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Uncertainty of Measurement Results in the Process of Product Qualitative Level Identification

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Abstract — **The method of processing the several series' results of direct measurement with repeated observations aimed at determining the standard uncertainty during the research of a product qualitative level is proposed**

Keywords—product qualitative level, measurement error, calculation of uncertainty, dispersive analysis

I. Introduction

The most probable way of product estimation is the identification of its technical state due to its exploitation characteristics that are quantitatively expressed as the totality of product parameters. The paramters that among whole parameter totality are used in the process of product qualitative level (QL) identification are, according to [1], the indices of product quality [2].

In the process of QL determining, a measuring experiment as well as the measured value identification is one of the most objective information sources concerning product qualitative properties.

II. Sources of Measurement Errors

It is known that quality is being formed in the process of production manufacturing and is directly related to the perfection of a technological process. In its turn, any process of series production is accompanied with production errors, and it is practically impossible to gain the same parameter value in two products that are supposed to be identical. It befalls because of the influence of casual and determined factors that define the obtaining of any parameter value in the technological process. Factor influence leads to a certain distribution of production errors which in its turn causes the dispersion of characteristic values, and subsequently leads to our doubt or uncertainty during product QL assessment [3].

On the other hand, the process of measurement itself is accompanied with errors. Since the assessment of measurement quality is usually based not only on the error value but its probability characteristics as well, it is quite expedient to form product quality assessment on the basis of probability characteristics.

The result of measurement could be (independently of our knowledge about it) very close to the value of the measured quantity but meanwhile could not evoke relevant credence. In the same way, product quality indices could be maximally approximated to their optimum values, but regarding our knowledge limitation, we estimate this product not too highly or rather quite the reverse.

III. Uncertainty Origin

Thus the quantitative estimation of product parameters must rest on the analysis of their measurement or control results. To determine the probability characteristics of product parameters, it is necessary to perform the statistical analysis of their value dispersion. The totality of parameters that determine a product as the object of quality monitoring must comprise, if it is possible, just independent controlled parameters. The indices of product quality, in its turn, should be expressed directly or functionally through product parameters. Under such conditions only, we could determine the belonging of this product to a certain QL as well as the main characteristics of distribution laws of the controlled parameter probabilities. The parametric structure of the product could be determined in case of the possibility to estimate and manage a product QL through quality indices.

Obviously, greater importance is allotted to the degree of credence to high product characteristics than to the degree of these characteristics' approximation to optimal values. Therefore, quality assessment must be based not on a genuine characteristic value (or the deviation from this genuine value) but on the observed (estimated) changeability (spread) of this characteristic identification results. From product to product the observed changeability mathematically could be expressed by dispersion or by the distribution law of index characteristic value probabilities and indirectly could affirm this index essentiality or the given degree of credence. The positive second root of dispersion is interpreted in metrology as the standard uncertainty.

According to [3], uncertainty is a value which characterizes the dispersion of observation results that is the issue of non perfection of measurement methods and means, as well as the property of the measured value itself with regard to an accepted metrological model. The type A estimation is calculated statistically due to the results of multiple observations [3], [4].

IV. Nestling Design for a Single Quality Index

Let us take into consideration the indices, whose values could alter within the given boundaries of the certain range, so the upper QL should be matched with the approximation of an index value to some optimum value from this range. The value of the product X diameter with admissible deviations x_{j-} and x_{j+} and the optimum value x_{j0}, which must not always coincide with the middle of the admissible deviation range $\overline{x}_j$, belong to such indices.

The gaining of index characteristic values during product QL identification takes place, as a rule, using several examples from lot or group of products, and different lots of products, moreover, measurement could be performed by different laboratories. Accordingly, measurement is conducted at different times, by different methods and operators. Such research should be interpreted as interlaboratory, and the special measurement model attained in consequence, according to requirements [3], is called the nested design. The nested design is characterized by several groups of results of direct measurement with repeated observations and different evaluations of the measured value and standard uncertainty. To form the united result with adequate dispersion estimation, we should consolidate the received groups, applying a dispersive analysis [5]. Measurement should be organized in such a way that an equalized cell structure with the same observation quantity in each group is attained.

V. Type A Uncertainty Calculation

Let us assume that n groups of direct multiple observations of the quantity X with m observations in every group are gained. The best estimation of the measured quantity X is the arithmetical mean $\overline{x}$ of arithmetical means $\overline{x}_i$ of every i-th group:

$$X = \overline{x} = \frac{1}{n}\sum_{i=1}^{n}\overline{x}_i = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m}x_{ij}\,, \qquad (1)$$

here x_{ij} – the j-th observation in the i-th group.

And the evaluation of standard uncertainty of group arithmetical means, which in general is expressed by the formula:

$$u_A(\overline{x}) = \sqrt{\frac{1}{nm(nm-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij}-\overline{x})^2}\,, \qquad (2)$$

could be determined considering the results of dispersive analysis that helps to conclude the essentiality of between-group dispersion in comparison with a within-group component [4]. For this purpose the total dispersion evaluation should be represented as

$$u_A^2 = u_1^2 + u_2^2$$

here u_1^2 - a component that reflects the between-group spread of observation; u_2^2 - a within-group component.

The values u_1^2 and u_2^2 are easily determinable, if to rewrite the formula (2) in the following way:

$$\begin{aligned} u_A^2(\overline{x}) &= \frac{1}{nm(nm-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(x_{ij}-\overline{x})^2 = \\ &= \frac{1}{nm(nm-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}\left[(x_{ij}-\overline{x}_i)+(\overline{x}_i-\overline{x})\right]^2 = \qquad (3) \\ &= \frac{1}{nm(nm-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}\begin{bmatrix}(x_{ij}-\overline{x}_i)^2 + \\ +2(x_{ij}-\overline{x}_i)(\overline{x}_i-\overline{x}) + \\ +(\overline{x}_i-\overline{x})^2\end{bmatrix} \end{aligned}$$

In the gained formula:

$$u_1^2 = \frac{1}{nm(nm-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(\overline{x}_i-\overline{x})^2 = \frac{1}{nm(nm-1)}\sum_{i=1}^{n}m(\overline{x}_i-\overline{x})^2$$

- between-group dispersion;

$$u_2^2 = \frac{1}{nm(nm-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(\overline{x}_{ij}-\overline{x}_i)^2$$

- within-group dispersion, and the third member of the equation is neglected, since

$$\begin{aligned} &\sum_{i=1}^{n}\sum_{j=1}^{m}2(x_{ij}-\overline{x}_i)(\overline{x}_i-\overline{x}) = \\ &= 2\sum_{i=1}^{n}(\overline{x}_i-\overline{x})\sum_{j=1}^{m}(x_{ij}-\overline{x}_i) = \\ &= 2\sum_{i=1}^{n}(\overline{x}_i-\overline{x})(m\overline{x}_i-m\overline{x}_i) = 0 \end{aligned} .$$

To evaluate the hypothesis about the uncertainty of the u_1^2 component, statistical criteria [5] are used, particularly, the ratios of selective between-group and within-group dispersion evaluations. According to Fisher's theorem for the normally distributed elements of samples divided on *n* groups, each one of which contains *m* elements, the quantity $Q_1 = \sum_{i=1}^{n} m(\bar{x}_i - \bar{x})^2$ has $(n-1)$ freedom degrees, and quantity $Q_2 = \sum_{i=1}^{n}\sum_{j=1}^{m}(\bar{x}_{ij} - \bar{x}_i)^2$ has $(nm-n)$ freedom degrees. Due to correlation of Q_1 and Q_2 the function $F_{n-1,nm-n} = \dfrac{Q_1/(n-1)}{Q_2/(nm-n)}$ could be found. The table value of Fisher's distribution could be taken from Fisher's table for different credible probabilities with $(n-1)$ and $(nm-n)$ freedom degrees.

A statistical criterion is formulated in such a way:

<u>if</u> $F_{n-1,\, nm-n} > C$,

here C - $F_{n-1,\, nm-n}$ – the function value taken from the Fisher's distribution table,

<u>then</u> the hypothesis is rejected and $u_A^2 = u_1^2 + u_2^2$. If the hypothesis is true, then $\bar{x}_i$ and $\bar{x}$ are relevant estimations of the same value, therefore they are close to each other, the value Q_2 is small and correspondently could be neglected. Thus the evaluation of experimental dispersion u_A^2 is determined only by the within-group component $u_A^2 = u_2^2$.

With making decisions concerning between-group dispersion existence, dispersion evaluation must be calculated by the formula (2).

VI. Identification of a Product Qualitative Level

When the only research goal is the derivation of the unified measurement result with adequate dispersion estimating, the gained groups should be merged using the described dispersion analysis. In case the general information on the product QL constitutes our target, both components should be taken into account separately.

Evidently, the subject of QL assessment in the industrial conditions is a lot or group (totality) of wares with the certain proper uncertainty or instability of parameters. The point of the proposed research roots in the consideration of emphasized heterogeneity or instability when assessing the industrial production QL. For this special purpose, the between-group dispersion employment is highly recommended here. Since the between-group dispersion characterizes the spread of measurement results caused by investigating different product-pattern selections, its value reveals the level of researched object quality-index spread. To define such a spread level, the notion of *qualimetric uncertainty (QU)* could be introduced. The value of QU indirectly characterizes the quality or qualimetry properties of ware totality [7], [8].

In the best case, the experiment of qualimetry uncertainty evaluation should be planned so that a within-group component should be less than the between-group dispersion. Therefore the component u_1^2 could be distinguished due to the experiment results, and consequently, included in the input information for ware QL assessment.

Since the participants of a QL assessment process as a rule belong to different manufacturers of the similar production, which is characterized by the unified taxonomy of indices, no doubt the results of identifying the quantitative quality-estimates of various manufacturers and the estimates of qualimetric uncertainty accompanied any QL assessment could be compared. In this case, the magnitude of any estimate of qualimetric uncertainty concerning the certain object in the general totality of compared products could be used as the criterion for this object QL assessment [9].

VII. Conclusions

Anyway the trustful result of industrial production QL assessment could be realized only on the basis of multiple observations (measurement, assessment, analysis and diagnostics) due to the quality indices of several ware-patterns. As a result the matrix of statistical data of index values is being formed that also could be used for the estimation of appropriate uncertainty components, namely qualimetric uncertainty characteristics. Moreover, the gained estimate does not require specialized investigations but additional processing of observation results and appropriate dispersion evaluation. Consequently, the principal significance could belong to the qualimetric uncertainty whose magnitude indirectly characterizes the quality level of the whole assessed ware lot – the higher dispersion, the lower quality.

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