A New Method for Automatic Gross Error Detection in Remote Sensing Image Geometric Correction

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Abstract—Conventional gross error detection methods are mainly based on Gauss-Markov model and Least Squares Estimation, and are not adapted to gross error detection for control points of satellite remote sensing images, due to the serious ill-condition of satellite remote sensing imaging model and many iterations in the solving process. This paper proposed a method automatically detecting gross error of control points for geometric correction of satellite remote sensing images. This method substitutes the simple Least Squares Estimation with Levenberg-Marquardt (LM) algorithm, and removes one control point with the maximum standardized residual before updating the imaging model each time, until all the gross errors are eliminated. The disadvantages of traditional data detecting methods based on hypothesis testing were analyzed first, then a new approach determining the stopping point of gross error detection was put forward, that is clustering the absolute values of standardized residual differences. Experimental results and comparisons with other methods confirmed the validity of the proposed method.

Keywords-component; gross error; geometric correction; control point; ill-conditioned; Levenberg-Marquardt algorithm; difference clustering

I. INTRODUCTION

The study of classical geodetic and photogrammetric adjustment is based on the assumption that the observation contains only accidental error, while the current trend is to better consider the possible existence of gross error and systematic error in the adjustment process, and to insure the accuracy and reliability of adjustment results. For the adjustment of photogrammetry, the phase of solving the imaging model merely considering accidental error ended early in the 70s of last century, and since the late 60s last century, two types of error, which are both systematic error and accidental error or both gross error and accidental error, had begun to be processed by many scholars.

The systematic error of satellite imagery can be removed by the system-level geometric correction, and therefore the solving process of satellite geometric correction model only involves the accidental error and gross error. In theory, many studies focus on reliability of adjustment models, namely, the ability to find gross error and the impact of the undetectable gross error on the result of adjustment. In the practical side, efforts have been made to find effective and automatic method for gross error detection.

Studies for reliability are based on mathematical statistics and hypothesis testing. Classical hypothesis testing theory was proposed by Neymann and Pearson at 1933, while in the field of surveying adjustment, the reliability theory was first advanced by a Dutch professor Baarda at 1967[1,2]. Starting from a single one-dimensional alternative hypothesis, Baarda’s reliability theory studied the adjustment system’s capacity of finding a single model error and the effect of the undetectable model error on adjustment result. In addition, Baarda also deduced the snooping method, which uses standardized residuals subjecting to the normal distribution as the amount of statistical test. In 1985, Li Deren proposed the separability and reliability theory of adjustment model from the Gauss-Markov model that contains two multi-dimensional alternative hypotheses [3].

A practical aspect of reliability theory is to find a viable way to identify gross error, which diverges into two collateral branches. Derived from Baarda’s reliability theory, the first approach uses the data detection method or distribution detection method and brings gross error into the function model to discover and eliminate gross error. The other approach is to bring gross error into the adjustment random model, and use option function method, which gives small weights to the observations with gross error in the successive iterations, automatically removing gross errors. In this paper, we focus on the method automatically detecting gross error of control points with the first approach, which can be applied to satellite remote sensing image geometric correction.

The remainder of this paper is organized as follows. The conventional gross error detection model is described in Section2, a gross error detection method based on LM algorithm is described in Section3, and the method of determining the hold point of gross error detection is described in Section4, experiment analysis and estimation are provided in Section5. Finally, conclusions are drawn in Section6.

II. CONVENTIONAL GROSS ERROR DETECTION METHOD

In the process of satellite remote sensing image geometric correction, there are mainly two steps, the first step is to solve the satellite remote sensing image geometric correction model...
using the ground control points (GCPs), and the second is to correct the image with the geometric corrected model. Geometric accuracy of corrected image is relevant not only to the choice of correction model, but also to the accuracy of model solution, which depends on the quality of control points, the accuracy of initial value and the accuracy of model solution algorithm.

A GCP observation includes three ground coordinates and its corresponding image pixel coordinates. As satellite remote sensing image geometric correction requires a great number of GCP observations, an important source of which is image auto-matching, so there may be gross error in observations. If gross error in the observation is not effectively detected and rejected, the accuracy of the remote sensing image geometric correction model will be reduced, and then the corrected image will also be lack of accuracy.

In order to overcome the impact of blunder observations, the traditional data detection method performs the gross error detection in the following steps:

Firstly, we need to build an adjustment model, and the most common linear model is Gauss-Markov Model, (for the nonlinear model, the most common method is to linearize the model according to Taylor's theorem.)

\[ V = AX - L \]
\[ \Sigma_i = \sigma^2 P^{-1} \]  
(2.1)

where:

- \( L \) is the n-by-1 observation vector,
- \( \Sigma_i \) is the n-by-n covariance vector,
- \( P \) is the n-by-n observation weight matrix,
- \( \sigma^2 \) is the unit weight variance,
- \( A \) is the n-by-m coefficient matrix,
- \( X \) is the m-by-1 unknown parameter vector,
- \( V \) is the n-by-1 correction vector.

Then seek the least-squares solution \( \hat{X} \) of unknown parameter vector \( X \):

\[ \hat{X} = (A^T P A)^{-1} A^T P V \]  
(2.2)

Correction vector \( V \) can be calculated by substituting \( \hat{X} \) into formula (2.1), and the estimation of the unit weight variance \( \sigma^2 \) can be calculated as:

\[ \hat{\sigma}^2 = \frac{V^T P V}{r} \]  
(2.3)

where \( r \) is the number of redundant observations.

Calculate the correlation coefficients matrix \( Q_{vv} \),

\[ Q_{vv} = P^{-1} - A(A^T P A)^{-1} A^T \]  
(2.4)

Calculate standardized residuals \( w_i \) of observations according to the results above, and apply the Student's t-test to the standardized residuals \( w_i \),

\[ w_i = \frac{v_i}{\hat{\sigma}_0 \sqrt{q_{v_i}}} \sim t_{n-r} \]  
(2.5)

where \( v_i \) is the \( i \)-th observation correction,

\( q_{v_i} \) is the \( i \)-th diagonal entry of matrix \( Q_{vv} \),

\( t_{n-r} \) denotes t-distribution with \( n - r \) degrees of freedom.

Giving a significance level \( \alpha_0 \) (for instance \( \alpha_0 = 0.05 \)), look up the critical value \( c \) in the Student’s t-distribution table. To determine whether any gross error is present at observation \( l_i \), there are following hypotheses:

- \( H_0 \): no gross error is present at observation \( l_i \)
- \( H_1 \): observation \( l_i \) is a gross error

If the standardized residuals \( |w_i| \leq c \), the zero hypothesis is admitted at a level of significance \( \alpha_0 \), and observation \( l_i \) is a normal value. Otherwise the zero hypothesis is rejected and the observation \( l_i \) is considered as a gross error at the level of significance \( \alpha_0 \).

However, the existing study has manifested that when applied to solution of satellite remote sensing imaging model, the unknown parameters in the formula (2.1) are strongly correlated, resulting in the near-linear dependency between the column vectors of matrix \( A \) in the Gauss-Markov model. Thus, normal matrix \( A^T P A \) will be severely ill-conditioned, even singular. In this case, the Least Square solution might be quite unstable and greatly deviate from the true value, and then we can not correctly obtain standardized residuals, which are the requisites of the gross error detection.

III. IMPROVEMENT OF GROSS ERROR DETECTION METHOD

A. Levenberg-Marquardt Algorithm

The LM algorithm was first created by Levenberg, and modified by Marquardt. It significantly outperforms gradient descent method and conjugate gradient method, and has been
widely applied in nonlinear optimization. As a combination of the steepest decent and the Gauss-Newton method, it inherits the global-search of gradient descent as well as the local-fast-converge of Gauss-Newton. At the same time, it overcomes the obstacle of ill-conditioned normal equations [4]. Making use of the second derivative, LM algorithm converges even much faster than the gradient descent method.

For a nonlinear error equation:

\[ V = f(X) - L \]  \( (3.1) \)

Suppose \( X_{(k)} \) is the \( k \) th iterative solution vector, then recurrence formula can be described as follow:

\[
\begin{align*}
X_{(k+1)} &= X_{(k)} + \Delta X_{(k)} \\
\Delta X_{(k)} &= -(\nabla^2 S_{(k)})^{-1}\nabla S_{(k)}
\end{align*}
\]

where \( S_{(k)} = V^T(k)V_{(k)} \) is the error criterion function, \( V_{(k)} \) is the error vector in the \( k \) th iteration, \( \Delta X_{(k)} \) is the solution vector of LM algorithm in the \( k \) th iteration, \( \nabla^2 S_{(k)} \) is the Hessian matrix of \( S_{(k)} \), \( \nabla S_{(k)} \) is the gradient vector of \( S_{(k)} \).

In the \( k \) th iteration, the solution of LM Algorithm is (D.W. Marquardt, 1963)

\[
\Delta X_{(k)} = -(J^T_{(k)}J_{(k)} + \mu_{(k)}I)^{-1}J^T_{(k)}V 
\]

where \( J_{(k)} \) the Jacobian matrix of \( S_{(k)} \), \( \mu_{(k)} > 0 \) is the damping coefficient in the \( k \) th iteration, \( I \) is the identity matrix.

If the damping coefficient \( \mu_{(k)} \equiv 0 \), the LM algorithm degenerates into Gauss-Newton method; if \( \mu_{(k)} \) is quite great, the LM algorithm is close to gradient descent algorithm. When \( \mu_{(k)} \) is great enough, we can have \((J^T_{(k)}J_{(k)} + \mu_{(k)}I)^{-1}\) positive definite and therefore reversible, so LM algorithm is quite stable when applied to solve an ill-conditioned model. In this paper, \( \mu_{(k)} \) is calculated by the formula \( \mu_{(k)} = \|J^T_{(k)}V\| \), and such \( \mu_{(k)} \) ensures the convergence of LM algorithm [5].

B. Gross Error Detection based on LM Algorithm

The geometric models of satellite remote sensing image are usually nonlinear and severely ill-conditioned, thus the conventional method based on least squares estimation cannot be applied to the GCPs' gross error detection. However, by substituting the least square method with LM algorithm, which has superiority in nonlinear optimization and is stable in ill-condition, we can develop a gross error detection method adjusted to the satellite remote sensing imaging models.

Considering the geometric model of satellite remote sensing images is calculated by all the data of control points, the presence of gross error control points can lead to inaccuracies in the model solution, thus affecting the accuracy of standardized residuals. In order to reduce the possibility of misjudgment, this paper adopts the strategy eliminating gross error points one by one. Each time we exclude a control point of the largest standardized residual, and recalculate the model with new standardized residuals by the remaining control points, until all the gross error control points are removed.

Taking SPOT5 data as an example, the process of automatic gross error detection of GCP is shown in Figure 1.

![Flow chart of gross error GCP detection taking SPOT5 image as an example](image)

**Figure 1. Flow chart of gross error GCP detection taking SPOT5 image as an example**

IV. STOPPING CONDITION FOR GROSS ERROR DETECTION

A. Traditional Decision Method

In conventional detection methods, the classical hypothesis-testing theory is the basic tool for discerning gross error, and there are specifically three ways of hypothesis-testing.

The first one is to use all the data of control points to perform the hypothesis testing, and determine all the gross errors once. Considering that the geometric model of satellite
remote sensing images is calculated by all the data of control points, the presence of gross error control points can lead to inaccuracies in the model solution, thus affecting the accuracy of standardized residuals.

The second approach is to eliminate gross errors one by one. Each time we exclude a control point of the largest standardized residual, and recalculate the model with new standardized residuals by the remaining control points, until all the gross error control points are removed. The difficulty for this tactics is how to choose of confidence level.

In the third approach, multi-dimensional hypothesis testing is used to find observations that are gross errors. As the number of combinations for multi-dimensional hypothesis testing is extremely large, the exhaustivity of the combinations is generally impossible. Sarjakoski was the first to propose that artificial intelligence can be applied to the heuristic search of gross error [6]. If multiple blunders are involved in the observation data and the test statistics containing gross error are correlated strongly, Sarjakoski’s method evidently excels the traditional data detection method [3]. Unfortunately, the heuristic search of gross error is still time consuming.

B. Hold Point Auto-determination Based on Difference Clustering

In order to reduce the possibility of misjudgment, this paper adopts the strategy eliminating gross error points one by one. Then, through the simulating test, we established the process of successive removal of the control point with largest standardized residual, and got the standardized residuals curves, which demonstrate significant characteristics.

When the control points without gross error are abundant, the standardized residual curve is consequently flat and horizontal; when the data of control points contains outliers, the tendencies of curve vary at the blunders ---- may be a sharp decrease, or a sharp increase, or almost no change; at the last gross error, however, the curve is bound to descend suddenly; when the number of control points is quite small, the curve fluctuates very much, even if no gross error exist in the observation data.

According to the characteristics of standardized residual curve, this paper developed a new approach based on difference clustering to determining the stopping point.

Firstly, we need to make a hypothesis for the maximum proportion of gross error in the data of control points. In error theory, breakdown point is the fraction of data that can be given arbitrary values without making the estimator arbitrarily bad. The notion of breakdown point is proposed by Hampel and developed by Donoho and Huber, has a relatively mature theoretical system now. And existing studies have shown that, for different estimation methods, the breakdown point is generally 30% -50% [8]. Consequently, we can assume that the proportion of blunder is no greater than 30%; otherwise the observation data will be too unreasonable to use.

On the basis of analysis of standardized residual curve, denoting the curve trends with the absolute value of difference between two adjacent standardized residuals, the stopping point will appear in the last point where the absolute value of difference is anomalous large. Therefore, the clustering method can be used to divide absolute values of difference into different classes, and the hold point will be in the classes of large values. In this paper, this method is named “difference clustering” for short. Taking into account that the values’ dispersity is uncertain, a self-adaptive clustering algorithm is proposed with the ISODATA method. Comparing with other clustering methods, this algorithm has some further refinements by splitting and merging of clusters. Clusters are merged if either the number of members in a cluster is less than a certain threshold or if the centers of two clusters are closer than a certain threshold. Clusters are split into two different clusters if the cluster standard deviation exceeds a predefined value and the number of members is twice the threshold for the minimum number of members [9].

The following is the specific procedure of stopping point auto-determination based on difference clustering:

1) Remove the top 30% control points one by one, following the principle of rejecting the maximum standardized residual, and record the excluded standardized residuals $\sigma_1, \sigma_2, \ldots, \sigma_n$ in sequence.

2) Successively calculate absolute value of difference between two adjacent standardized residuals:

$$\delta_i = |\sigma_{i+1} - \sigma_i|, \quad (i = 1, 2, \ldots, n-1)$$

3) Cluster the absolute values of difference using ISODATA clustering algorithm, and suppose $\delta_1, \delta_2, \ldots, \delta_{n-1}$ can be clustered into $k$ clusters:

$$\{\delta_{n_{i1}}, \delta_{n_{i2}}, \ldots, \delta_{n_{ik}}\}, \quad (i = 1, 2, \ldots, k)$$

where $n_{i1} < n_{i2} < \ldots < n_{ik}$.

4) Get the clustering centers of the $k$ clusters $c_1, c_2, \ldots, c_k$, and make sure that $c_1$ is the minimal among the clustering centers.

5) If $k = 1$, no gross error is present in the data of control points; if $k > 1$, find the maximum $n_{i_{1j}}$ in $n_{22j}, n_{33j}, \ldots, n_{k3j}$, and it is the stopping point for gross error detection.

V. TEST AND RESULT ANALYSIS

Two Landsat5 multi-spectral images were chosen in the test, and we have the data of ground points, some of which contain gross error. The GCPs were collected automatically by image matching with ERDAS 9.0’s AutoSync module which gives hundreds of points. In data-A, 60 points were selected as GCPs adding gross errors (4-6 pixels) into 7 points. In data-B, 35 points were selected as GCPs, adding gross errors (3-4 pixels) into 5 points.
For each data set, we used three approaches based on the LM algorithm to detect the outliers. The first two methods are based on hypothesis testing. One is to determine all the gross errors once, in which two different confidence levels (0.95 and 0.99) were selected, and the other is to reject the gross errors one by one until the success of hypothesis testing, with respectively three different confidence levels (0.95, 0.97 and 0.99). The last approach is to exclude the outliers one by one, and determine the stopping point by the method of difference clustering. The results are shown in Table 1 and Table 2.

<table>
<thead>
<tr>
<th>Gross Error Detection Method</th>
<th>Confidence Level</th>
<th>Correct Judgments</th>
<th>Erroneous Judgments</th>
<th>Missed Gross Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>determine all the gross errors once</td>
<td>0.95</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>remove gross errors one by one</td>
<td>0.95</td>
<td>7</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.97</td>
<td>7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>based on difference clustering</td>
<td>NULL</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE II. TEST RESULT OF DATA-B

<table>
<thead>
<tr>
<th>Gross Error Detection Method</th>
<th>Confidence Level</th>
<th>Correct Judgments</th>
<th>Erroneous Judgments</th>
<th>Missed Gross Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>determine all the gross errors once</td>
<td>0.95</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>remove gross errors one by one</td>
<td>0.95</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.97</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>based on difference clustering</td>
<td>NULL</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In the last approach, we used difference clustering to classify the absolute values of difference of data. Figure 2 and figure 3 respectively show the standardized residual curves of data-A and data-B, which show the change trends of largest standardized residuals in the process of successive eliminations. Vertical coordinates represent the largest standardized residual in each iteration. The 7th point is the last point with gross error in figure 2, and the 5th point is the last point with gross error in figure 3. However, in the graph of standardized residual curve, it is difficult to identify the last control point with gross error, but we can easily identify it through clustering of the absolute values of difference. Figure 4 and Figure 5 respectively show the clustering results of the absolute values of differences between adjacent standardized residuals in figure 2 and figure 3. Clusters of large values are inside the solid circles and the cluster of small values is inside the dashed circle. The absolute values of difference of data-A are classified into 4 classes, and the 7th value is the last one in the clusters of large values, so it is the last anomalously large difference, thus 7th control point is the hold point in gross error detection, and the first 7 points removed are the control points with gross error detected with this method; the values of data-B are classified into two classes, and the last anomalously large value is the 5th, so the first 5 points removed are the control points with gross error detected. Both of the two clustering results successfully identified the stopping points of the gross error detection process.

We can conclude the followings from the tests. Firstly, by applying the LM algorithm, three detecting methods all overcome the ill-condition of satellite remote sensing geometric model. Secondly, the results in Table 1 show the first method is more likely to misjudge the gross errors than the others, and the second method makes right detection if confidence level is appropriately chosen. However, the proper confidence level varies in different tests. The third method is shown better stability than the first two methods.
VI. CONCLUSIONS

1) Instead of least squares estimation, we used LM algorithm in this paper to improve the traditional gross error identification method based on Baarda reliability theory. The improved method adapts to ill-conditioned satellite remote sensing imaging model, and can be effectively applied to the gross error detection of observed values for the control points.

2) The hypothesis testing method based on statistics is a simple and absolute decision rule, and we have to choose different hypothesis testing methods and the corresponding significance level according to different needs. According to the tests, when the gross error control points are greater than 3 pixels, a significance level at around 3% is relatively proper for effectively detecting the gross error control points.

3) The method of automatically determining the stopping point based on difference clustering can simulate the process of successive removal of the control point with largest standardized residual, and determine the stopping point automatically according to characteristics of standardized residuals curve. ISODATA clustering algorithm allows for different number of clusters, this method guarantees that all of the small differences (normal differences) of standardized residuals can be distinguished accurately. Tests showed that the gross error detection method based on LM algorithm and difference clustering can automatically identify all the outliers larger than three pixels in the control points used for satellite remote sensing image geometric correction.

4) When correlation between the gross error observations is strong, standardized residuals of the normal values may also be large, so the method only using the absolute values of standardized residuals to judge gross error may result in misjudgments.

5) The method of automatically determining the stopping point based on difference clustering needs to generate the standardized residuals curve first, which may cause the calculation of excess information, and lead to inefficiency. When the difference between gross error and the true value is small, this unsupervised method may cause misjudgments, and in that case, we need to modify the parameters of ISODATA algorithm. So how to improve the adaptability and robustness of the algorithm has yet to be further studied.

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