Modelling of coupled heat and moisture transfer in porous construction materials

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\textbf{A B S T R A C T}

In this paper, a mathematical model is proposed for predicting air temperature and humidity in a two-room region. The model contains a coupled relationship between temperature and humidity within the constructions and can be solved by using the numerical method. However, the two-room region can be reduced to a single region when the region with no ventilation is considered, and then the room temperature and relative humidity can be obtained analytically. The solution obtained in this paper is verified by comparing with the result of the analytical method. It shows that the two results are in agreement. In addition, the proposed model can also be applied to simultaneously obtain the transient temperature and humidity of a two-room region for different porous construction materials.

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1. Introduction

In general, house constructions involve the use of porous materials, which have the effect of stabilizing room air humidity by their moisture absorption or desorption, and room air temperature by their small heat diffusivity under external varying weather. The comfortable room temperature and relative humidity ranges for residential living in summer are 26 °C–28 °C and 40%–60%, respectively, and 23 °C–25 °C and 50%–70% in winter, respectively. A room climate that is too humid and too hot affects human health\textsuperscript{3}. Air conditioning is the most convenient method to maintain a desirable room climate, but it is too energy-consuming. Therefore, an alternative energy-saving method for maintaining comfortable room temperature and relative humidity is desirable.

There are many mathematical models to predict room temperature\textsuperscript{4–6} and calculate room humidity\textsuperscript{7–9}. However, houses are constructed of porous materials in which the existing coupled effect of the heat and moisture is not negligible\textsuperscript{10,11}, and the room temperature and humidity conditions are also coupled with the constructions. However, only a limited portion of the literature is concerned with coupled heat and moisture in buildings. In this paper, a mathematical model and its solution simulating indoor air temperature and relative humidity of the two-room region is proposed. The model takes into account a coupled effect of heat and moisture in house constructions, and the unknown room temperature and humidity are also coupled with the constructions. The model can be used to predict the indoor temperature and relative humidity with ventilation for single layer constructions of various materials which have been used in Taiwan.

2. Description of the model

A new model has been developed for predicting the temperature and relative humidity of the two-room region as shown in Fig. 1. The model combines the heat and moisture balance of the indoor air and the hygrothermal performance of the
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>wall cross section area [$m^2$]</td>
</tr>
<tr>
<td>$c_p$</td>
<td>heat capacity [$J/kg K$]</td>
</tr>
<tr>
<td>$D$</td>
<td>equivalent diffusion coefficient of moisture content [$m^2/s$]</td>
</tr>
<tr>
<td>$D_m$</td>
<td>diffusion coefficient of moisture content [$m^2/s$]</td>
</tr>
<tr>
<td>$D_{mV}$</td>
<td>vapor diffusion coefficient due to moisture gradient [$m^2/s$]</td>
</tr>
<tr>
<td>$D_{mV}$</td>
<td>vapor diffusion coefficient due to temperature gradient [$m^2/s K$]</td>
</tr>
<tr>
<td>$\dot{G}$</td>
<td>ventilation rate between room and external atmosphere [$m^3/s$]</td>
</tr>
<tr>
<td>$h_c$</td>
<td>convective heat transfer coefficient [$W/m^2 K$]</td>
</tr>
<tr>
<td>$h_m$</td>
<td>convective moisture transfer coefficient [$m/s$]</td>
</tr>
<tr>
<td>$h_{LV}$</td>
<td>heat of phase change [$kJ/kg$]</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity [$W/m K$]</td>
</tr>
<tr>
<td>$L$</td>
<td>equivalent diffusion coefficient of temperature [$m^2/s$]</td>
</tr>
<tr>
<td>$\ell$</td>
<td>thickness of the wall [$m$]</td>
</tr>
<tr>
<td>$m$</td>
<td>moisture content [$kg/kg$]</td>
</tr>
<tr>
<td>$m_0$</td>
<td>initial moisture content [$kg/kg$]</td>
</tr>
<tr>
<td>$\tilde{m}$</td>
<td>Laplace transformation of $m$</td>
</tr>
<tr>
<td>$M$</td>
<td>input moisture [$kg/s$]</td>
</tr>
<tr>
<td>$p_s$</td>
<td>saturated pressure at constant temperature [$kPa$]</td>
</tr>
<tr>
<td>$\dot{P}$</td>
<td>ventilation rate between two rooms [$m^3/s$]</td>
</tr>
<tr>
<td>$S$</td>
<td>Laplace transformation parameter</td>
</tr>
<tr>
<td>$t$</td>
<td>time [$s$]</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature [$K$]</td>
</tr>
<tr>
<td>$T_0$</td>
<td>initial temperature [$K$]</td>
</tr>
<tr>
<td>$\tilde{T}$</td>
<td>Laplace transformation of $T$</td>
</tr>
<tr>
<td>$w$</td>
<td>room capacity [$m^3$]</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\rho_0$</td>
<td>material density [$kg/m^3$]</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>air density [$kg/m^3$]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>coupling coefficient due to moisture migration [$K$]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>coupling coefficient due to heat conduction [$1/K$]</td>
</tr>
<tr>
<td>$\phi$</td>
<td>relative humidity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>heat of adsorption or desorption [$kJ/kg$]</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>initial state</td>
</tr>
<tr>
<td>E</td>
<td>external walls and ceiling</td>
</tr>
<tr>
<td>F</td>
<td>floor</td>
</tr>
<tr>
<td>i</td>
<td>room ($i = 1, 2$)</td>
</tr>
<tr>
<td>I</td>
<td>internal wall</td>
</tr>
<tr>
<td>L</td>
<td>liquid</td>
</tr>
<tr>
<td>m</td>
<td>moisture</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>$\infty$</td>
<td>ambient atmosphere.</td>
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</tbody>
</table>

building constructions. The temperature and humidity in the constructions are coupled and the room temperature and humidity of the rooms are also coupled with those of the constructions. To simplify the problem, the following assumptions are made.

1. All material properties in the simulation are constant.
2. Heat flow and moisture migration in the building are assumed one-dimensional.
3. The dwelling is assumed to be flat-roofed.
4. The effect of gravity is considered to be negligible.
5. The pressure is considered to be constant and uniform throughout the constructions.
6. No doors are included in the model.
7. The effect of radiation is not considered.
3. The coupled system

Based on the above assumptions, the basic equations with coupled temperature and moisture content in a porous construction can be obtained [1]. Thus all the constructions in a building can be expressed as follows:

**Internal wall**

\[
L_i \frac{\partial^2 T_i}{\partial x^2} = \frac{\partial T_i}{\partial t} - \nu_i \frac{\partial m_i}{\partial t}, \quad 0 < x < L_i
\]  
\[1a\]

\[
D_i \frac{\partial^2 m_i}{\partial x^2} = \frac{\partial m_i}{\partial t} - \lambda_i \frac{\partial T_i}{\partial t}, \quad 0 < x < L_i
\]  
\[1b\]

\[
k_i \frac{\partial T_i(0, t)}{\partial x} = h_{CI} [T_i(0, t) - T_1(t)]
\]  
\[2a\]

\[-k_i \frac{\partial T_i(L_i, t)}{\partial x} = h_{CI} [T_i(L_i, t) - T_2(t)]
\]  
\[2b\]

\[
\rho_{oE}D_{mi} \frac{\partial m_i(0, t)}{\partial x} = h_{mi} [m_i(0, t) - m_1(t)]
\]  
\[2c\]

\[-\rho_{oE}D_{mi} \frac{\partial m_i(L_i, t)}{\partial x} = h_{mi} [m_i(L_i, t) - m_2(t)]
\]  
\[2d\]

\[
T_i(x, 0) = T_{i0}(x)
\]  
\[3a\]

\[
m_i(x, 0) = m_{i0}(x)
\]  
\[3b\]

**External walls and ceiling**

\[
L_{Ei} \frac{\partial^2 T_{Ei}}{\partial y^2} = \frac{\partial T_{Ei}}{\partial t} - \nu_{Ei} \frac{\partial m_{Ei}}{\partial t}, \quad 0 < y < L_{Ei} \quad i = 1, 2
\]  
\[4a\]

\[
D_{Ei} \frac{\partial^2 m_{Ei}}{\partial y^2} = \frac{\partial m_{Ei}}{\partial t} - \lambda_{Ei} \frac{\partial T_{Ei}}{\partial t}, \quad 0 < y < L_{Ei} \quad i = 1, 2
\]  
\[4b\]

\[
k_{Ei} \frac{\partial T_{Ei}(0, t)}{\partial y} = h_{CEi} [T_{Ei}(0, t) - T_i(t)], \quad i = 1, 2
\]  
\[5a\]

\[-k_{Ei} \frac{\partial T_{Ei}(L_{Ei}, t)}{\partial y} = h_{CEi} [T_{Ei}(L_{Ei}, t) - T_\infty(t)], \quad i = 1, 2
\]  
\[5b\]

\[
\rho_{oE}D_{mEi} \frac{\partial m_{Ei}(0, t)}{\partial y} = h_{mEi} [m_{Ei}(0, t) - m_i(t)], \quad i = 1, 2
\]  
\[5c\]

\[-\rho_{oE}D_{mEi} \frac{\partial m_{Ei}(L_{Ei}, t)}{\partial y} = h_{mEi} [m_{Ei}(L_{Ei}, t) - m_\infty(t)], \quad i = 1, 2
\]  
\[5d\]

\[
T_{Ei}(y, 0) = T_{Ei0}(y), \quad i = 1, 2
\]  
\[6a\]

\[
m_{Ei}(x, 0) = m_{Ei0}(y), \quad i = 1, 2
\]  
\[6b\]
In addition to the above equations, the heat and mass in the air of the rooms must remain in balance. For room 1, we have

$$w_1 \rho_{a1} c_{p1} \frac{dT_1}{dt} - A_1 k_1 \frac{dT_1}{dx} |_{x=0} - 4A_{E1} k_{E1} \frac{dT_{E1}}{dy} |_{y=0} - A_{F1} k_{F1} \frac{dT_{F1}}{dz} |_{z=0} - \dot{C}_1 \rho_{a1} c_{p1}(T_{\infty} - T_1) - \dot{P} \rho_{a1} c_{p1}(T_2 - T_1) = H_1(t)$$

and

$$w_1 \rho_{a1} m_{1} \frac{dm_1}{dt} - A_1 \rho_{a1} D_{m1} \frac{dm_1}{dx} |_{x=0} - 4A_{E1} \rho_{a1} D_{mE1} \frac{dm_{E1}}{dy} |_{y=0} - A_{F1} \rho_{a1} D_{mF1} \frac{dm_{F1}}{dz} |_{z=0} - \dot{C}_1 \rho_{a1}(m_{\infty} - m_1) - \dot{P} \rho_{a1}(m_2 - m_1) = M_1(t)$$

where $\dot{P}$ represents ventilation rate between two rooms and $\dot{C}_1$ is the ventilation rate between room 1 and external atmosphere; $H_1(t)$ and $M_1(t)$ are input heat and moisture in room 1, respectively.

Substituting the corresponding boundary conditions into the above equations, we can obtain the following forms:

$$w_1 \rho_{a1} c_{p1} \frac{dT_1}{dt} - \xi_1 T_1(t) + A_1 h_{E1} T_1(0, t) + 4A_{E1} h_{CE1} T_{E1}(0, t) + A_{F1} h_{CF1} T_{F1}(0, t)$$

$$- \dot{C}_1 \rho_{a1} c_{p1}[T_{\infty}(t) - T_1(t)] - \dot{P} \rho_{a1} c_{p1}[T_2(t) - T_1(t)] = H_1(t)$$

and

$$w_1 \rho_{a1} m_{1} \frac{dm_1}{dt} - \psi_1 m_1(t) + A_1 h_{m1} m_{11}(0, t) + 4A_{E1} h_{CE1} m_{E1}(0, t) + A_{F1} h_{CF1} m_{F1}(0, t)$$

$$- \dot{C}_1 \rho_{a1}(m_{\infty} - m_1) - \dot{P} \rho_{a1}(m_2 - m_1) = M_1(t).$$

Similarly, for room 2, the following equations exist

$$w_2 \rho_{a2} c_{p2} \frac{dT_2}{dt} - \xi_2 T_2(t) + A_1 h_{E2} T_2(\ell_1, t) + 4A_{E2} h_{CE2} T_{E2}(0, t) + A_{F2} h_{CF2} T_{F2}(0, t)$$

$$- \dot{C}_2 \rho_{a2} c_{p2}[T_{\infty}(t) - T_2(t)] - \dot{P} \rho_{a2} c_{p2}[T_1(t) - T_2(t)] = H_2(t)$$

and

$$w_2 \rho_{a2} m_{2} \frac{dm_2}{dt} - \psi_2 m_2(t) + A_1 h_{m2} m_{12}(0, t) + 4A_{E2} h_{CE2} m_{E2}(0, t) + A_{F2} h_{CF2} m_{F2}(0, t)$$

$$- \dot{C}_2 \rho_{a2}(m_{\infty} - m_2) - \dot{P} \rho_{a2}(m_1(t) - m_2(t)) = M_2(t)$$

where

$$\xi_i = A_1 h_{E1} + 4A_{E1} h_{CE1} + A_{F1} h_{CF1}, \quad i = 1, 2$$

$$\psi_i = A_1 h_{m1} + 4A_{E1} h_{mE1} + A_{F1} h_{mF1}, \quad i = 1, 2.$$
4. The solution of the problem

Taking Laplace transformation to Eqs. (11) and (12), we can obtain

\[
\begin{align*}
\mathcal{W}_{ni} & \mathcal{R}_{\text{at}} \mathcal{C}_{\text{p}} \left[ \mathcal{S} \mathcal{T}_t(S) - \mathcal{T}_t(0) \right] - \xi_t \mathcal{T}_t(S) + A_i \mathcal{H}_{\text{CH}} \mathcal{T}_t(S) + 4A_i \mathcal{H}_{\text{CH}} \mathcal{T}_t(S) + A_i \mathcal{H}_{\text{CH}} \mathcal{T}_t(S) \\
& - \mathcal{G}_i \mathcal{R}_{\text{at}} \mathcal{C}_{\text{p}} \left[ \mathcal{T}_t(0) - \mathcal{T}_t(S) \right] - \mathcal{P}_i \mathcal{R}_{\text{at}} \mathcal{C}_{\text{p}} \left[ \mathcal{T}_t(0) - \mathcal{T}_t(S) \right] (-1)^{i+1} \\
& = \mathcal{H}_i(S), \quad i = 1, 2 \\
\mathcal{W}_{ni} & \mathcal{R}_{\text{at}} \left[ \mathcal{S} \mathcal{\tilde{m}}_i(S) - \mathcal{\tilde{m}}_i(0) \right] + \mathcal{G}_i \mathcal{R}_{\text{at}} \mathcal{C}_{\text{p}} \left[ \mathcal{\tilde{m}}_i(S) - \mathcal{\tilde{m}}_i(0) \right] - 4A_i \mathcal{H}_{\text{CH}} \mathcal{\tilde{m}}_i(S) + A_i \mathcal{H}_{\text{CH}} \mathcal{\tilde{m}}_i(S) \\
& - \mathcal{G}_i \mathcal{R}_{\text{at}} \mathcal{C}_{\text{p}} \left[ \mathcal{\tilde{m}}_i(S) - \mathcal{\tilde{m}}_i(0) \right] - \mathcal{P}_i \mathcal{R}_{\text{at}} \mathcal{C}_{\text{p}} \left[ \mathcal{\tilde{m}}_i(S) - \mathcal{\tilde{m}}_i(0) \right] (-1)^{i+1} \\
& = \mathcal{\tilde{M}}_i(S), \quad i = 1, 2
\end{align*}
\]  

\[(14a)
\]

where

\[
\mathcal{T}_t = \begin{cases} 
\mathcal{T}_t(0, S) & \text{for } i = 1 \\
\mathcal{T}_t(t, S) & \text{for } i = 2 
\end{cases}
\]

and

\[
\mathcal{\tilde{m}}_i = \begin{cases} 
\mathcal{\tilde{m}}_i(0, S) & \text{for } i = 1 \\
\mathcal{\tilde{m}}_i(t, S) & \text{for } i = 2 
\end{cases}
\]

(14b)

The approach taken in this work is to first find solutions to the problems with the temperatures and the moistures of buildings written as functions of $$T_i(t)$$ and $$m_i(t)$$. Then these results are added to the application of Duhamel’s Theorem [13] to the differential equations for $$T_i(t)$$ and $$m_i(t)$$. We can obtain the following equations:

\[
\begin{align*}
T_j(x, t) &= \sum_{j=1}^{2}(T_j(t) - T_{j0})T_j(x, 0) + \int_0^t (T_j(\tau) - T_{j0}) \frac{\partial T_j^i(x, t - \tau)}{\partial \tau} d\tau \\
& + \sum_{i=3}^{4}(m_{j-2}(t) - m_{j0})T_j^i(x, 0) + \int_0^t (m_{j-2}(\tau) - m_{j0}) \frac{\partial T_j^i(x, t - \tau)}{\partial \tau} d\tau + T_{j0} \\
& \quad \text{for } j = 1, 2
\end{align*}
\]

\[(15a)
\]

\[
\begin{align*}
m_j(t, x) &= \sum_{j=1}^{2}(T_j(t) - T_{j0})m_j(x, 0) + \int_0^t (T_j(\tau) - T_{j0}) \frac{\partial m_j^i(x, t - \tau)}{\partial \tau} d\tau \\
& + \sum_{i=3}^{4}(m_{j-2}(t) - m_{j0})m_j^i(x, 0) + \int_0^t (m_{j-2}(\tau) - m_{j0}) \frac{\partial m_j^i(x, t - \tau)}{\partial \tau} d\tau + m_{j0} \\
& \quad \text{for } j = 1, 2
\end{align*}
\]

\[(15b)
\]

\[
\begin{align*}
T_{Ei}(y, t) &= (T_i(t) - T_{E0})T_{Ei}(y, 0) + \int_0^t (T_i(\tau) - T_{E0}) \frac{\partial T_{Ei}(y, t - \tau)}{\partial \tau} d\tau \\
& + (T_{E0}(t) - T_{E0})T_{Ei}^2 + \int_0^t (T_{E0}(\tau) - T_{E0}) \frac{\partial T_{Ei}^2(y, t - \tau)}{\partial \tau} d\tau \\
& + \int_0^t (m_i(\tau) - m_{E0}) \frac{\partial T_{Ei}^3(y, t - \tau)}{\partial \tau} d\tau + (m_{E0}(t) - m_{E0})T_{Ei}^3(y, 0) \\
& + \int_0^t (m_{E0}(\tau) - m_{E0}) \frac{\partial T_{Ei}^4(y, t - \tau)}{\partial \tau} d\tau + m_{E0}, \quad i = 1, 2
\end{align*}
\]

\[(15c)
\]

\[
\begin{align*}
m_{Ei}(y, t) &= (T_i(t) - T_{E0})m_{Ei}(y, 0) + \int_0^t (T_i(\tau) - T_{E0}) \frac{\partial m_{Ei}(y, t - \tau)}{\partial \tau} d\tau \\
& + T_{E0}(t)m_{Ei}^2(y, 0) + \int_0^t (T_{E0}(\tau) - T_{E0}) \frac{\partial m_{Ei}^2(y, t - \tau)}{\partial \tau} d\tau + (m_{i}(t) - m_{E0})m_{Ei}^3(y, 0) \\
& + \int_0^t (m_{i}(\tau) - m_{E0}) \frac{\partial m_{Ei}^3(y, t - \tau)}{\partial \tau} d\tau + m_{E0} \\
& + \int_0^t (m_{E0}(\tau) - m_{E0}) \frac{\partial m_{Ei}^4(y, t - \tau)}{\partial \tau} d\tau + m_{E0}, \quad i = 1, 2
\end{align*}
\]

\[(15d)
\]

\[
\begin{align*}
T_{Fi}(z, t) &= (T_i(t) - T_{F0})T_{Fi}(z, 0) + \int_0^t (T_i(\tau) - T_{F0}) \frac{\partial T_{Fi}(z, t - \tau)}{\partial \tau} d\tau \\
& + (m_i(t) - m_{F0})T_{Fi}^2(z, 0) + \int_0^t (m_i(\tau) - m_{F0}) \frac{\partial T_{Fi}^2(z, t - \tau)}{\partial \tau} d\tau + T_{F0}, \quad i = 1, 2
\end{align*}
\]

\[(15e)
\]
\[ m_{Fi}(z, t) = (T_i(t) - T_{Fo})m_{0i}^2(z, 0) + \int_0^t (T_i(\tau) - T_{Fo}) \frac{\partial m_{Fi}^1(z, t - \tau)}{\partial \tau} d\tau + (m_i(t) - m_{Fo})m_{0i}^2(z, 0) + \int_0^t (m_i(\tau) - m_{Fo}) \frac{\partial m_{Fi}^1(z, t - \tau)}{\partial \tau} d\tau + m_{Fo}, \quad i = 1, 2 \]  

(15f)

where \( T_i^1 \) and \( m_i^1 \) can be evaluated from Eqs. (1) and (2) with \( T_1(t) = 1 \) and \( T_2(t) = m_1(t) = m_2(t) = 0; T_i^2 \) and \( m_i^2 \), \( T_i^3 \) and \( m_i^3 \), and \( T_i^4 \) and \( m_i^4 \) are also determined with \( T_2(t) = 1, m_1(t) = 1, \) and \( m_2(t) = 1, \) respectively, and the others are zero. Using the same procedures, we can obtain \( T_{Fi}^j, m_{Fi}^j \) \((i = 1, 2, j = 1, 2, 3, 4)\) and \( T_{Fi}^j, m_{Fi}^j \) \((i = 1, 2, j = 1, 2)\) from Eqs. (4), (5), (7) and (8), respectively.

Applying Laplace transformation to Eq. (15), they become

\[ \tilde{T}_i(x, S) = \sum_{j=1}^{2} \tilde{T}_{Fi}^j(x, S)[S\tilde{T}_j(S) - T_{Fo}] + \sum_{j=3}^{4} \tilde{T}_{Fi}^j(x, S)[S\tilde{m}_{j-2}(S) - m_{0j}] + \frac{T_{Fo}}{S} \]  

(16a)

\[ \tilde{m}_i(x, S) = \sum_{j=1}^{2} \tilde{m}_{Fi}^j(x, S)[S\tilde{T}_j(S) - T_{Fo}] + \sum_{j=3}^{4} \tilde{m}_{Fi}^j(x, S)[S\tilde{m}_{j-2}(S) - m_{0j}] + \frac{m_{0j}}{S} \]  

(16b)

\[ \tilde{T}_{Fi}(y, S) = \tilde{T}_{Fi}^1(y, S)[S\tilde{T}_i(S) - T_{E0}] + \tilde{T}_{Fi}^2(y, S)[S\tilde{T}_\infty(S) - T_{E0}] \]  

\[ + \tilde{T}_{Fi}^4(y, S)[S\tilde{m}_i(S) - m_{E0}] + \frac{T_{E0}}{S} \]  

\[ i = 1, 2 \]  

(16c)

\[ \tilde{m}_{Fi}(y, S) = \tilde{m}_{Fi}^1(y, S)[S\tilde{T}_i(S) - T_{E0}] + \tilde{m}_{Fi}^2(y, S)[S\tilde{T}_\infty(S) - T_{E0}] \]  

\[ + \tilde{m}_{Fi}^3(y, S)[S\tilde{m}_i(S) - m_{E0}] + \frac{m_{E0}}{S} \]  

\[ i = 1, 2 \]  

(16d)

\[ \tilde{T}_{Fi}(z, S) = \tilde{T}_{Fi}^1(z, S)[S\tilde{T}_i(S) - T_{Fo}] + \tilde{T}_{Fi}^2(z, S)[S\tilde{m}_i(S) - m_{Fo}] + \frac{T_{Fo}}{S} \]  

\[ i = 1, 2 \]  

(16e)

\[ \tilde{m}_{Fi}(z, S) = \tilde{m}_{Fi}^1(z, S)[S\tilde{T}_i(S) - T_{Fo}] + \tilde{m}_{Fi}^2(z, S)[S\tilde{m}_i(S) - m_{Fo}] + \frac{T_{Fo}}{S} \]  

\[ i = 1, 2 \]  

(16f)

Substituting the above equations into Eq. (14), we can obtain the following matrix form:

\[ \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} \tilde{T}_i(S) \\ \tilde{T}_j(S) \\ \tilde{m}_1(S) \\ \tilde{m}_2(S) \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ I_1 \\ I_2 \end{bmatrix} \]  

(17)

where

\[ K_{ij} = (w_{1i} \rho_{1i} c_{P1} S + \hat{C}_{1i} \rho_{1i} c_{P1} + \hat{P}_{1i} \rho_{1i} c_{P1} + \xi_{1i}) \delta_{ij} - \hat{P}_{1i} \rho_{1i} c_{P1} \delta_{ij} - A_i h_{ci} S \tilde{T}_1^1(0, S) - 4 A_i h_{ci} S \tilde{T}_1^3(0, S) (\delta_{ij} + \delta_{ij}) - A_i h_{ci} S \tilde{T}_1^5(0, S) (\delta_{ij} + \delta_{ij}) \]  

\[ j = 1, 2, 3, 4 \]  

(18a)

\[ K_{2j} = (w_{1j} \rho_{1j} c_{P2} S + \hat{C}_{1j} \rho_{1j} c_{P2} + \hat{P}_{1j} \rho_{1j} c_{P2} + \xi_{1j}) \delta_{ij} - \hat{P}_{1j} \rho_{1j} c_{P2} \delta_{ij} - A_i h_{ci} S \tilde{T}_2^1(0, S) - 4 A_i h_{ci} S \tilde{T}_2^3(0, S) (\delta_{ij} + \delta_{ij}) - A_i h_{ci} S \tilde{T}_2^5(0, S) (\delta_{ij} + \delta_{ij}) \]  

\[ j = 1, 2, 3, 4 \]  

(18b)

\[ K_{3j} = (w_{1j} \rho_{1j} S + \hat{C}_{1j} \rho_{1j} + \hat{P}_{1j} \rho_{1j} + \psi_{1j}) \delta_{ij} - \hat{P}_{1j} \rho_{1j} \delta_{ij} - A_i h_{ci} S \tilde{m}_1^1(0, S) - 4 A_i h_{ci} S \tilde{m}_1^3(0, S) (\delta_{ij} + \delta_{ij}) - A_i h_{ci} S \tilde{m}_1^5(0, S) (\delta_{ij} + \delta_{ij}) \]  

\[ j = 1, 2, 3, 4 \]  

(18c)

\[ K_{4j} = (w_{1j} \rho_{1j} S + \hat{C}_{1j} \rho_{1j} + \hat{P}_{1j} \rho_{1j} + \psi_{1j}) \delta_{ij} - \hat{P}_{1j} \rho_{1j} \delta_{ij} - A_i h_{ci} S \tilde{m}_1^1(0, S) - 4 A_i h_{ci} S \tilde{m}_1^3(0, S) (\delta_{ij} + \delta_{ij}) - A_i h_{ci} S \tilde{m}_1^5(0, S) (\delta_{ij} + \delta_{ij}) \]  

\[ j = 1, 2, 3, 4 \]  

(18d)

\[ F_i = \tilde{H}_i(S) + \hat{C}_{i1} \rho_{1i} c_{F1} \tilde{T}_\infty + w_{1i} \rho_{1i} c_{F1} T_i(0) - A_i h_{ci} \begin{bmatrix} \tilde{T}_1^1 T_{Fo} + \tilde{T}_2^1 T_{Fo} + \tilde{T}_3^1 m_{Fo} + \tilde{T}_4^1 m_{Fo} \end{bmatrix} \]  

\[ - 4 A_i h_{ci} \begin{bmatrix} \tilde{T}_1^1(0, S) T_{Fo} + \tilde{T}_2^1(0, S) T_{Fo} + \tilde{T}_3^1(0, S) m_{Fo} + \tilde{T}_4^1(0, S) m_{Fo} \end{bmatrix} \]  

\[ - S \tilde{T}_\infty(S) \tilde{T}_1^2(0, S) - S \tilde{m}_\infty(S) \tilde{T}_1^4(0, S) - \frac{T_{Fo}}{S} \]  

\[ - A_i h_{ci} \begin{bmatrix} \tilde{T}_1^1(0, S) T_{Fo} + \tilde{T}_2^1(0, S) m_{Fo} - \frac{T_{Fo}}{S} \end{bmatrix}, \quad i = 1, 2 \]  

(18e)
The material properties and parameters used in the analysis are listed in Table 1 [11,16–18]. The convective heat transfer coefficient, \( h_C \), depends slightly on the nature of surface, particularly on their relative orientation. According to the Chartered Institute of Building Service Engineers Guide [18] and to simplify the manipulation process, in this paper we take \( h_{CE} = 3.5 \text{ W/m}^2 \text{ K} \) for the wall and ceiling and \( h_{CF} = 1.5 \text{ W/m}^2 \text{ K} \) for the floor. On the other hand, the measurement of mass transfer coefficient, \( h_m \), is a difficult task. The relationship between \( h_c \) and \( h_m \) can be approximately expressed as \( h_m \approx 0.001 h_c \) [19], accordingly, we use \( h_{mf} = h_{me} = 3.5 \times 10^{-3} \text{ m/s} \) and \( h_{mf} = 1.5 \times 10^{-3} \text{ m/s} \).

The time step calculation is taken as one hour. The relationship between relative humidity and moisture content is [19]

\[
\phi = \frac{0.1013 \times m}{(0.622 + m) \times p_s}
\]

where \( p_s \) represents the saturated pressure at constant temperature.

When the two-room region with no ventilation is considered, the two rooms can be reduced to a single room and then the room temperature and relative humidity can be obtained analytically [1]. The analytical solution can be used to verify the numerical solution presented in the paper. It is noted that the analytical method is only suitable for a single room and cannot apply to the two-room region. The comparison of the two solutions for the room temperature and relative humidity of brick construction with no ventilation are given in Table 2. It shows that the two results are in agreement.

The temperature and relative humidity of room 1 for various construction materials with small ventilation rate \( \dot{\gamma} \) are shown in Figs. 2 and 3, respectively. In addition, room 2 is subjected to the same external conditions except ventilation rate, as shown in Figs. 4 and 5. All simulations assume uniform initial temperature of 21.8 °C and initial relative humidity of 75%.
Table 2
Comparison of two solutions for room temperature and relative humidity of brick construction with no ventilation.

<table>
<thead>
<tr>
<th>Temperature and relative humidity</th>
<th>Time of day (h)</th>
<th></th>
<th></th>
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<td>7</td>
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<td>19.5</td>
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<td>19.2</td>
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<td>87</td>
<td>87</td>
<td>88</td>
<td>89</td>
<td>90</td>
<td>89</td>
<td>87</td>
<td>71</td>
<td>61</td>
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<tr>
<td>Room relative humidity (%)</td>
<td>Numerical solution⁴</td>
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<td>75.1</td>
<td>75.3</td>
<td>75.4</td>
<td>75.6</td>
<td>75.8</td>
<td>75.9</td>
<td>76.1</td>
<td>76.2</td>
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<tr>
<td></td>
<td>Analytical solution⁴</td>
<td>75.0</td>
<td>75.2</td>
<td>75.3</td>
<td>75.5</td>
<td>75.6</td>
<td>75.9</td>
<td>76.0</td>
<td>76.1</td>
<td>76.3</td>
</tr>
</tbody>
</table>

⁴ The numerical solution is obtained using the method described in this paper. The analytical solution is obtained based on Cramer’s rule [1,2].

Fig. 2. The air temperature of room 1 for various construction materials with ventilation rate 0.02 m³/s.

The actual outdoor temperature was measured in Tainan, Taiwan on January 1 and 2, 2009, during winter. After the effect of ventilation, heat conduction, and coupled heat and moisture within the constructions, the room temperature in Fig. 2 of brick, concrete, and wood constructions are similar. On the other hand, the relative humidity values of room 1 for various constructions are shown in Fig. 3, in general, due to the slow moisture diffusion within the construction, the ventilation rate has a larger effect than moisture diffusion on the room moisture. When the external air temperature is cool at night, its moisture content is low, resulting in low room moisture due to ventilation. Moreover, the moisture in the construction can be transferred to the room when the room humidity is low. But the moisture in the wood construction diffuses slowly, resulting in low relative humidity at night. Air temperature and humidity are both high at daytime, so the daytime relative humidity of the room of wood construction is different from that at night.

The temperature and relative humidity of room 2 are shown in Figs. 4 and 5, respectively. The temperature and relative humidity of indoor air are affected significantly by the outdoor weather conditions with high ventilation rate of ˙G = 0.3 m³/s at noon. Hence, the changes in room temperature and relative humidity are smaller than those of room 1. The apparent discrepancies between the brick and wood constructions since heat and moisture diffusion within the wood construction are slower than that within the brick and concrete constructions. Comparing Fig. 2 with Fig. 4, the temperature of room 2 for the wood construction increases 3–4 °C, while the temperature for the brick and concrete constructions increases 2–3 °C. Comparing Fig. 3 with Fig. 5, the relative humidity of room 2 of the wood construction decreases as much as 15% and that of the brick and concrete constructions decreases as much as 12%, so that the temperature and relative humidity values of room 2 can reach the comfortable range.
Fig. 3. The relative humidity of room 1 for various construction materials with ventilation rate 0.02 m³/s.

Fig. 4. The air temperature of room 2 for various construction materials with ventilation rate 0.3 m³/s.

6. Conclusion

In this paper, a mathematical model for predicting the air temperature and humidity of a two-room region was proposed. The model, including different ventilation rates and considering simultaneous heat and moisture transport of a building construction, may be used to conserve energy and to improve room air quality. According to the analysis, the room temperature and relative humidity values for the brick and concrete constructions were similar, and differed greatly from the values of the wood construction. In addition, during winter, the room temperature increased 3–4 °C for the wood construction with a high ventilation rate at noon and increased 2–3 °C for the brick and concrete constructions. The room
relative humidity of the wood construction dropped as much as 15%, and that of the brick and concrete constructions dropped as much as 12%.

References