>FGN</td>
<td>fractal Gaussian noise</td>
</tr>
<tr>
<td>LRD</td>
<td>long-range dependent</td>
</tr>
<tr>
<td>MOC</td>
<td>main overflow cluster</td>
</tr>
<tr>
<td>MOCL</td>
<td>main overflow cluster length</td>
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Fig. A-1 Outline of the heuristic algorithm of TOCL calculation.
Appendix D: List of Symbols and Variables

\[ X_t \] \quad \text{input traffic at time slot } t \\
\[ \mu \] \quad \text{traffic mean} \\
\[ \delta \] \quad \text{traffic standard deviation} \\
\[ B \] \quad \text{buffer size} \\
\[ C \] \quad \text{link capacity} \\
\[ Q_t \] \quad \text{queue length in the infinite buffer at time } t \\
\[ Q'_t \] \quad \text{queue length in the finite buffer at time } t \\
\[ X_{toC} \] \quad \text{input traffic at buffer overflow point} \\
\[ T_{TOCL} \] \quad \text{total overflow cluster length} \\
\[ T_{MOCL} \] \quad \text{main overflow cluster length} \\
\[ B_{MOCP} \] \quad \text{main overflow cluster peak} \\
\[ T_p \] \quad \text{MOC peak point} \\
\[ T_f \] \quad \text{MOC start point} \\
\[ T_e \] \quad \text{MOC end point} \\
\[ T_{pf} \] \quad \text{MOC formation interval} \\
\[ T_{pe} \] \quad \text{MOC erosion interval} \\
\[ \bar{T}_f \] \quad \text{typical MOC start point} \\
\[ \bar{T}_{pf} \] \quad \text{typical MOC formation interval} \\
\[ \bar{T}_{pe} \] \quad \text{typical MOC erosion interval} \\
\[ R_f \] \quad \text{traffic rate in interval of length } \bar{T}_f \\
\[ R_{pf} \] \quad \text{traffic rate in interval of length } \bar{T}_{pf} \\
\[ R_{pe} \] \quad \text{traffic rate in interval of length } \bar{T}_{pe} \\
\[ \bar{R}_f \] \quad \text{traffic rate in interval } (0, \bar{T}_f) \\
\[ \bar{R}_{pf} \] \quad \text{traffic rate in interval } (\bar{T}_f, \bar{T}_p) \\
\[ \bar{R}_{pe} \] \quad \text{traffic rate in interval } (\bar{T}_{p}, \bar{T}_e) \\
\[ R^*_f \] \quad \text{conditional random variable of } R_{pf} \\
\[ R^*_e \] \quad \text{conditional random variable of } R_{pe} \\
\[ f_{pf} \] \quad \text{density function of } R^*_f \\
\[ f_{pe} \] \quad \text{density function of } R^*_e \\
\[ R_{TOCL} \] \quad \text{traffic rate in interval } (\bar{T}_f, \bar{T}_e) \\
\[ P_{OC} \] \quad \text{probability of OC occurrence} \\
\[ P_{MOC} \] \quad \text{probability of MOC occurrence} \\
\[ U_j \] \quad \text{MWM traffic solution at scale } j \\
\[ M_j \] \quad \text{cascade generator of MWM traffic at scale } j

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