Constrained Least Square Pre-Distortion
Scheme for Multiuser UWB

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Abstract

The paper proposes a Constrained Least Square (CLS) pre-distortion scheme for multiple-input single-output (MISO) multiple access ultra-wideband (UWB) systems. In such a scheme, a simple objective function is defined, which can be efficiently solved by a gradient based algorithm. For the performance evaluation, scenarios CM1 and CM3 of the IEEE 802.15.3a channel model are considered. Results show that the CLS algorithm has a fast convergence and a good trade-off between intersymbol interference (ISI) and multiple access interference (MAI) reduction and signal-to-noise ratio (SNR) preservation, performing better than Time Reversal (TR) pre-distortion.

Index Terms

CLS, MISO, pre-distortion, time reversal, UWB.

I. INTRODUCTION

Impulsive Ultra-wideband (I-UWB) employs ultra short pulses to transmit information, resulting in a very large bandwidth. Therefore, the UWB channel is characterized by a dense multipath environment. In order to effectively capture the energy spread over the multipath components,
the transmit based time reversal technique (sometimes called pre-Rake) has been investigated [1], [2], [3], [4], [5]. In baseband time reversal, the channel impulse response (CIR) is estimated from a probe signal, and the data is convolved with the complex conjugate time reversed version of the estimated CIR (namely, time reversal coefficients) prior to transmission.

MISO Multiuser TR I-UWB is shown to be efficient when the transmission rate is moderated [2], [6], [7]. However, for high transmission rates, the residual ISI and MAI will still degrade the system performance [8]. In this work, a CLS pre-distortion scheme is proposed for MISO Multiuser TR I-UWB.

II. Channel and System Model

A discrete-time complex baseband version of the IEEE 802.15.3a model is adopted with two scenarios: CM1 and CM3. The $k$th channel impulse response (CIR) with sampling interval $T$ and length $L$ is obtained by [9]

$$h_k[m] = g_T(t) * h_k'(t) * g_R(t)\big|_{mT},$$

where $*$ denotes convolution, $g_R(t)$ is matched to the pulse $g_T(t)$, which has a square-root raised-cosine (RRC) shape, and $h_k'(t)$ is a baseband version of the CIR in the IEEE 802.15.3a model. The effective symbol rate is controlled by the space between consecutive symbols, $T_s = \kappa T$, where $\kappa$ is an integer. The multipath components are independent across antennas, but the shadowing terms $\chi_k$ are correlated according to the following correlation matrix [10]:

$$R_x = \begin{bmatrix}
1 & 0.86 & 0.54 & 0.25 \\
0.86 & 1 & 0.86 & 0.54 \\
0.54 & 0.86 & 1 & 0.54 \\
0.25 & 0.54 & 0.86 & 1
\end{bmatrix}. $$

The CIR is truncated according to the BER analysis in Sec. IV. The maximum relative delay among CIRs is fixed at 2.5 ns.

CIR estimation errors model is based on [4]. A sequence of $N_P$ probe pulses with repetition period longer than the maximum effective delay spread of the channel, $\tau_{ef}$, is transmitted from the receiver to the transmitter side, and the $N_P$ CIRs received on each antenna are coherently averaged. The estimation noise on the $\ell$th resolvable path of the $k$th antenna is a complex Gaussian random variable with variance of the in-phase and quadrature components given by $N_0/(2N_P)$, where $N_0/2$ is the double-sided power spectral density of the AWGN.
Fig. 1 illustrates the system model with $N_t$ transmitted antennas and $U$ users. With BPSK modulation, the transmitted signal of the $u$th user on the $k$th antenna is given by

$$s_{u,k}[m] = \sqrt{E_b^u} \sum_{i=-\infty}^{\infty} b_i^u \gamma_{u,k}[m - i \kappa],$$  \hspace{1cm} (3)$$

where $\gamma_{u,k}[m]$ is the pre-filter coefficients. Assuming perfect synchronism, the matched filter (MF) output sampled at the effective symbol rate, $1/T_s$, is written as:

$$y_u[n] = \sum_{i=-\infty}^{\infty} b_i^u x_u[n - i] + \sum_{i=-\infty}^{\infty} \sum_{q=1 \atop q \neq u}^{U} b_i^q f_{u,q}[n - i] + z[n],$$  \hspace{1cm} (4)$$

with $x_u[n]$ and $f_{u,q}[n]$ being obtained by sampling

$$x_u[m] = \sqrt{E_b^u} \sum_{k=1}^{N_t} \gamma_{k,u}[m] * h_{k,u}[m],$$  \hspace{1cm} (5)$$

and

$$f_{u,q}[m] = \sqrt{E_b^u} \sum_{k=1}^{N_t} \gamma_{k,q}[m] * h_{k,u}[m],$$  \hspace{1cm} (6)$$

respectively, by a factor $\kappa = T_s/T$. Equation (4) can be rewritten as

$$y_u[n] = b_0^u x_u[0] + \sum_{i=0}^{\infty} b_{n-i}^u x_u[i] + \sum_{i=-\infty}^{\infty} \sum_{q=1 \atop q \neq u}^{U} b_i^q f_{u,q}[i] + z[n].$$  \hspace{1cm} (7)$$

Eq. (7) can be written in a vector matrix form as follows

$$y_{u,n} = b_{u,n}^\top x_u + \sum_{q=1 \atop q \neq u}^{U} b_{q,n}^\top f_{u,q} + z_{u,n},$$  \hspace{1cm} (8)$$

where the vector $b_{u,n} = \left[ b_{n+L_\gamma-1}^u, \ldots, b_n^u, \ldots, b_{n-L_C+1}^u \right]^\top$ has $p = L_\gamma + L_C - 1$ elements.

An estimative of the received signal after MF is given by

$$\tilde{y}_{u,n} = b_{u,n}^\top \tilde{x}_u + \sum_{q=1 \atop q \neq u}^{U} b_{q,n}^\top \tilde{f}_{u,q} + z_{u,n}$$

$$= \left[ b_1^u \cdots b_{u,n}^\top \cdots b_{U,n}^\top \right] \cdot \begin{bmatrix} \tilde{f}_{u,1} \\ \vdots \\ \tilde{x}_u \\ \vdots \\ \tilde{f}_{u,U} \end{bmatrix} + z_{u,n}$$  \hspace{1cm} (9)
Defining \( \tilde{y}^\top_n = [\tilde{y}_{1,n} \; \tilde{y}_{2,n} \; \cdots \; \tilde{y}_{U,n}] \), it follows that
\[
\tilde{y}^\top_n = \begin{bmatrix} b_{1,n}^\top & b_{2,n}^\top & \cdots & b_{U,n}^\top \end{bmatrix} \begin{bmatrix} \bar{x}_1 & \bar{f}_{2,1} & \cdots & \bar{f}_{U,1} \\
\bar{f}_{1,2} & \bar{x}_2 & \cdots & \bar{f}_{U,2} \\
\vdots & \vdots & \ddots & \vdots \\
\bar{f}_{1,U} & \bar{f}_{2,U} & \cdots & \bar{x}_U \end{bmatrix} + z^\top_n. \tag{10}
\]

Note that \( A_c \) matrix in (10) is given by
\[
A_c = \begin{bmatrix} \bar{H}_1 \gamma_1 & \bar{H}_2 \gamma_1 & \cdots & \bar{H}_U \gamma_1 \\
\bar{H}_1 \gamma_2 & \bar{H}_2 \gamma_2 & \cdots & \bar{H}_U \gamma_2 \\
\vdots & \vdots & \ddots & \vdots \\
\bar{H}_1 \gamma_U & \bar{H}_2 \gamma_U & \cdots & \bar{H}_U \gamma_U \end{bmatrix}, \tag{11}
\]
where \( \bar{x}_u = \bar{H}_u \gamma_u \), \( \bar{f}_{u,q} = \bar{H}_u \gamma_q \), with \( u \neq q \), and
\[
\bar{H}_u = \begin{bmatrix} \left( \bar{h}_u^0 \right)^\top & 0^\top & \cdots & 0^\top \\
\left( \bar{h}_u^1 \right)^\top & \left( \bar{h}_u^0 \right)^\top & \ddots & \vdots \\
\vdots & \left( \bar{h}_u^1 \right)^\top & \ddots & 0^\top \\
0^\top & \left( \bar{h}_u^{L_C-1} \right)^\top & \left( \bar{h}_u^0 \right)^\top & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0^\top & \cdots & 0^\top \left( \bar{h}_u^{L_C-1} \right)^\top \end{bmatrix} \tag{12}
\]
is a circulant matrix with \( \bar{h}_u^\ell = [\bar{h}_{1,u}[\ell] \; \cdots \; \bar{h}_{N_t,u}[\ell]]^\top \).

III. PRE-DISTORTION SCHEMES

In this section, multiuser zero-forcing (ZF) and the proposed multiuser (CLS) constrained least square are presented.

A. ZF Pre-distortion

With the above formulation, a ZF scheme can be directly obtained finding \( \gamma_1, \cdots, \gamma_U \), such that
\[
A_c = \begin{bmatrix}
d_\delta & 0 & \cdots & 0 \\
0 & d_\delta & \cdots & \vdots \\
\vdots & \vdots & \ddots & 0 \\
0 & \cdots & 0 & d_\delta
\end{bmatrix} = D_\delta,
\]

with \( d_\delta = [0 \cdots 1 \cdots 0]^T \) and 1 at the \( L_\gamma \)th position. Note, however, that Eq. (13) is equivalent to

\[
\begin{bmatrix}
\tilde{H}_1 \gamma_1 & \tilde{H}_1 \gamma_2 & \cdots & \tilde{H}_1 \gamma_U \\
\tilde{H}_2 \gamma_1 & \tilde{H}_2 \gamma_2 & \cdots & \tilde{H}_2 \gamma_U \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{H}_U \gamma_1 & \tilde{H}_U \gamma_2 & \cdots & \tilde{H}_U \gamma_U
\end{bmatrix} = \tilde{H}_c \Gamma_c = D_\delta,
\]

where \( \tilde{H}_c = \begin{bmatrix} \tilde{H}_1 \\ \vdots \\ \tilde{H}_U \end{bmatrix} \), with dimension \( (U p \times q) \), and \( \Gamma_c = \begin{bmatrix} \gamma_1 & \cdots & \gamma_U \end{bmatrix} \), with dimension \( (q \times U) \), being \( p = L_c + L_\gamma - 1 \) and \( q = N_t L_\gamma \). Hence, the concatenated matrix of coefficients in the ZF pre-filter is given by

\[
\Gamma_c^{\text{tmp}} = \tilde{H}_c^{\dagger} D_\delta,
\]

such that \( \Gamma_c^{\text{tmp}} = \begin{bmatrix} \gamma_1^{\text{tmp}} & \gamma_2^{\text{tmp}} & \cdots & \gamma_U^{\text{tmp}} \end{bmatrix} \) and \((\cdot)^{\dagger}\) indicates the pseudo-inverse or Moore-Penrose inverse operator. The subscript \( \text{tmp} \) in \( \Gamma_c^{\text{tmp}} \) means temporary, because Eq. (15) has to be normalized in order to obtain a constrained total power \( P_{\text{tot}} \leq U N_t \), i.e.,

\[
\Gamma_c^{\text{ZF}} = \begin{bmatrix} \gamma_1^{\text{ZF}} & \gamma_2^{\text{ZF}} & \cdots & \gamma_U^{\text{ZF}} \end{bmatrix} = \sqrt{U N_t} \frac{\Gamma_c^{\text{tmp}}}{\|\Gamma_c^{\text{tmp}}\|_F^2},
\]

where \( \|\cdot\|_F \) is the Frobenius norm.

\[\text{B. CLS Pre-distortion}\]

The CLS objective function can be defined as

\[
\arg\min_{\Gamma_c} \| \tilde{H}_c \Gamma_c - D_\delta \|_F^2,
\]

s. t. \( \| \Gamma_c \|_F^2 \leq U N_t \)

where \( D_\delta = N_t D_\delta \). The \( \tilde{H}_c \) has the same form as \( \tilde{H}_u \), but with \( \tilde{H}_u \), formed by normalized coefficients per antenna, instead of \( \tilde{H}_u \).
Let $J_{CLS}$ be defined as $J_{CLS} = \| \tilde{H}_c \Gamma_c - D_\delta \|^2_F$. Hence $\tilde{H}_c$ can be decomposed as $U^H \tilde{H}_c V = \Sigma_{\tilde{H}_c}$, with $U$ and $V$ being unitary matrices and $\Sigma_{\tilde{H}_c}$ a diagonal matrix with nonnegative real numbers (singular values) on the diagonal. Applying this singular value decomposition (SVD),

$$
\| \tilde{H}_c \Gamma_c - D_\delta \|^2_F = \left\| U^H \left( \tilde{H}_c \Gamma_c - D_\delta \right) \right\|^2_F = \left\| U^H \left( \tilde{H}_c V V^H \Gamma_c - D_\delta \right) \right\|^2_F
$$

$$
= \left\| U^H \tilde{H}_c V V^H \Gamma_c - U^H D_\delta \right\|^2_F = \left\| \Sigma_{\tilde{H}_c} \tilde{\Gamma} - \tilde{D}_\delta \right\|^2_F,
$$

(18)

where $\tilde{\Gamma} = V^H \Gamma$ and $\tilde{D}_\delta = U^H D_\delta$. Using Lagrange multipliers, the following problem is formulated:

$$
\mathcal{L} \left( \tilde{\Gamma}, \lambda \right) = \operatorname{tr} \left( \Sigma_{\tilde{H}_c} \tilde{\Gamma} - D_\delta \right)^T \left( \Sigma_{\tilde{H}_c} \tilde{\Gamma} - D_\delta \right) + \lambda \left( \operatorname{tr} \left( \tilde{\Gamma}^T \tilde{\Gamma} \right) - U N_t \right)
$$

$$
= \operatorname{tr} \left( \tilde{\Gamma}^T \Sigma_{\tilde{H}_c} \Sigma_{\tilde{H}_c} \tilde{\Gamma} \right) - \operatorname{tr} \left( \tilde{\Gamma}^T \Sigma_{\tilde{H}_c} D_\delta \right) - \operatorname{tr} \left( D_\delta^T \Sigma_{\tilde{H}_c} \tilde{\Gamma} \right) + \operatorname{tr} \left( D_\delta^T D_\delta \right) + \lambda \left( \operatorname{tr} \left( \tilde{\Gamma}^T \tilde{\Gamma} \right) - U N_t \right)
$$

(19)

Taking the derivative of $\mathcal{L} \left( \tilde{\Gamma}, \lambda \right)$ regarding $\tilde{\Gamma}^*$ and making the result equal to zero,

$$
\frac{\partial \mathcal{L} \left( \tilde{\Gamma}, \lambda \right)}{\partial \tilde{\Gamma}^*} = 0 \Rightarrow \Sigma_{\tilde{H}_c}^T \Sigma_{\tilde{H}_c} \tilde{\Gamma} - \Sigma_{\tilde{H}_c}^T D_\delta + \lambda \tilde{\Gamma} = 0
$$

$$
\Rightarrow \left[ \lambda I + \Sigma_{\tilde{H}_c}^T \Sigma_{\tilde{H}_c} \right] \tilde{\Gamma} = \Sigma_{\tilde{H}_c}^T \tilde{D}_\delta
$$

(20)

Therefore

$$
\tilde{\Gamma} = \left[ \lambda I + \Sigma_{\tilde{H}_c}^T \Sigma_{\tilde{H}_c} \right]^{-1} \Sigma_{\tilde{H}_c}^T \tilde{D}_\delta
$$

$$
= \begin{bmatrix}
\frac{s_1}{\lambda + \varsigma_j} \bar{d}_\delta^{1,1} & \frac{s_1}{\lambda + \varsigma_j} \bar{d}_\delta^{1,2} & \cdots & \frac{s_1}{\lambda + \varsigma_j} \bar{d}_\delta^{1,U} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{s_r}{\lambda + \varsigma_j} \bar{d}_\delta^{r,1} & \frac{s_r}{\lambda + \varsigma_j} \bar{d}_\delta^{r,2} & \cdots & \frac{s_r}{\lambda + \varsigma_j} \bar{d}_\delta^{r,U}
\end{bmatrix},
$$

(21)

with the following constraint:

$$
\sum_{j=1}^U \sum_{i=1}^r \left( \frac{s_i}{\lambda + \varsigma_j} \right)^2 \left| \bar{d}_\delta^{i,j} \right|^2 \leq U N_t,
$$

(22)

where $r = \text{rank} \left( \tilde{H}_c \right)$. If $\sum_{j=1}^U \sum_{i=1}^r \left| \bar{d}_\delta^{i,j} \right|^2 > U N_t$, hence,

$$
\tilde{\Gamma} = \begin{bmatrix}
\frac{s_1}{\lambda^0 + \varsigma_j} \bar{d}_\delta^{1,1} & \frac{s_1}{\lambda^0 + \varsigma_j} \bar{d}_\delta^{1,2} & \cdots & \frac{s_1}{\lambda^0 + \varsigma_j} \bar{d}_\delta^{1,U} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{s_r}{\lambda^0 + \varsigma_j} \bar{d}_\delta^{r,1} & \frac{s_r}{\lambda^0 + \varsigma_j} \bar{d}_\delta^{r,2} & \cdots & \frac{s_r}{\lambda^0 + \varsigma_j} \bar{d}_\delta^{r,U}
\end{bmatrix},
$$

(23)

March 6, 2012

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and \( \lambda^o \) is obtained solving the following equation in terms of \( \lambda \):

\[
U \sum_{j=1}^{r} \sum_{i=1}^{r} \left( \frac{S_i}{\lambda + \zeta_i^2} \right) \left| \tilde{d}_{ij} \right|^2 = U N_t, \tag{24}
\]

which can be done, for instance, by the bisection method. Else,

\[
\tilde{\Gamma} = \begin{bmatrix}
\dfrac{\tilde{d}_{1,1}}{\zeta_1} & \dfrac{\tilde{d}_{1,2}}{\zeta_1} & \ldots & \dfrac{\tilde{d}_{1,r}}{\zeta_1} \\
\dfrac{\tilde{d}_{2,1}}{\zeta_2} & \dfrac{\tilde{d}_{2,2}}{\zeta_2} & \ldots & \dfrac{\tilde{d}_{2,r}}{\zeta_2} \\
\vdots & \vdots & \ddots & \vdots \\
\dfrac{\tilde{d}_{r,1}}{\zeta_r} & \dfrac{\tilde{d}_{r,2}}{\zeta_r} & \ldots & \dfrac{\tilde{d}_{r,r}}{\zeta_r}
\end{bmatrix}.
\tag{25}
\]

Therefore, \( \Gamma = V \tilde{\Gamma} \). In the sequel, a simple algorithm, based on the gradient algorithm, is adapted in order to solve the CLS problem. \( J_{CLS} \) can be written as

\[
J_{CLS} = \text{tr} \left( \tilde{H}^H \Gamma_c - D_\delta \right) \left( \tilde{H}^H \Gamma_c - D_\delta \right)
= \text{tr} \left( \Gamma_c^H \tilde{H}^H \Gamma_c \right) - \text{tr} \left( \Gamma_c^H D_\delta \right) - \text{tr} \left( \tilde{D}_\delta \tilde{H}^H \Gamma_c \right) - \text{tr} \left( D_\delta \tilde{D}_\delta \right)
\tag{26}
\]

The gradient of \( J_{CLS} \) is defined as \( \nabla J_{CLS} = 2 \frac{\partial J_{CLS}}{\partial \Gamma_c^*} \). Its derivative regarding \( \Gamma_c^* \) is given by

\[
\frac{\partial J_{CLS}}{\partial \Gamma_c^*} = \tilde{H}^H \tilde{H}^H \Gamma_c - \tilde{H}^H D_\delta = \tilde{H}^H \left( \tilde{H}^H \Gamma_c - D_\delta \right).
\tag{27}
\]

Hence, \( \nabla J_{CLS} = -2 \tilde{H}^H E \), where \( E = D_\delta - \tilde{H} \Gamma_c \). Algorithm 1 is then obtained.

**Algorithm 1 Gradient based CLS Algorithm**

\[
\Gamma_c^{CLS}(i) = \left[ \gamma_1^{CLS}(i) \ldots \gamma_u^{CLS}(i) \right]^T;
\]

\[
\Gamma_c^{tmp} = \left[ \gamma_1^{tmp} \ldots \gamma_u^{tmp} \right]^T;
\]

**Initialization:** \( \Gamma_c^{CLS}(0) = \Gamma_c^{TR} \) (TR: time reversal (TR) solution)

**for** \( i = 1, 2, \ldots \), ITER (number of iterations)

\[
E(i-1) = D_\delta - \tilde{H} \Gamma_c^{CLS}(i-1); \quad \text{(Error calculation)}
\]

\[
\Gamma_c^{tmp} = \Gamma_c^{CLS}(i-1) + \mu \tilde{H}^H E(i-1); \quad \text{(Temporary actualization)}
\]

\[
\Gamma_c^{CLS}(i) = \sqrt{U \cdot N_t} \frac{\Gamma_c^{tmp}}{\|\Gamma_c^{tmp}\|_F}; \quad \text{(Normalization \( \rightarrow \) total power constraint)}
\]

**end**

with \( D_\delta = N_t D_\delta \).

Figure 2 presents a comparison between the analytical and the gradient based CLS methods.

Results confirm the validation of the gradient based algorithm.
IV. SIMULATION CONFIGURATION AND RESULTS

Performance results are obtained considering Monte Carlo simulation (MCS) with at least 50 errors per simulated point. Channel state is assumed to be static during the frame transmission. Two, three and four antenna elements are adopted in a multiuser scenario with $U = 2$ and $U = 3$ users. In order to reduce complexity without compromise the system performance significantly, the CIR was truncated according to a BER criterion shown in Figure 3. It was considered $N_t = 3$ antennas and $U = 1$ user, i.e., without multiple user interference, in order to emphasize the effect of the estimated CIR length. Hence, from Figure 3, it is possible to see that $L_C = 35$ and $L_C = 60$ represent good choices for CM1 and CM3, respectively. Simulation specification and parameters are summarized in Tab. I.

Figure 4 illustrates some cases of the CLS convergence for $U = 2$ users. It is possible to see that $\mu = 0.1$ and $ITER = 20$ are enough for the algorithm to achieve convergence. The same values were observed for $U = 3$ users.

The term $\langle \|E\|_F^2 \rangle$ is the mean square of the error matrix. BER results as a function of $E_b/N_0$ in dB are presented in Figures 5 to 9.

Note that for $N_t \geq U$ the scheme CLS has a good performance, mainly for $N_t > U$. It is worth mentioning that, if $N_t > U$, the proposed CLS pre-distortion technique does not suffer with the BER floor degradation effect, as the ZF and TR approaches. When $N_t < U$, the performance results are not satisfactory at all, as shown in Fig. 7.

Note that the CLS gradient based algorithm converges with few iterations. This iterative solution presents a relatively low complexity when compared to the theoretical CLS version, which requires SVD decomposition of a large matrix, as well as the ZF scheme, that needs to implement a large matrix inversion.

V. CONCLUSIONS

This work proposed a new constrained least square pre-distortion scheme for a multi-user MISO UWB and compared the results to the well known time reversal and zero-forcing schemes. Results showed that CLS has a good performance, even for CIR estimation errors (noise and truncation) when the number of antennas is, at least, equal to the number of users. The multiuser CLS scheme can be iteratively solved using a gradient based algorithm, which has a lower complexity compared to the analytical CLS version and the analytical ZF scheme. The proposed
iterative CLS multi-user detector can possibly be a good solution for an application that has a good computational capacity at the UWB transmitter and requires a very low-complexity receiver, in no fast varying fading channels.

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Fig. 2. Comparison of CLS grad and CLS SVD for $U=2$, $N_t=3$, CM1 and $\mu = 0.1$.

Fig. 3. CIR truncation for CM1 and CM3, with $N_t = 3$ and $U = 1$. 
Fig. 4. Multiuser gradient based CLS convergence for $U = 2$. 

Fig. 5. BER performance for $U = 2, N_t = 2$. 
Fig. 6. BER performance for $U = 2, N_t = 3$.

Fig. 7. BER performance for $U = 3, N_t = 2$.

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