Perfect Blind-Channel Shortening for Multicarrier Systems

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Blind equalization, cyclic prefix (CP), orthogonal, frequency-division multiplexing (OFDM), time-domain equalizer
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ABSTRACT

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Perfect Blind-Channel Shortening for Multicarrier Systems

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Abstract—In multicarrier systems, when the order of a channel impulse response is larger than the length of the cyclic prefix (CP), there is a significant performance degradation due to interblock interference (IBI). This paper proposes a blind-channel shortening method in which the equalizer parameter vector is formed by the noise subspace of the received signal correlation matrix so that the output power is maximized. The proposed method can not only shorten the effective channel impulse response to within the CP length but also maximize the output signal-to-interference-and-noise ratio while eliminating the IBI. We point out that the performance depends on the choice of a decision delay and propose a simple method for determining the appropriate delay. We propose both a batch algorithm and an adaptive algorithm and show by simulation that they are superior to the conventional algorithms.

Index Terms—Blind equalization, cyclic prefix (CP), orthogonal frequency-division multiplexing (OFDM), time-domain equalizer.

I. INTRODUCTION

MULTICARRIER modulation, such as orthogonal frequency-division multiplexing (OFDM), is an attractive technique for high-speed data transmission and holds great potential for audio/video broadcasting, wireless local area networks, and future wideband cellular systems. In multicarrier systems, a cyclic prefix (CP) is inserted between data blocks to prevent interblock interference (IBI) caused by a dispersive channel. When the channel impulse response is extremely long, a long CP is not always available since it reduces the transmission efficiency. If the channel impulse response is longer than the length of CP plus one, IBI remains. There have been some works to cope with the IBI in multicarrier applications such as Digital Subscriber Line (DSL) [1], [2], the Digital Terrestrial Television Broadcasting (DTTB) using Single Frequency Network (SFN) [3], 5-GHz wireless LAN [4], and WiMAX [5]. Channel shortening filters are known to be relatively simple and effective techniques [1], [2]. They shorten the impulse response of the effective channel, which is a convolution of a channel and a linear equalizer, to within the CP length plus one. A number of channel shortening methods have been proposed in the past decade, but most of them require training sequences that reduce channel throughput.

The recently proposed blind-channel shortening techniques [6]–[12] are attractive because they do not require the transmission of training sequences. The sum-squared autocorrelation minimization (SAM) algorithm [7] is an adaptive algorithm that tries to minimize the sum of the squares of autocorrelations of all lags greater than the desired window. The SAM cost function has undesired local minima, though, and its global convergence is not assured. As demonstrated in [10], it may fail to shorten a channel impulse response. The single lag autocorrelation minimization (SLAM) algorithm [8] minimizes only a single autocorrelation term. It can be regarded as a simplified version of the SAM algorithm and its cost function also has undesired local minima. An adaptive algorithm exploiting the existence of null subcarriers [9] tries to force the receiver’s fast Fourier transform (FFT) output corresponding to the null subcarriers to be zero. Although this algorithm requires frequency-hopping transmission for global convergence, frequency-hopping null tones are impractical. Second-order statistical methods [10] search for an equalizer parameter vector in the null space of correlation matrices, but their computational complexity is high. Unlike the other methods, they can be implemented only by batch processing algorithms. To deal with both static and time varying environments, an approach should be implemented by both batch and adaptive algorithms. The multicarrier equalization by restoration of redundancy (MERRY) algorithm in [11] is a simple blind algorithm, is globally convergent, and can also be implemented by a batch algorithm.

This paper shows that although the MERRY algorithms using over-sampled channel outputs can achieve perfect channel shortening, which means that IBI can be canceled completely, their output signal-to-interference-and-noise ratio (SINR) can be unsatisfactory. We therefore propose a method that maximizes the output SINR while eliminating the IBI and that can be implemented by both a batch algorithm and an adaptive algorithm. We also consider the choice of a decision delay that affects the SINR performance.

This paper is organized as follows. In Section II, we first describe a communication model using a channel shortening equalizer and then describe the MERRY algorithms. Section III describes our method for maximizing the output SINR while eliminating the IBI and also presents a simple method for choosing an appropriate decision delay. Section IV presents batch and adaptive algorithms implementing our methods. Section V shows simulation results, and Section VI concludes the paper by summarizing it briefly.
II. PROBLEM FORMULATION

A. Communication Model

We consider a baseband multicarrier communication system using the CP (Fig. 1). The M-ary-QAM data sequences \( \{s_k\} \) generated at a rate \( 1/T_x \) are i.i.d. with variance \( \sigma_s^2 \). The \( n \)th data block of length \( N \) is

\[
s_n = \begin{bmatrix} s_{nN} & s_{nN+1} & \cdots & s_{(n+1)N-1} \end{bmatrix}^T
\]

(1)

where the superscript \( T \) represents the transpose of a matrix. Applying an inverse fast Fourier transform (IFFT) to \( s_n \) produces a time-domain block vector of length \( N \)

\[
x_n = \begin{bmatrix} x_0[n] & x_1[n] & \cdots & x_{N-1}[n] \end{bmatrix}^T = F_s x_n
\]

(2)

where \( F \) is an \( N \)-dimensional IFFT matrix that has a unitary property \( FF^H = I_N \), where the superscript \( H \) represents the conjugate transpose of a matrix and \( I_N \) denotes an identity matrix of size \( N \). A CP of length \( P \) is added to the beginning of the block \( x_n \) to form the \( n \)th multicarrier block \( u_n \) of length \( Q = N + P \)

\[
u_n = \begin{bmatrix} u_{nQ} & \cdots & u_{nQ+P-1} \end{bmatrix}
\]

\[
= \begin{bmatrix} x_{N-P}[n] & \cdots & x_{N-1}[n] \\
\vdots & \ddots & \vdots \\
x_0[n] & \cdots & x_{N-1}[n] \\
\end{bmatrix}
\]

(3)

Obviously, the transmitted signal \( u_k \) is cyclostationary. The received signal is sampled at \( t = kT_x/p \), where the integer \( p \) is the over-sampling factor. The channel output samples can be then expressed as [10]

\[
r_k = \sum_{n=-\infty}^{\infty} u_n h_{k-np} + w_k
\]

(4)

where \( h_k \) is the sampled impulse response of the original channel, which includes transmitter and receiver filters as well as the physical channel, and the AWGN \( w_k \) is stationary and independent of data sequences \( s_k \). Assume that \( h(t) \) has joint finite support \( [0, (M+1)T_x] \), where the integer \( M+1 \) is length of the channel impulse response. Thus, the channel output samples are

\[
r_k = \begin{bmatrix} r_{kp} \\
r_{kp+1} \\
\vdots \\
r_{(k+p-1)p} \\
\end{bmatrix} = \sum_{i=0}^{M} h[i] u_{k-i} + w_k
\]

(5)

where

\[
h[i] = \begin{bmatrix} h_{kp} & h_{kp+1} & \cdots & h_{(i+1)p-1} \end{bmatrix}^T
\]

(6)

\[
w_k = \begin{bmatrix} w_{kp} & w_{kp+1} & \cdots & w_{(k+p-1)p-1} \end{bmatrix}^T
\]

(7)

The input to an equalizer consists of channel outputs sampled over an \( L \) symbol period. Then, let \( L_P \) be the number of sampled channel outputs to be collected in a block. We define the following notations:

\[	u[k] = \begin{bmatrix} u_k & u_{k-1} & \cdots & u_{k-(L+M)+1} \end{bmatrix}^T
\]

(8)

\[
w[k] = \begin{bmatrix} w_k & w_{k-1} & \cdots & w_{k-(L+1)+1} \end{bmatrix}^T
\]

(9)

Moreover, we form an \( L_P \times (L+M) \) block Toeplitz matrix

\[
H = \begin{bmatrix}
h[0] & h[1] & \cdots & h[M] & 0 & \cdots & 0 \\
0 & h[0] & h[1] & \cdots & h[M] & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & h[0] & h[1] & \cdots & h[M] \\
\end{bmatrix}
\]

(10)

A sampled-channel output signal vector of length \( L_P \) can be expressed as

\[
r_k = \begin{bmatrix} r_k \\
r_{k-1} \\
\vdots \\
r_{k-(L+1)} \\
\end{bmatrix} = H u[k] + w[k]
\]

(11)

We let the parameter vector of a channel shortening be \( g = [g_0 \ g_1 \ \cdots \ g_{L_P-1}]^T \). The output of the equalizer, known as time-domain equalizer (TEQ), can be described by

\[
y_k = g^T r[k] = g^T (H u[k] + w[k]).
\]

(12)
Let the effective channel impulse response be denoted by \( \mathbf{c} = [c_0, c_1, \ldots, c_{L+M-1}]^T = \mathbf{H}^H \mathbf{g} \). The design of a perfect channel shortening equalizer can be expressed as choosing \( \mathbf{g} \) to force all samples \( \mathbf{c} \), except for preselected part of \( P+1 \) consecutive samples, to be zero. Thus, the perfect channel shortening equalizer can completely eliminate IBI. Our immediate goal is to find the vector \( \mathbf{g} \) performing the perfect channel shortening from only the channel output.

### B. MERRY Algorithms

MERRY algorithms [11] minimize the cost function given by

\[
J_\delta(\mathbf{g}) = E \left[ |y_{P-1+\delta} - y_{Q-1+\delta}|^2 \right], \text{ s.t. } \|\mathbf{g}\| = 1
\]

(13)

where \( \delta \in \{0,1,\ldots,Q-1\} \) is the decision delay parameter and \( E[\cdot] \) represents the ensemble average. We refer to this function to as the MERRY cost function.

The original MERRY algorithm in [11] is a stochastic gradient algorithm of the MERRY cost function. The adaptive MERRY algorithm is given by

\[
\tilde{\mathbf{r}}[k] = \mathbf{r}[(k-1)Q + P - 1 + \delta] - \mathbf{r}[(k-1)Q + Q - 1 + \delta]
\]

\[
\tilde{\mathbf{c}}[k] = \mathbf{g}^H \tilde{\mathbf{r}}'[k]
\]

\[
\mathbf{g}[k] = \frac{\mathbf{g}[k-1] - \mu \mathbf{c}^*[k] \tilde{\mathbf{r}}'[k]}{\|\mathbf{g}[k-1] - \mu \mathbf{c}^*[k] \tilde{\mathbf{r}}'[k]\|}
\]

where \( \mu \) is the step gain and the superscript * represents complex conjugation. When \( p = 1 \), the adaptive MERRY algorithm is global convergent [11].

The MERRY cost function can be rewritten as

\[
J_\delta(\mathbf{g}) = \mathbf{g}^H \mathbf{R}_\delta \mathbf{g}, \text{ s.t. } \|\mathbf{g}\| = 1
\]

(14)

where \( \mathbf{R}_\delta \) is an \( LP \times LP \) matrix defined by

\[
\mathbf{R}_\delta \triangleq E \left[ (\mathbf{r}[P - 1 + \delta] - \mathbf{r}[Q - 1 + \delta]) \right.
\]

\[
	imes (\mathbf{r}[P - 1 + \delta] - \mathbf{r}[Q - 1 + \delta])^H \bigg] .
\]

(15)

The parameter vector minimizing the MERRY cost function is given by an eigenvector corresponding to the minimum eigenvalue of \( \mathbf{R}_\delta \). In the batch MERRY algorithm, the correlation matrix \( \mathbf{R}_\delta \) is estimated by time averaging, and then the parameter vector is determined by the eigendecomposition of the estimated correlation matrix.

Let us look further into the MERRY cost function. We let \( \mathbf{c}_{\text{des}} \) be the desired signal component which consists of \( P \) samples of the impulse response of the total system \( \mathbf{c} \) and let \( \mathbf{c}_{\text{ibi}} \) be the IBI component that is the remaining part of \( \mathbf{c} \). Let \( \mathbf{H}_{\text{des}} \) be a matrix consisting of \( P \) columns of \( \mathbf{H} \) such that \( \mathbf{c}_{\text{des}} = \mathbf{H}_{\text{des}}^H \mathbf{g} \), and let \( \mathbf{H}_{\text{ibi}} \) be the remaining part of \( \mathbf{H} \) such that \( \mathbf{c}_{\text{ibi}} = \mathbf{H}_{\text{ibi}}^H \mathbf{g} \). When \( \delta = 0 \), \( \mathbf{H}_{\text{des}} \) is the first \( P \) column vectors of \( \mathbf{H} \) and \( \mathbf{H}_{\text{ibi}} \) is the remaining part. That is, \( \mathbf{H} = [\mathbf{H}_{\text{des}}, \mathbf{H}_{\text{ibi}}] \). Taking into account that the channel input sequence \( \{u_k\} \) can be regarded as an uncorrelated sequence as long as \( L \leq N - M \), we can express the correlation matrix \( \mathbf{R}_0 \) as

\[
\mathbf{R}_0 = 2\sigma_w^2 \mathbf{H}_{\text{des}} \mathbf{H}_{\text{des}}^H + 2\sigma_n^2 \mathbf{I}_P
\]

(16)

and thus the MERRY cost function becomes

\[
J_0(\mathbf{g}) = 2\sigma_w^2 \| \mathbf{c}_{\text{ibi}} \|^2 + 2\sigma_n^2
\]

(17)

where \( \sigma_w^2 \triangleq E \left[ |u_k|^2 \right] \). This implies that the minimization of the MERRY cost function minimizes both the IBI and the noise component.

The MERRY cost function is reasonable, but its performance is limited. When the noise is absent, the correlation matrix \( \mathbf{R}_0 \) is semi-positive definite, \( \mathbf{R}_0 \succeq \mathbf{0} \), from its definition. Moreover, when \( p = 1 \), \( \mathbf{R}_0 \) has full rank because

\[
\text{rank}(\mathbf{R}_0) = \text{rank}(\mathbf{H}_{\text{des}}) = L.
\]

Thus, \( \mathbf{R}_0 \) is positive definite. Then, since \( J_0(\mathbf{g}) > 0 \) for any \( \mathbf{g} \neq \mathbf{0} \), \( \| \mathbf{c}_{\text{ibi}} \| \neq 0 \). This implies that the equalizer with \( p = 1 \) using the MERRY algorithms cannot shorten the channel perfectly even in the absence of the noise.

### III. PERFECT BLIND-CHANNEL SHORTENING

#### A. Shortenable

Let us consider the shortenable of the MERRY approach using over-sampled channel outputs. That is, let us consider it when \( p \geq 2 \). For simplicity assume for the moment that \( \delta = 0 \). Also assume that \( \mathbf{H} \) has full column rank, which is a common assumption in blind equalization and identification [16]. We then get the following result.

**Proposition 1:** When \( p \geq 2 \) and \( \mathbf{H} \) has full column rank, the IBI component can be canceled completely by minimizing \( J_0(\mathbf{g}) \).

**Proof:** Let \( \lambda_0 > \lambda_1 > \cdots > \lambda_{LP-1} \) be the eigenvalues of \( \mathbf{R}_0 \) and \( \mathbf{g}_i \), \( i = 0, \ldots, LP-1 \), be the corresponding eigenvectors. From (14), the minimization of \( J_0(\mathbf{g}) \) under the norm constraint \( \| \mathbf{g} \| = 1 \) can be done by using \( \mathbf{g} = \mathbf{g}_{LP-1} \). Since \( \mathbf{H}_{\text{ibi}} \) has full column rank, the minimum eigenvalue of \( \mathbf{R}_0 \) is \( \lambda_{LP-1} = 2\sigma_n^2 \). And since \( J_0(\mathbf{g}) = \lambda_i \), the minimum of \( J_0 \) is \( J_0(\mathbf{g}_{LP-1}) = 2\sigma_n^2 \). From (17), this implies that \( \| \mathbf{c}_{\text{ibi}} \|^2 = 0 \). That is, it implies that the IBI is canceled completely.

Shortenability issues for more general cases have been discussed in [13].

#### B. SINR Maximization

Although both the IBI and the noise power can be minimized by minimizing the MERRY cost function, this does not mean that the desired signal component is not reduced. As shown later in simulation results, a drawback of the MERRY algorithm using over-sampled channel outputs is that it can weaken the desired signal component, i.e., noise enhancement. Our simulation result shown in Section V suggests that when the signal-to-noise ratio (SNR) is low this problem is more serious when \( p \geq 2 \) than when \( p = 1 \). To overcome this problem we propose a method...
that maximizes the output SINR while completely removing the IBI.

Since the rank of $H_{\text{del}}$ is $L + M - P$, there are $Lp = (L + M - P)$ eigenvectors $g_i$ spanning the noise subspace of $R_0$. The following linear combination of the $q$ eigenvectors is used as an equalizer parameter vector

$$g = g_{Lp-1}a_0 + g_{Lp-2}a_1 + \cdots + g_{Lp-q}a_{q-1} = G_qa_q$$  \hspace{1cm} (18)

where

$$G_q = [g_{Lp-1} \cdots g_{Lp-q}]$$ \hspace{1cm} (19)

$$a_q = [a_0 \cdots a_{q-1}]^T.$$ \hspace{1cm} (20)

The coefficient vector $a_q$ is chosen so that the output power is maximized, and the cost function is given by

$$J(a_q) = E \left[ |y_k|^2 \right], \text{s.t. } ||a_q|| = 1$$

$$= \alpha_q^H G_q^H R_q G_q a_q$$ \hspace{1cm} (21)

where $R_q \triangleq E \left[ r[k]r^H[k] \right]$. When $q \leq Lp - (L + M - P)$, $H_{\text{del}}^H g a_q = 0$ for any $a_q$ and the cost function can thus be rewritten as

$$J(a_q) = \sigma_w^2 a_q^H G_q^H H_{\text{del}} H_{\text{del}}^H G_q a_q + \sigma_w^2.$$ \hspace{1cm} (22)

The first term corresponds to the desired signal power and the second term corresponds to the noise power. Maximizing $J(a_q)$ enables the desired signal power to be maximized without changing the noise power. The basic idea of the proposed method is that IBI is canceled by choosing $g$ from the noise subspace of $R_0$ and the output SINR is maximized by maximizing the output power. We obtain the following proposition.

**Proposition 2:** The equalizer parameter vector is given by (18). Let $\tilde{u}[k] = [u_k \ u_{k-1} \cdots u_{k-P+1}]^T$. When $q \leq Lp - (L + M - P)$, the output SINR defined by

$$\text{SINR} \triangleq \frac{E \left[ |c_{\text{del}}^H \tilde{u}[k]|^2 \right]^2}{E \left[ |y_k|^2 - c_{\text{del}}^H \tilde{u}[k]|^2 \right]^2}.$$ \hspace{1cm} (23)

is maximized by choosing the coefficient vector $a_q$ so that the output power $J(a_q)$ is maximized.

**Proof:** Let $u_k$ and $u_k$ be uncorrelated sequences and are uncorrelated with each other and $||g|| = 1$, the SINR can be rewritten as

$$\text{SINR} = \frac{\sigma_w^2||c_{\text{del}}||^2}{\sigma_w^2||c_{\text{del}}||^2 + \sigma_w^2}.$$ \hspace{1cm} (24)

Since $g$ is chosen from the noise subspace of $R_0$, IBI is canceled completely i.e., $||c_{\text{del}}||^2 = 0$ and thus the output power becomes

$$E \left[ |y_k|^2 \right] = E \left[ |c_{\text{del}}^H \tilde{u}[k] + g^H w[k]|^2 \right]$$

$$= \sigma_w^2||c_{\text{del}}||^2 + \sigma_w^2.$$ \hspace{1cm} (25)

Then, since

$$\text{SINR} = \frac{J(a_q) - \sigma_w^2}{\sigma_w^2}$$

the SINR can be maximized by maximizing $J(a_q)$.

We can expect the SINR to increase as the number of $q$ of the combined vectors increases. We then obtain the following proposition.

**Proposition 3:** Denote by $S_q$ the maximum output power obtained by optimally combining $q$ eigenvectors. Then $S_q \geq S_i$ for $0 < i < q$ as long as $q \leq Lp - (L + M - P)$.

**Proof:** Let $a_q$ be the coefficient vector that maximizes the output power. That is

$$a_q = \arg \max_{||a_q||=1} J(a_q)$$

and we have

$$S_q = J(a_q) = a_q^H G_q^H H_{\text{del}} H_{\text{del}}^H G_q a_q + \sigma_w^2.$$ \hspace{1cm} (26)

The best coefficient $a_q$ for $q < i < q$ can be represented by a vector with $q$ entries: $a_q = \pmatrix{a_{q_0}^T \ 0 \cdots 0}$. Then

$$S_i = a_q^H G_q^H H_{\text{del}} H_{\text{del}}^H G_q a_q + \sigma_w^2.$$ \hspace{1cm} (27)

As will be discussed in Section IV, the number $q$ of combined eigenvectors, or equivalently the dimension of the signal space, $L + M - P$, should be estimated by using an information theoretic criterion such as the minimum description length (MDL) [14], [15].

### C. Choice of Decision Delay

For simplicity, we have so far considered the case of zero delay $\delta = 0$. In practice, however, we have to set the decision delay appropriately because the delay affects the performance. We expect that, as in blind equalization [16], an appropriately chosen delay will improve performance.

Let us first consider the influence of the delay $\delta$ on the effective channel impulse response. Depending on $\delta$, the effective channel impulse response $c$ can be classified into four cases.

**Case 1:** $0 \leq \delta \leq L + M - P$

In this case, $H_{\text{del}}$ takes the following form:

$$H_{\text{del}} = [H(:,0: \delta - 1) \ H(:, \delta + P:L + M - 1)]$$

where $H(:,i:j)$ is the MATLAB notation denoting the $i$th to $j$th columns of $H$. Then the interference component becomes

$$c_{\text{del}} = [c_0 \cdots c_{\delta - 1} \ c_{\delta + P} \cdots c_{L + M - 1}]^T.$$ \hspace{1cm} (28)

Since $H_{\text{del}}$ has full column rank, $c_{\text{del}}$ can be reduced to zero by minimizing the MERRY cost function $J_0(g)$. Consequently, the effective channel impulse response becomes

$$c = [0 \cdots 0 \ c_{\delta} \cdots c_{\delta + P - 1} \ 0 \cdots 0]^T.$$ \hspace{1cm} (29)
Case 2: $L + M - P + 1 \leq \delta \leq L + M - 1$

$$H_{\delta i} = H(\cdot ; 0 \mid \delta - 1)$$

$$c_{\delta i} = [c_0 \cdots c_{\delta - 1}]^T$$

$$c = [0 \cdots 0 c_{L+M-1}]^T.$$  

Case 3: $L + M \leq \delta \leq N$

$$H_{\delta i} = H, \ c_{\delta i} = c, \ c = 0.$$  

Case 4: $N + 1 \leq \delta \leq Q - 1$

$$H_{\delta i} = H(\cdot ; \delta - N \mid L + M - 1)$$

$$c_{\delta i} = [c_{\delta - N} \cdots c_{L+M-1}]^T$$

$$c = [c_0 \cdots c_{\delta - N - 1} 0 \cdots 0]^T.$$  

Table 1 summarizes the influence of $\delta$ on the position $d_0$ that the nonzero part of the effective channel impulse response starts from and the length $L_0$ of the nonzero part. The length $L_0$ should be long as possible because the power of the desired component at the equalizer output will increase as this length becomes longer. Since we can see in Table 1 that the length $L_0$ is affected by the delay $\delta$, the choice of the delay is important.

Exhaustive determination of the delay that provides the highest output SINR would impose a heavy computational burden, so delay is better determined by a computationally simple technique like that used in [12]. Here we propose a simple technique to determine the delay without estimating the channel impulse response directly. Our heuristic idea is to base the search for a delay on the observation of the original channel, instead of the effective channel. Consider the autocorrelation of the received signal defined by

$$\rho_d \triangleq E[r_{i+d} r_{i+d}^H].$$  

Taking into account the cyclostationarity of the transmitted signal $u_{ik}$, we can rewrite this function as

$$\rho_d = \begin{cases} 
\frac{1}{N_d} \sum_{i=0}^{N-d-1} |h[i]|^2, & 0 \leq d \leq M - P + 1 \\
\frac{1}{M - P + 2} \sum_{i=d}^{M+1} |h[i]|^2, & M - P + 2 \leq d \leq M \\
0, & M + 1 \leq d \leq N \\
\frac{1}{Q - 1} \sum_{i=0}^{Q - d - 1} |h[i]|^2, & N + 1 \leq d \leq Q - 1.
\end{cases}$$  

The quantity $\rho_d$ measures how the channel impulse response concentrates. A delay is determined as follows:

$$\delta = \arg \max_{0 \leq \delta \leq Q - 1} \rho_d.$$  

The best delay is thus determined by computing the energy of a part of the channel impulse response and searching for the point from which the maximum energy part starts. Unlike the method used in [12], this method is robust to the channel noise because the noise component vanishes in $\rho_d$. The validity of this method is shown by computer simulation in Section V.

IV. IMPLEMENTATION

The blind-channel shortening method presented in the previous section can force the IBI component at the equalizer output to be zero even in the presence of noise. Since this means that the channel can be shortened within the CP length, the method can perform perfect channel shortening. The proposed method can be summarized as follows: After the matrix $G_{\delta}$ specified in (19) is formed by linearly independent vectors that minimize $J_{\delta}(g)$, the vector $\alpha_{\delta}$ that maximizes $J(\alpha_{\delta})$ is computed. Then the equalizer parameter vector $g$ is computed according to (18). In this section, we present batch and adaptive algorithms implementing the proposed method. Before the equalizer parameter vector is computed in the proposed algorithms, the decision delay is determined by using the algorithm in (25) with the time-average of $\rho_d$ over $B_0$ blocks

$$\hat{\rho}_d = \frac{1}{B_0} \sum_{k=0}^{B_0 - 1} r_{k+P-1+d} r_{k+Q-1+d}.$$  

A. Batch Algorithm

The batch algorithm estimates the correlation matrices $R_{\delta}$ and $R_r$ by time-averaging and then eigendecomposes them.

Step 1: Obtain the time average estimate of $R_{\delta}$ over $B_1$ blocks as

$$\hat{R}_{\delta} = \frac{1}{B_1} \sum_{k=1}^{B_1} \hat{r}[k] \hat{r}[k]^H,$$

where

$$\hat{r}[k] = r[(k - 1)Q + P - 1 + \delta] - r[(k - 1)Q + Q - 1 + \delta].$$

Step 2: Eigendecompose $\hat{R}_{\delta}$ and obtain eigenvalues $\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{Lp-1}$ and corresponding eigenvectors $g_i$.

Step 3: Estimate the dimension of the noise subspace of $\hat{R}_{\delta}$ by using the MDL signal rank test [15]

$$q = Lp - \arg \min_{0 \leq q \leq Lp - 1} \text{MDL}[q]$$

and form $G_{\delta}$.  

TABLE I

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$d_0$</th>
<th>$L_0$</th>
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<tbody>
<tr>
<td>Case 1</td>
<td>$0 \leq \delta \leq L + M - P$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Case 2</td>
<td>$L + M - P + 1 \leq \delta \leq L + M - 1$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Case 3</td>
<td>$L + M \leq \delta \leq N$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Case 4</td>
<td>$N + 1 \leq \delta \leq Q - 1$</td>
<td>$\delta - N$</td>
</tr>
</tbody>
</table>
Step 4: Obtain the time average estimate of $R_r$ over $B_2$ blocks as

$$
\hat{R_r} = \frac{1}{B_2} \sum_{k=1}^{B_2} r[k]r^H[k].
$$

Step 5: Eigendecompose $G_q^H \hat{R}_r G_q$ and obtain the eigenvector $a_q$ corresponding to the maximum eigenvalue.

Step 6: Obtain the parameter vector as $g = G_q a_q$.

B. Adaptive Algorithm

In the adaptive algorithm, vectors that minimize $J_0$ are obtained by using the adaptive MERRY algorithm to update $L_p$ vectors with different initial values in parallel. A stochastic gradient technique is used to obtain $a_q$.

Step 1: Initialization:

Determine step gains $\mu$ and $\mu_a$ appropriately. Let $e_i$ be the $i$th coordinate unit vector. Set the variables as follows:

$$
ge_i[0] = e_{i+1}, \quad i = L_p - 1, L_p - 2, \ldots, 0$$

$$
a_i[0] = [a_0[0] \ a_1[0] \ \cdots \ a_{L_p-1}[0]]^T = [1 \ 0 \ \cdots \ 0]^T \quad k = 1.$$

Step 2: Estimation of eigenvectors:

$$
r[k] = r[(k-1)Q + P - 1 + \delta] - r[(k-1)Q + Q - 1 + \delta]$$

$$
\varepsilon_i[k] = g_i^H[(k-1)\hat{r}[k]]$$

$$
g_i[k] = \frac{g_i[k-1] - \mu \varepsilon_i[k] r[k]}{||g_i[k-1] - \mu \varepsilon_i[k] r[k]||}, \quad i = 0, 1, \ldots, L_p - 1.$$

Step 3: Equalizer output power maximization:

$$
G[k] = [g_0[k] \ g_1[k] \ \cdots \ g_{L_p-1}[k]]$$

$$
y_{k|Q} = a_i^H(k-1)G^H[k]r[(k-1)Q + Q - 1 + \delta]$$

$$
a_i[k] = a_i[k-1] + \mu a_i G^H[k]r[(k-1)Q + Q - 1 + \delta]$$

$$
g[k] = G[k]a_i[k].$$

Step 4: Set $k = k + 1$ and repeat Steps 2 through 3 until convergence.

In the algorithm, there is not any procedure for making the estimated eigenvectors linearly independent. One might be concerned that the vectors could be the same. Interestingly, in our simulation of static channels, the vectors differed from each other by setting their initial values different from each other. Moreover, in the algorithm, all possible $L_p$ vectors are updated at the same time and the estimation procedure of the dimension of noise subspace is omitted. Indeed, there are useless vectors among $L_p$ vectors since the dimension of the noise subspace of $R_S$ is less than $L_p$. This would slightly increase the computational complexity. Nevertheless, it is highly desirable that it need not estimate the noise subspace dimension. There would be more sophisticated algorithms, but this simple algorithm works as shown in the next section and is meaningful for the adaptive implementation of our SINR maximization approach.

We now consider the convergence property of the proposed adaptive algorithm. The proposed adaptive algorithm consists of two parts, i.e., the adaptation of $L_p$ $g$s and that of $a$. Both of them are based on the minimum (maximum) variance method with norm constraint as in the MERRY algorithm [11]. The convergence analysis of the MERRY algorithm has been shown in [11] and can be applied to ours directly. Consequently the proposed algorithm can be shown to be global convergent.

C. Computational Complexity

In the batch algorithm, the computational complexities of each step are $O(L_p^2P^2B_2)$ for Step 1, $O(L_p^3P^3)$ for Step 2, $O(L_p)$ for Step 3, $O(L_p^2P^2B_2)$ for Step 4, $O(L_p^2P^2q)$ for Step 5, and $O(L_pq)$ for Step 6. The major computational cost is to compute eigendecomposition of the correlation matrices. Its total computational complexity is proportional to $L_p^3P^3$. Although, the complexity of the batch algorithm is high, our preliminary simulation results showed that its performance is better than that of the proposed simpler adaptive algorithm when an amount of data is limited. In the adaptive algorithm, the computational complexities of Steps 2 and 3 are $O(L_p^2P^2)$ that is higher than that of the adaptive MERRY algorithm. Although the computational complexity of the proposed algorithms is not low, computational complexity should not be the major concern since microprocessors are getting faster all the time. Most mobile cost lies in RF and analog signal processing [17], [18].

V. SIMULATION RESULTS

A. Simulation Conditions

In this section, we show computer simulation results illustrating the performance of the proposed algorithms. The information bits were mapped onto QPSK symbols. A raised-cosine pulse $p(t)$ with a roll-off factor of 0.1 was used. The pulse was truncated to $6T_s$. The performance measure was the SINR defined as follows:

$$
\text{SINR} \triangleq \frac{E[|c_d^H u_{d^*}[k]|^2]}{E[|y_k - c_d^H u_{d^*}[k]|^2]} = \frac{\sigma_d^2||c_d||^2}{\sigma_d^2(||c_d||^2 - ||c_{d^*}||^2) + \sigma_w^2||g||^2} \quad (27)
$$

where $c_{d^*} \triangleq [c_d \ \cdots \ c_{d+P}]^T$, $u_{d^*}[k] \triangleq [u_{d,k-1}^* \ \cdots \ u_{d,k-P-1}^*]^T$, and $d^* \triangleq \arg \max_d ||c_d||^2$. Thus, $c_{d^*}$ represents the signal component that is the largest energy consecutive $P + 1$ samples. Average SINR was obtained by averaging the SINR over 100 different channels. The received SNR was defined by

$$
\text{SNR} \triangleq \frac{E[|r_k - w_k|^2]}{E[|w_k|^2]} = \frac{\sigma_w^2 \sum_{i=0}^{M} ||b[i]||^2}{\sigma_w^2}. \quad (28)
$$
for the determination of 
were zero-mean complex Gaussian
is capable of canceling IBI, its SINR performance
of combined 
which is incapable of the perfect IBI can-
 Unless otherwise stated, 
figures, the SINR tends to be high when 
This result shows that the proposed
algorithm is superior to the MERRY algorithm, which corresponds to \( q = 1 \).
We next compared the performance of the proposed algorithm with those of the MERRY algorithms. We had various algorithms use the decision delay determined by using in (25) with the value of \( \rho_d \) specified by (26). Fig. 5 shows the average SINR performance of various algorithms. Since IBI is dominant in high-SNR regions, the performance of the batch MERRY algorithm with \( p = 1 \), which is incapable of the perfect IBI cancellation, is very poor. Although the batch MERRY algorithm with \( p = 2 \) is capable of canceling IBI, its SINR performance is not good enough high because of noise enhancement especially in low-SNR region. The proposed batch algorithm performs best.

An original channel impulse response and the corresponding effective channel impulse response obtained by various batch algorithms are shown in Fig. 6. In the case of MERRY with \( p = 1 \), residual IBI can be seen. In the case of MERRY with \( p = 2 \), although IBI could be canceled, the magnitude of the effective channel impulse response equivalently, that of the desired signal component was not large enough. The proposed algorithm, however, could cancel IBI completely and provided a large effective channel response.

The CP length could be insufficient by accident or design. SINRs are plotted in Fig. 7 as a function of CP length. From

\[
h(t) = b_0 p(t) + b_1 p \left( t - \frac{T_s}{3} \right) + b_2 p(t - 5T_s) + b_3 p(t - 10T_s) + b_4 p(t - 15T_s) + b_5 p(t - 20T_s) + b_6 p(t - 25T_s) + b_7 p(t - 30T_s) + b_8 p(t - 35T_s).
\]

Channel coefficients \( b_i \) were zero-mean complex Gaussian random variables, and the length of the original channel impulse response was \( M + 1 = 41 \). Unless otherwise stated, SNR = 50 dB, \( B_0 = 10 \), and \( B_1 = B_2 = 1000 \).

We first examined the validity of \( \rho_d \) for the determination of a delay. For a given channel, SINR is plotted as a function of \( \delta \) in Fig. 2, where we can see that the SINR strongly depends on \( \delta \). Fig. 3 shows the autocorrelation \( \rho_d \). As can be seen in these figures, the SINR tends to be high when \( \rho_d \) is high.

We then examined the effect of combining linearly independent vectors in the noise subspace of \( \mathbf{R}_d \). The average SINR is shown in Fig. 4 as a function of the number \( q \) of combined vectors. In this case the dimension of the noise subspace of \( \mathbf{R}_d \) is theoretically 21. As expected, the SINR increases monotonically as long as \( q \leq 21 \). This result shows that the proposed

\[
\rho_d = \frac{1}{M+1} \sum_{t=0}^{M} h(t) h^*(t-d).
\]
C. Adaptive Algorithm

We set $N = 64$, $P = 8$, $L = 20$, $p = 2$, and $\text{SNR} = 30$ dB, and we considered four-ray multi-path channels resulting in an overall response of

$$h(t) = b_0 p(t) + b_1 p \left( t - \frac{T_s}{3} \right) + b_2 p(t - 5T_s) + b_3 p(t - 10T_s).$$

The length of the channel impulse response was $M + 1 = 16$. The step gains for the proposed algorithm were set to $\mu = 2 \times 10^{-3}$ and $\mu_e = 10^{-2}$, and the step gains of MERRY with $p = 1$ and MERRY with $p = 2$ were respectively set to $\mu = 2 \times 10^{-3}$ and $\mu = 2 \times 10^{-3}$. These step gains were determined such that the IBI component in the effective channel impulse response was as small as possible and SNR is as large as possible after 2,000 iterations. Average SNR for a given channel was obtained by averaging the SNR for 10 different ensembles.

The SINR curves are shown in Fig. 9, where we can see that the proposed adaptive algorithm can achieve a higher SINR than the adaptive MERRY algorithm. Fig. 10 shows an example of the original channel impulse response and the corresponding effective channel impulse responses obtained by various adaptive algorithms. As in the case of batch algorithms, the proposed adaptive algorithm is superior to the conventional algorithms. From these results, we could show the possibility of adaptive implementation of our SINR maximization approach.
As for adaptive algorithms, the application to dynamic channels is important. At this moment, the proposed adaptive algorithm is not aimed to cope with the time variation of channels. Hence, the algorithm should be modified to improve the tracking performance on fast time-varying channels.

The SNR in both simulations is rather high to show the superiority of the proposed algorithms clearly. We confirmed that the proposed algorithms work in a lower SNR as well.

VI. CONCLUSION

This paper proposed blind-channel shortening algorithms based on both batch and adaptive processing. Unlike the conventional algorithms, the proposed methods can not only remove IBI but also maximize the output SINR. We have considered the impact of the decision delay on the performance and proposed a simple method for determining the appropriate delay. Our simulation results demonstrate the superiority of the proposed methods over conventional ones.

REFERENCES

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