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Comparison of three expert elicitation methods for logistic regression on predicting the presence of the threatened brush-tailed rock-wallaby *Petrogale penicillata*

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SUMMARY

Numerous expert elicitation methods have been suggested for generalised linear models (GLMs). This paper compares three relatively new approaches to eliciting expert knowledge in a form suitable for Bayesian logistic regression. These methods were trialled on two experts in order to model the habitat suitability of the threatened Australian brush-tailed rock-wallaby (*Petrogale penicillata*). The first elicitation approach is a geographically assisted indirect predictive method with a geographic information system (GIS) interface. The second approach is a predictive indirect method which uses an interactive graphical tool. The third method uses a questionnaire to elicit expert knowledge directly about the impact of a habitat variable on the response. Two variables (slope and aspect) are used to examine prior and posterior distributions of the three methods. The results indicate that there are some similarities and dissimilarities between the expert informed priors of the two experts formulated from the different approaches. The choice of elicitation method depends on the statistical knowledge of the expert, their mapping skills, time constraints, accessibility to experts and funding available. This trial reveals that expert knowledge can be important when modelling rare event data, such as threatened species, because experts can provide additional information that may not be represented in the dataset. However care must be taken with the way in which this information is elicited and formulated. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: expert elicitation; Bayesian statistical modelling; logistic regression; habitat suitability modelling; threatened species

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1. INTRODUCTION

Elicitation of expert information and its representation as prior distributions are key steps in informative Bayesian analysis. Expert elicitation is a formal method of obtaining experts' prior beliefs about possible values of parameters in terms of probability; a comprehensive review is provided by O'Hagan *et al.* (2006). Kynn (2008) reviews psychological research on assessing probabilities and provides guidelines for eliciting expert knowledge.

This paper examines three fairly new elicitation approaches for logistic regression—two indirect (Kynn, 2005; Denham and Mengersen, 2007) and one direct (O'Leary *et al.*, 2008). These methods differ in the type of elicitation, elicitation tool, prior distribution and the requirement of a facilitator. Three approaches were trialled on a case study, modelling the habitat suitability of the threatened Australian brush-tailed rock-wallaby *Petrogale penicillata*, in which limited data are available but experts could provide potentially valuable additional information. The rock-wallaby is a cryptic species, preferring remote rocky terrain making reliable detection not only difficult but expensive (Murray *et al.*, 2008). Logistic regression was selected as a habitat suitability model, to relate the presence/absence with the environmental habitat variables (Guisan and Zimmermann, 2000; Scott *et al.*, 2002).

One of the most important issues of elicitation is the selection of the experts. An expert is someone who has knowledge of the subject of interest, for example knowledge on a particular species, although they do not necessarily have to know everything about the subject (e.g. Garthwaite *et al.*, 2005). Their knowledge is gained through education, training and experience in the field (Chi *et al.*, 1988). There are many types of experts (Hogarth, 1975; Shanteau, 1988; DeGroot, 1988). For example Denham and Mengersen (2007) identified two types of experts for species distribution modelling. One has good knowledge of where the species is present or absent, although they may not necessarily be aware of the reasons why the phenomenon occurs. The other has expertise in the physiological requirements of the species, but is less able to synthesise this spatially in order to identify where the species is likely to be found.

There are numerous elicitation methods for generalised linear models (GLMs) now available (O'Hagan *et al.*, 2006). Various methods of including informative priors into a logistic regression model have been proposed, but the majority of these use historical data (e.g. Ibrahim *et al.*, 1998; Chen and Dey, 2003) or emphasise computational methods (e.g. Albert and Chib, 1993; Chen *et al.*, 2003). Information can be elicited from the expert using either direct or indirect methods (Winkler, 1967). Direct elicitation involves asking the expert to quantify their beliefs directly on the coefficients in the regression model. Specific models for which this has been developed include cumulative logistic regression (Borsuk, 2004), zero inflated Poisson regression (Kuhnert *et al.*, 2005; Martin *et al.*, 2005) and multiple criteria analysis (e.g. Paruccini, 1994). Alternatively, indirect elicitation identifies the experts' knowledge by eliciting values of the response given values of covariates or eliciting values of the covariates given values of the response. This opinion is then transformed mathematically into a prior distribution for the parameters in the model. For logistic regression, two types of indirect elicitation methods have been proposed. First is the predictive P-method which elicits the probability of the response variable given values of the explanatory variables (Spetzler and Staël von Holstein, 1975). This method has been applied using an interview approach for normal linear models (Kadane *et al.*, 1980), and provides the basis for extension to logistic regression that includes conditional mean priors (Bedrick *et al.*, 1996), a questionnaire approach (Gill and Walker, 2005) and geographically assisted methods (Denham and Mengersen, 2007). Second is the PV-method, in which the expert first specifies several values of an explanatory variable and for each value provides a probability of the response variable (Spetzler and Staël von Holstein, 1975). The latter has been used in graphically assisted approaches (Kynn, 2005; Al-Awadhi and Garthwaite, 2006).

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There are potential pitfalls and biases when eliciting expert opinion; a comprehensive review is provided by Kynn (2008). These are conscious and subconscious inconsistencies between expert's answers and the true portrayal of their opinion, which include: displacement bias when the expert overestimates or underestimates the expected value; variability bias when the expert underestimates the certainty of their response and motivational biases when opinions are influenced for personal or research reasons (Spetzler and Staël von Holstein, 1975). Cognitive biases of an expert have been summarised by Kadane and Wolfson (1998) and O'Hagan *et al.* (2006); these include: availability when recent or important information is remembered more readily (Tversky and Kahneman, 1974); anchoring and adjustment is when an expert anchors on an initial probability estimate and adjusts the rest of their responses from this initial estimate (Tversky and Kahneman, 1974; Alpert and Raiffa, 1982); representativeness when the probability of an event is mistaken for its representativeness to some main component in the population (Kahneman and Tversky, 1972); overconfidence when the majority of the true values are in the extreme tails of the elicited distribution (Alpert and Raiffa, 1982); conjunction fallacy or coherence bias when elicited probabilities are not consistent with the laws of probability (Mullin, 1986) and hindsight bias if the expert has seen the observed data and revised their knowledge (Morgan and Henrion, 1990).

If multiple experts are available, then their opinions can be elicited as a group or obtained separately and then combined either mathematically or behaviourally (Clemen and Winkler, 1999). A mathematical aggregation involves eliciting opinions individually and then combining them into a single probability distribution using statistical, process or analytical models. Behavioural combination methods endeavour to reach an agreement between the experts through some type of interaction, being either face-to-face discussion, interaction by computer or anonymous convergence by methods such as Delphi (e.g. DeGroot, 1974; O'Hagan *et al.*, 2006). Biases arising from eliciting from multiple experts as a group include motivational, cognitive and systematic bias (Spetzler and Staël von Holstein, 1975; Gill and Walker, 2005; O'Hagan *et al.*, 2006).

In addition to determining what is to be elicited, it is also important to determine how it will be elicited. When designing an elicitation method, it is crucial to identify quantities that are meaningful to both the expert and the facilitator. In particular, questions should be phrased in terms familiar to the experts (Hogarth, 1975; Kadane and Wolfson, 1998). Experts with limited statistical knowledge, particularly of probabilities and distributions, may have difficulty with understanding and giving accurate statistical assessments. Therefore, such experts may provide responses which are internally inconsistent, particularly when asked about continuous responses (Winkler, 1967; Hogarth, 1975).

Numerous authors have identified various stages that are important in an expert elicitation process (Spetzler and Staël von Holstein, 1975; Shepherd and Kirkwood, 1994; Clemen and Reilly, 2001; Walls and Quigley, 2001; Jenkinson, 2005). Choy *et al.* (2008) identified a six-stage elicitation process necessary for an informative Bayesian modelling process by the statistician: determine purpose and motivation for using prior information; specify available prior knowledge from experts or other sources; formulate a statistical model representing the conceptual model; design numerical encoding; manage uncertainty for accurate and robust elicitation; design an elicitation protocol to manage logistics of implementing elicitation. These stages provides a basis for comparing elicitation methods and will be used to compare the three elicitation methods trialled in this paper.

In ecology, large datasets are often required to provide complete coverage of the species geographic distribution (Austin and Meyers, 1996; Thuiller *et al.*, 2004). By virtue of being rare or threatened, densities are low making detection difficult, hence data for such species are usually limited (Manel *et al.*, 1999; Engler *et al.*, 2004) and predominantly consist of either presences or absences (Radeloff *et al.*, 1998; Pearce *et al.*, 2001) or comprise presences only (e.g. Brotons *et al.*, 2004; Engler *et al.*,

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2004). This typically arises because obtaining surveys that sufficiently represent the variability of ecological response across the spatial area is difficult, time consuming and costly, particularly in remote areas (e.g. Manel *et al*., 1999; Moritz *et al*., 2001; Austin, 2002). Furthermore, the available data are usually acquired from casual and non-systematic surveys (Rushton *et al*., 2004; Graham *et al*., 2004). Therefore developing a habitat suitability model from limited data can be problematic and may yield prediction biases (Hosmer and Lemeshow, 1989; Manel *et al*., 2001). In cases such as these, expert knowledge can contribute by addressing these information gaps (Martin *et al*., 2005). In the ecological community, expert opinion has indeed been acknowledged as having some value in modelling, particularly when the observed data are limited or unreliable (e.g. Pearce *et al*., 2001; Yamada *et al*., 2003). There are many other rare event contexts in which expert knowledge could assist in modelling. Published examples include examining severe accident management strategy to prevent failure in boiling water reactors in engineering (Yu, 2002); and investigating survival attributable to various indicators such as in coronary artery disease therapy in medicine (Sachdev *et al*., 2004).

Motivated by a case study (Subsection 2.1), this paper reviews three recent elicitation approaches (Subsection 2.3) for logistic regression, as defined in Subsection 2.2. These methods are described in Subsection 2.3. A comparison and summary of the advantages and disadvantages of the methods are presented in Subsection 2.5. Results from the three elicitation methods to the case sudy are presented in Section 3. A discussion of the outcomes and implications of this project appears in Section 4.

2. METHODS

2.1. Case study

The Australian brush-tailed rock-wallaby is a medium sized wallaby with a total length about 1.2 m. It has a coastal to sub-coastal distribution and ranges from just north of Brisbane, Queensland, to western Victoria (Maynes and Sharman, 1983). This species is listed as threatened in Queensland, endangered in New South Wales, critically endangered in Victoria and extinct in the Australian Capital Territory.

Greater understanding of its habitat requirements is necessary for better management and conservation of this species (Eldridge, 1997; Carter and Goldizen, 2003). At the time of analysis, the available data on this species comprised of only 41 presence and nine absence sites in Queensland, with data on 42 variables for each site (e.g. slope and temperature). A rich source of additional information about habitat suitability for the rock-wallaby rested with ecologists who have studied these animals. In light of this, a trial was carried out on two experts as part of an Expert Elicitation Workshop at Queensland University of Technology, Australia. Expert I had many years of research experience with macropods but had no local knowledge of the species (has knowledge of the species in New South Wales), no geographic information system (GIS) experience and some statistical knowledge. In contrast, expert II had recent, local knowledge of the species, GIS experience and some statistical knowledge. The aim was to elicit their opinion about the species distribution of the brush-tailed rock-wallaby and use it to inform the priors in a predictive model.

The three expert elicitation methods that were trialled on the two experts were: (A) a geographically assisted predictive elicitation method (Denham and Mengersen, 2007), (B) a graphically assisted predictive elicitation approach (Kynn, 2005) and (C) a simple questionnaire approach (O'Leary *et al*., 2008). A standardised methodology was followed in the trial. Each expert used all three approaches. Expert I commenced with A, followed by B. Expert II commenced with B, followed by A. After a break, approach C was trialled on both experts independently at the same time.

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2.2. Bayesian logistic regression

Logistic regression with over-dispersion was chosen to model the habitat suitability of the rock-wallaby. The observed data are modelled as

$$y_i \sim \text{Bernoulli}(p_i), \tag{1}$$

where y_i is the observed presence (1) or absence (0) of the species at site $i = 1, \ldots, n$ and p_i is the probability of presence at site i. Then

$$\text{logit}(p_i) = \log \frac{p_i}{1 - p_i} = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_J x_{iJ} + \varepsilon_i \tag{2}$$

where β_0 is the intercept; $\beta_j,\ j = 1, \ldots, J$ are the coefficients associated with each environmental habitat variable $x_{i1}, x_{i2}, \ldots, x_{iJ}$ and $\varepsilon_i \sim \text{N}(0, \sigma^2)$ is the extra binomial variation (Besag *et al*., 1995). Independent multivariate normal (MVN) priors are placed on

$$\beta_j \sim \text{MVN}(\mu_j, \Sigma), \qquad \Sigma = \text{diag}(\sigma_{\beta_1}, \ldots, \sigma_{\beta_j}) \tag{3}$$

If no information about a coefficient is available or 'non-informative' priors are desired, it is typical to let $\mu_j = 0$, and either set each element of the diagonal of the dispersion matrix Σ to a large constant or allow it to have a uniform (non-negative) distribution with a large range (e.g. Spiegelhalter *et al*., 2003). If, however, expert opinion is available about the coefficients, either directly or indirectly, this can be used to refine the values of the hyperparameters μ_j and Σ, or indeed to revise the normal distribution assumption.

2.3. Expert elicitation

The three elicitation methods are described here in terms of: specification, statistical formulation, design of numerical encoding, and use of technology.

2.3.1. Geographically assisted P-method elicitation. The approach of Denham and Mengersen (2007) is an indirect predictive P-method of elicitation (Spetzler and Staël von Holstein, 1975) that was designed for environmental modelling. This method capitalises on the geographic nature of these problems by embedding the elicitation in a GIS, a map-based software. The resultant interactive map-based tool is used to elicit site-based predictions from a single expert for a logistic regression model (although the approach can be extended to all GLM models). Statistical and graphical functions were included in the software to assist the expert in the elicitation.

This method indirectly elicits the regression coefficients β for the MVN distribution (Equation (3)). In the prototype software tool developed by Denham and Mengersen (2007) for predicting the presence of rare species, the expert firstly selects a subset of J variables within a GIS interface. Next, the expert selects a point k (where $k = 1, \ldots, K$) on the map and an interactive dialogue pops up, which displays the values of the selected variables (e.g. slope is 60°, and rock type is basalt) at that point. This dialogue also displays a plot of a beta distribution, taken to represent the elicited predicted probability p_k of presence at the kth location, with three adjustable points situated at the median m, lower q_L and upper q_U quantiles. To assist in the choice of these three values, the expert is asked to consider $d = 100$ sites similar to the kth location chosen; the median m can be expressed as the proportion of d sites in which

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is the expert's estimate of the probability of species presence, and they believe that the probability is equally likely to lie above or below this estimate. The quartiles q_L and q_U represent the lowest and highest number of d sites, respectively, where the expert thinks presence is believable but surprising. More precisely, the degree of surprise is that they believe that there is a one in four chance that the probability could lie below q_L and a one in four chance that the probability lies above q_U. When considering these three values, the expert should use all of the knowledge they have about the location. The elicited values m_k, q_{Lk} and q_{Uk} are then used to estimate the a_k and b_k values of the beta distribution $p_k \sim Be(a_k, b_k)$ (Jenkinson, 2005).

Once a minimum number of locations have been elicited, model feedback in the form of univariate response curves can be developed and viewed. These response curves show the univariate relationship between the probability of occurrence p (estimated from the elicited information) and each selected explanatory variable j. They can be used by the expert to assist their understanding of the relationship between the probability of presence and the particular environmental gradient. The residuals of the response curve are also evaluated by the expert to ensure consistency with their opinion and any influential or surprising points K_s are further explored. The map is updated by the expert either adding K_s points or editing the m, q_L and q_U for an existing point; consequently the univariate response curves are updated.

After the elicitation, the expert informed priors for β and σ^2 (Equation (3)) can be calculated using a simulation approach. The probability of presence $p_k \sim \beta(a_k, b_k)$ is simulated at each of the expert's points k and a linear model $\text{logit}(p_k) \sim N(X\beta, \sigma^2)$ is fitted to the simulated values using least squares. This approach was selected because 'it makes no assumptions to derive pseudo-sample sizes, or asymptotic normality assumptions in the GLM estimation stage' (Denham and Mengersen, 2007).

An advantage of this approach is that the explanatory variables in the Bayesian logistic model can be different from the variables used by the expert during the elicitation, since the GIS software can be used to obtain values of any variable of interest at the kth point. The elicited probabilities of presence at particular points/locations on the map reflect 'real observations' from the *holistic* viewpoint of the expert, whereby the selected statistical model provides a partial description of the data. Moreover, in doing this, the statistical model can be changed without a need for re-elicitation.

This map-based design for elicitation was chosen since ecologists are typically familiar with maps of the area and some experts are experienced with using the GIS software. A facilitator is required to elicit the information, explain the software and phrase the questions in words the expert understands well.

2.3.2. Graphically assisted predictive PV-method elicitation. Kynn (2005) developed an interactive indirect graphical tool called ELICITOR, which is now an add-in to WinBUGS (www.winbugs-development.org.uk). It elicits species response curve predictions from a single expert for the logistic regression model, and more recently for all GLM models. This PV-method (Spetzler and Staël von Holstein, 1975) was developed from the ideas of Bedrick *et al*. (1996) and Garthwaite (1998) (later version see Garthwaite and Al-Awadhi (2006)), and builds conditional mean priors on the scale of the response variable.

The approach models the response as the simple Bernoulli case and indirectly elicits from an expert the regression coefficients $\beta \sim \text{N}(\mu, \sigma)$. The species response is modelled as the difference from the optimum, with all other variables at their optimum, so that Equation (2) is rearranged as $\text{logit}(p_i) = \beta_0 + F_1(x_1) + F_2(x_2) + \cdots$, where the continuous variables are $F_j(x_j) = \beta_j |x_j - x_{j,\max}|^m$, $m = 1, 2$, and the categorical variables are $F_c(x_c) = \gamma_0 x_{c,0} + \gamma_1 x_{c,1} + \cdots$. The intercept is modelled as the logit of the probability of presence when all variables are at their optimum. The variables that are included

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in the model are selected by the expert. Each variable is elicited separately while keeping all other variables at their optimum. For each variable *j* the expert is initially asked to specify several values of the *j*th variable (such that these values range from low to high probability of presence). Then, for the intercept and each of these values of the *j*th variable, the expert is asked to provide the median, lower and upper quantile for the probability of presence.

For the intercept and each variable, ELICITOR provides a graph depicting the species response curve; the *x*-axis is the covariate (values specified by the expert) and the *y*-axis is the predicted probability or frequency of presence. Categorical variables are characterised by bar charts and continuous variables either by linear, quadratic or piecewise linear functions fit to the points. The median of the intercept is elicited by asking the expert to estimate—out of say 100 survey sites—the number with presence. The upper and lower percentiles or the credible interval (CI) are elicited by either direct fractile elicitation or a bisection method. The simplest method is to elicit the lower and upper quantile and then ask the expert the size of the CI. For each variable, the expert selects the best level for locating the species and then chooses a number of sub-optimal points. For each point, the median and CI are elicited using the same approach.

To assist in decreasing cognitive biases, model feedback can be provided during the elicitation. This includes displaying a different CI to check that these intervals are consistent with the expert's opinion. The software includes the capacity to display the prior probability density functions on the scale of the response variable.

Formulation of this expert information into prior distributions for the logistic regression requires complex computation. The intercept and covariate parameters are assumed to be independent. For categorical variables, a normal prior distribution is calculated for each level. Continuous variables are modelled as the difference between the sub-optimal level and the maximum value, that is $\beta_j|x - x_{\max}|^m$. If the *j*th continuous variable was elicited as a piecewise linear function then if $x \leq k_{\text{opt}}$ then $F(x) = s_{i-1}(x - k_i) + \sum_{j=i}^{\text{opt}} s_j(k_j - k_{j+1})$, where $k_{i-1} < x \leq k_i$, k is the knot (optimal and sub-optimal points elicited); s is the slope between the two knots and opt is the optimal probability of presence. Similarly, $F(x)$ can be calculated for $x > k_{\text{opt}}$, $F(x) = s_i(x - k_i) + \sum_{j=\text{opt}}^{i-1} s_j(k_{j+1} - k_j)$, where $k_i < x \leq k_{i+1}$.

The prior mean for the intercept is $\mu = \text{logit}(p_{\text{opt}})$, where p_{opt} is the expert's opinion of the probability of presence given all covariates are at optimum. The prior mean for the β and γ coefficients are calculated by $\mu = \Delta(p)/(\Delta x) = (\text{logit}(p_{\text{opt}}) - \text{logit}(p_\ell))/(x_{\text{opt}} - x_\ell)$, where x_{opt} is the optimal level of the x variable, and ℓ is the sub-optimal level. Since dummy variables are used for categorical variables, the calculation of the mean of each sub-optimal level does not include the denominator (Δx).

The standard deviation of each normal distribution can be calculated either using an approximation to the normal distribution (e.g. Walpole and Myers, 1993) or using the standard z score. ELICITOR uses the approximation $\sigma = (q_{(\alpha)X} - \mu)/(4.91[\alpha^{0.14} - (1 - \alpha)^{0.14}])$ where $q_{(\alpha)X}$ is the expert's estimation of the lower or upper probability of presence and α is the selected CI. Alternatively the z score can be used to calculate the standard deviation $\sigma = (q_{(\alpha)X} - \mu)/(\Phi^{-1}(\alpha))$, where $\Phi^{-1}(\alpha)$ is the α quantile for the standard normal distribution (with mean of zero and standard deviation of one).

The graphical design of this tool increases the flexibility of the elicitation: it offers a variety of elicitation procedures; any size CI can be elicited and medians and CIs can be elicited in any order. Tools to assist the expert in visualising and determining their probabilities include a probability wheel, a random sample represented by a box with blue squares, static display of probabilities and odds on a logarithmic scale and static display of some proposed scales for associated numeric and verbal probabilities. The design of the accompanying questions followed existing methods for asking about median values and CIs for future observable quantities (e.g. Kadane *et al.*, 1980; O'Hagan, 1998; Al-Awadhi and Garthwaite, 2001). Psychological literature reviewed by Kynn (2005) provided the basis

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for designing the software and questions. Similar to method A, this method requires a facilitator to elicit the information, explain the software and phrase the questions in words the expert best understands.

Al-Awadhi and Garthwaite (2006) developed a similar elicitation approach. Their piecewise linear function has a fixed optimal knot and number of knots, whereas in Kynn's (2005) approach the expert chooses which point is the optimal knot and number of knots (at least two). Kynn assumes that the prior of each β coefficient is independent and normally distributed, whereas Al-Awadhi and Garthwaite (2006) assume that the β parameters have a MVN distribution.

2.3.3. Questionnaire delivered direct simple elicitation method. Martin *et al*. (2005) proposed a direct questionnaire approach for eliciting opinion from multiple experts for zero inflated Poisson regression. O'Leary *et al*. (2008) adapted this approach for the logistic regression context and extended the questionnaire to include elicitation of measures of confidence. It is a simple elicitation method as it does not require the experts to have knowledge of probabilities or distributions. Additionally, this approach is appropriate for single or multiple experts.

This elicitation method models the response as Bernoulli and assumes that β regression coefficients are a mixture of three normal distributions reflecting increase, decrease and no substantive change in the response, so that

$$\beta_j \sim \sum_{m=-1,0,+1} w_{jm} \mathrm{N}(\mu_{jm}, \sigma^2_{jm}) \tag{4}$$

Unlike methods A and B, this method is a direct elicitation as it asks the expert to classify their beliefs directly about the coefficients.

Individually, the experts are asked to estimate the impact z_i of each habitat variable x_j on the response of site occupancy of the species. Questions are phrased differently depending on whether the variable is categorical or continuous. The experts are asked whether the response decreases ($z = -1$), does not substantively change ($z = 0$) or increases ($z = +1$) as the covariate also increases (if continuous variable) or as the covariate changes to level c in comparison to a baseline category (if categorical variable). The baseline category is identified by the expert. For each reported impact, the expert is also asked to rate their confidence: from 7 (100% confident) to 1 (not confident at all). To assist the expert in determining their confidence rating score, verbal descriptions (e.g. pretty confident) and odds ratios (better than $10 - 1$ or $20 - 1$ odds) are provided. These confidence scores are converted to a percentage of confidence w_j, defined below.

Because the experts are presented with all variables of interest at once, they are able to answer the questions in the order that suits them and make changes to their responses at any time during the elicitation. Due to the simple spreadsheet design no feedback is given to the experts during the elicitation. Since all variables are viewed at the same time, it is assumed (and encouraged in the supporting documentation/introduction) that when the expert is deciding on the impact of one variable, all other variables displayed are considered to be held at their optimum and each expert's set of responses is coherent.

An expert's impact and confidence score for each variable are formulated into a mixture of normal priors, where the *m*th mixture component reflects decreasing, no substantive change or increasing probability of presence, as each covariate changes with respect to a baseline value. No 'substantive' change was represented by the range $|\beta_j| < \delta_j$, $\delta_j > 0$. The lower and upper endpoints for the mixture are denoted by A_j and B_j, respectively, which are set at 'sensible' limits on the values of the regression coefficients. Given a stated confidence is w_j for impact z_j, the latent variable z_j is scaled to account

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for the endpoints (B and δ) as follows: if $z_j = -1$ then $-A_j \leq \beta_j \leq 0$; if $z_j = 0$ then $-\delta_j \leq \beta_j \leq \delta_j$ and if $z_j = +1$ then $0 \leq \beta_j \leq B_j$. The values of δ_j, A_j and B_j can be set at biologically or physically reasonable limits on the values of the regression coefficients, elicited from the experts themselves or based on published data. In this case study, the first of these options was adopted and in the absence of other information, we set $\delta_j = \delta$, $B_j = B$ and $A_j = -B \; \forall j = 1, \ldots J$.

Calculation of the remaining confidence amongst the other two possibilities depends on the stated impact, and taking into account the endpoints (A, B and δ). If the stated impact is positive ($z_j = +1$) then

$$\begin{aligned} w^*_{j,-1} &= p(z_j = -1) = \left(\frac{B-1}{B}\right)\left(1 - w_{j,+1}\right) \\ w^*_{j,0} &= p(z_j = 0) = 1 - w^*_{j,-1} - w^*_{j,+1} = \frac{1}{B} \\ w^*_{j,+1} &= p(z_j = +1) = \left(\frac{B-1}{B}\right) w_{j,+1} \end{aligned} \tag{5}$$

Similarly, when the stated impact is negative ($z_j = -1$), then $(1 - w_{j,+1})$ is replaced with $w_{j,-1}$ in the expressions for $w^*_{j,-1}$ and $(w_{j,+1})$ is replaced with $(1 - w_{j,-1})$ in expressions for $w^*_{j,+1}$. Alternatively if the stated impact is no change ($z = 0$) then $w^*_{j,0} = w_{j,0}$ and $w^*_{j,-1} = w^*_{j,+1} = \frac{1}{2}(1 - w_{j,0})$.

The expert informed prior for each variable j is then modelled as a mixture of three distributions corresponding to each possible impact $z_j = m$, with mixture parameters θ_j, where $\beta_j \sim \sum_{m=-1,0,+1} w_{jm}\, p(\beta_j | z_j = m, \theta_j)$. Here the endpoints of the β_j coefficient are assumed to be B and δ, therefore the mixture component $m = -1$, $\mu_j = -B/2$ and $\sigma_j = B/[2\Phi^{-1}(1 - \alpha/2)]$; for $m = 0$, $\mu_j = 0$ and $\sigma_j = \delta/\Phi^{-1}(1 - \alpha/2)$ and for $m = +1$, $\mu_j = B/2$ and $\sigma_j = (-B)/[2\Phi^{-1}(\alpha/2)]$. Since the intercept was not elicited for this method, a non-informative prior (normal distribution with a mean of zero and variance 10) was placed on the intercept.

The elicitation questionnaire was designed as a spreadsheet. The questionnaire was divided into three sections. Firstly, an introduction sets up the problem of habitat suitability modelling for rare and threatened species. This included a brief definition of the probability of occurrence and habitat, area of occurrence and detectability. Following this, there was a description of the questions, split into categorical and continuous variables. A table was provided so that the expert could provide their opinion on the impact of a variable and their confidence. Lastly, the expert was also asked to identify which variables they believed to be important in determining the occurrence of the species.

2.4. *Non-informative prior*

The sensitivity to prior knowledge was investigated by comparing posterior distributions under the model with expert informed priors to the model with non-informative priors. The non-informative prior selected for all coefficients in the model was a normal distribution with a mean of zero. A large though sensible range for coefficient values was achieved by setting the variance to 10.

2.5. *Comparison of the three expert elicitation approaches*

Although the three elicitation methods have a common purpose, they differ in substantial ways as summarised in Table 1. The form of the prior distribution on the regression coefficients β is different

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Table 1. Advantages and disadvantages of three elicitation methods

		Method A	Method B	Method C
1	Prior distribution for β coefficient	$\mathrm{MVN}(\mu, \Sigma)$	$\mathrm{N}(\mu, \sigma)$	$\sum_{m=1,2,3} w_{jm}\mathrm{N}(\mu_{jm}, \sigma^2_{jm})$
2	Types of elicitation	indirect P-method	indirect PV-method	direct
3	Elicitation tool	geographically assisted	graphically assisted	questionnaire
	Advantages			
4	Accomplished remotely			✓
5	Elicitation method repeatable	✓	✓	✓
6	Flexibility of model structure	✓	✓	
7	Easily handles multiple experts			✓
8	Important variables are identified by experts	✓	✓	✓
9	Quick and simple			✓
10	Model feedback during elicitation	✓	✓	
11	Do not need to understand statistical model	✓		✓
12	Takes advantages of spatial nature of data	✓ ✓		
13	Exploits quantifiable ecological knowledge	✓	✓	
	Disadvantages			
14	Knowledge of basic probability theory	✓	✓	
15	Limited to landscape scale variables	✓		
16	Complex elicitation software required	✓	✓	

in all three methods, with method A assuming a MVN distribution, method B also assuming a MVN distribution with independent errors and method C assuming a mixture of three normal distributions. Both methods A and B elicit expert opinion indirectly but differ in the type of indirect elicitation (P- and PV-methods, respectively), while method C elicits directly. Methods A and B both require specific complex software, based on GIS and graphical architecture, respectively. Method C has a simple spreadsheet design, so does not require specific software. The software of methods A and B provide feedback to the expert during the elicitation, whereas method C does not have this capability (although it could be developed). Model feedback of method A is in the form of univariate response curves. The feedback of method B is the ability to reflect back a different CI and plot the prior probability density function on the scale of the response. Cognitive biases were reduced in both methods A and B through feedback, whereas method C decreased these biases by means of the spreadsheet design and the simple approach of eliciting the model parameters.

A facilitator is needed for methods A and B to teach the expert how to use the specific software. Also interaction between the facilitator and expert is required, so that the elicitation questions can be phrased to suit the individual expert's understanding and knowledge of statistics and probabilities. Because method C requires no complex software, this elicitation method can be answered remotely without a facilitator, but if required the questionnaire can be supported by telephone and/or online discussions.

All three elicitation methods are repeatable and can also be used to elicit opinions of other rock-wallaby experts, and can be applied to other case studies which would benefit from expert elicitation.

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The expert is asked in all three elicitation methods to identify the important variables in predicting the presence of the rock-wallaby. After the elicitation, only method A can develop a model with different variables to those selected and used by the expert during the elicitation. However, the selection of variables in this method is limited to environmental variables available as GIS layers (thus landscape scale) such as topography and climate. Only method A acknowledges the spatial nature of the rock-wallaby data, by eliciting expert opinion via maps.

Method C can formulate priors from the opinion of a single or multiple experts, whereas complete priors can be built from one expert under methods A and B. An expert's understanding of the logistic regression model being applied is not required for methods A and C. Methods A and B quantify expert opinion of the presence of rock-wallabies as probabilities, whereas method C only elicits the impact of covariates on the response as one of three categories (decreasing, no change or increasing). Therefore methods A and B require a more complex understanding of probabilities, that is understanding of quantiles. A disadvantage of method C is that assumptions have to be made on the values of interval endpoints, in order to convert elicited information into a mixture of normal priors.

3. RESULTS

3.1. Results from elicitation

The variables aspect and slope are used here to illustrate and compare the prior and posterior distributions obtained under the three elicitation methods.

Table 2, Figure 1 and Table 3 display the elicitation results of experts I and II using methods A, B and C, respectively. For method A, 13 points on the map were elicited from expert I and 10 points from expert II. Aspect and slope were elicited as continuous variables. Elicited quantiles of expert II were broader than those of expert I, resulting in expert II's estimation of the probability of presence ($p_k \sim Be(a_k, b_k)$) having wider range than expert I (Table 2). The results from method A imply that both

Table 2. Elicitation results of experts I and II using method A; 13 points were elicited from expert I and 10 from expert II. Shown for each point are the value of aspect and slope provided to the expert and the estimation of $\hat{a}$ and $\hat{b}$ from the elicited beta distribution

	Expert I				Expert II			
Site	Aspect	Slope	$\hat{a}$	$\hat{b}$	Aspect	Slope	$\hat{a}$	$\hat{b}$
1	354.24	24.71	1.00	2.20	310.56	20.42	4.10	2.20
2	38.20	27.21	0.70	2.40	322.23	28.74	7.60	4.20
3	330.67	13.38	0.30	1.70	18.59	18.12	2.50	1.20
4	12.40	13.09	0.90	6.90	90.42	16.82	2.00	4.00
5	310.35	18.21	1.00	4.20	185.92	24.01	1.00	4.30
6	189.11	29.36	1.00	14.80	346.67	26.17	9.00	3.20
7	10.64	33.14	1.00	50.00	45.63	13.24	2.60	5.20
8	291.49	27.90	1.00	3.70	13.24	28.02	4.20	1.00
9	64.69	0.74	0.90	100.00	140.20	29.53	1.40	4.20
10	16.83	10.05	0.90	200.00	314.76	1.48	1.00	9.70
11	240.19	25.80	1.00	2.20				
12	181.55	0.21	1.00	200.00				
13	91.99	31.60	1.00	3.20				

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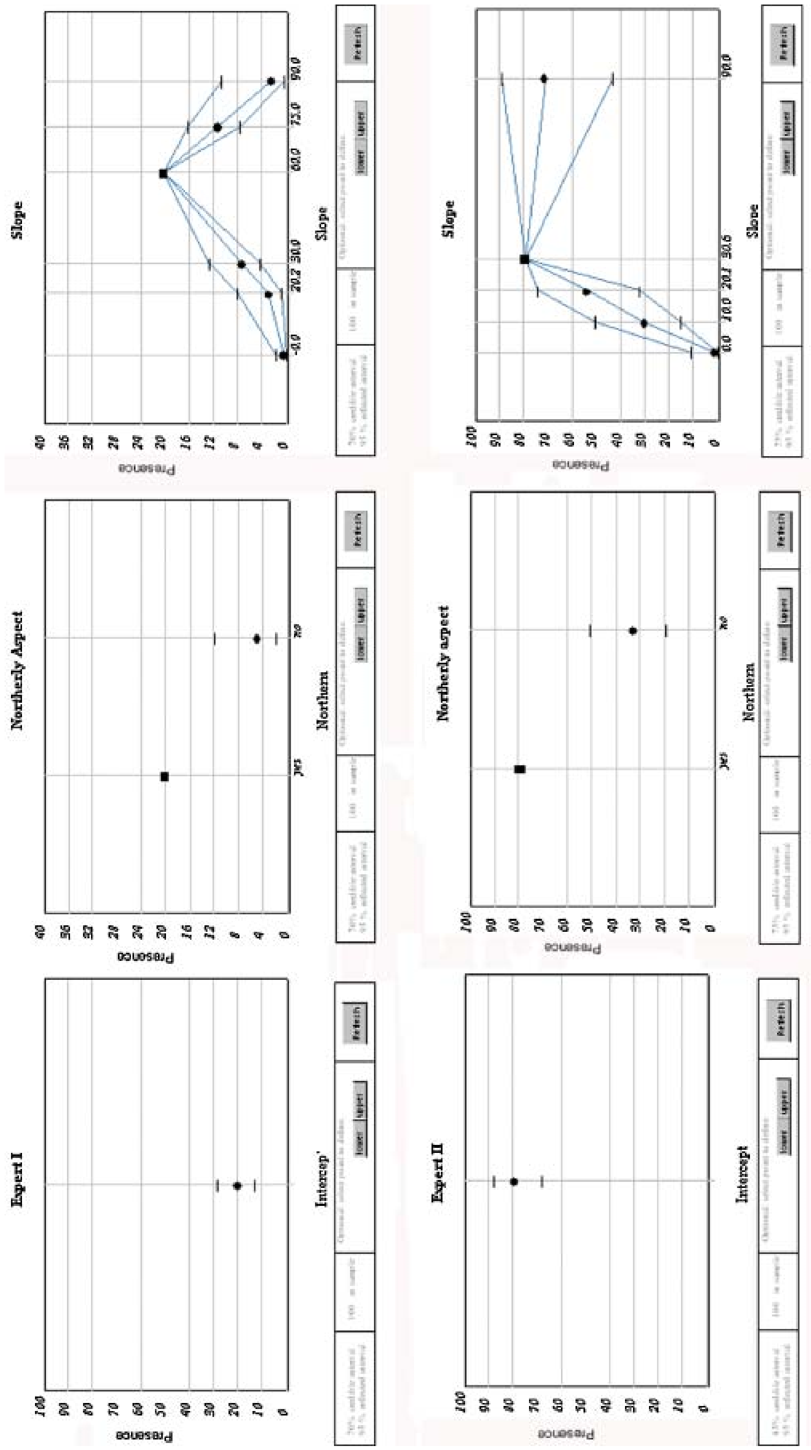

Figure 1. Elicitation results of experts I (row 1) and II (row 2) using method B—elicited intercept (first column), aspect (second column) and slope (third column)

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Table 3. Elicitation results of experts I and II using method C—elicited impact and confidence score for aspect and slope

	Expert I		Expert II	
Variable	Impact	Score	Impact	Score
North	+1	6	+1	6.6
Northwest	+1	4	+1	6
Northeast	+1	4	+1	6
West	0	3	0	5
Slope	0	4	+1	5

experts believe that there is a relationship between aspect and slope, since the majority of the points with a high estimated probability of presence ($\hat{y}$) have northerly aspects ($300^\circ - 360^\circ$ and $0^\circ - 45^\circ$) and steep slopes (>20). Expert I was unfamiliar with GIS but expert II was an experienced user; so expert I took longer to learn the elicitation software. Denham and Mengersen (2007) applied a second order polynomial to aspect that was flexible enough to capture the effect aspect might have on probability of occurrence. For further details of the results of method A for the rock-wallaby case study see Denham and Mengersen.

Using method B, expert I reflected that when all variables are at their optimum then the rock-wallaby would be present at 20 out of 100 survey sites (intercept), whereas expert II opined presence at 80 out of 100 sites (Figure 1). For both experts, aspect was elicited as a categorical variable, with the optimum category being northerly aspects. Experts I and II reflected that the effect of changing aspect (when all other variables are at their optimum) to aspects other than northerly was that the expected number of sites dropped by 5 and 34 sites, respectively. Slope was elicited as a piecewise linear function from both experts. The experts disagreed about the effect of slope, in particular the optimum level (expert I believed it is 60 whereas expert II opined 30) and magnitude of response to change (Figure 1). However, they agreed that sites with very flat slopes have a very low probability of presence.

For method C, aspect was elicited as a categorical variable and slope as a continuous variable. The questionnaire responses showed that both experts believed northern (north, northwest and northeast) aspects increased the probability of occurrence compared to the baseline of west aspect (Table 3), and the other aspect levels were believed to have a negative impact. The experts' opinion on these three aspects were averaged into a 'northern' aspect. The experts disagreed about the impact of slope: expert I believed there was no change in probability of presence when slope increased, while expert II believed this probability increased as slope increased. Expert II was more confident than expert I about the impact of slope.

3.2. *Comparison of the prior and posterior distributions*

For each method, the information elicited from each expert was translated into priors, using the methodology described in Subsection 2.3 and then combined with rock-wallaby data to form the posterior distributions. Table 4 shows the two experts' elicited means and 95% CIs of slope and aspect from the three elicitation methods. Figure 2 displays the non-informative priors, the expert informed priors (of both experts from the three elicitation methods) and the posterior distributions of slope generated from each of these priors. The prior and posterior distributions for all three methods have been standardised so that all distributions are on the same scale.

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Table 4. Summary of the expert I and II's standardised prior means and 95% CI of slope and aspect using the three elicitation methods

	Method A		Method B		Method C		
	Expert I	Expert II	Expert I	Expert II	Expert I	Expert II	Both expert
Slope	0.87	1.15	0.19	1.03	−0.01	0.92	0.41
	(−0.71, 2.46)	(−0.52, 2.82)	(−3.72, 4.51)	(−0.65, 4.59)	(−4.13, 4.15)	(−4.16, 4.74)	(−4.10, 4.46)
Aspect	0.06	−0.28	−1.50	−2.00	0.57	1.77	1.17
	(−1.91, 2.03)	(−1.31, 0.75)	(−4.80, 2.12)	(−4.52, 0.67)	(−4.24, 4.607)	(−2.61, 4.84)	(−3.83, 4.75)

The prior means for the slope coefficient are similar for experts I and II using method A and expert II using methods B and C, with all means concentrated around one. The prior mean for expert I, obtained using methods B and C are concentrated around zero. For method C, expert I believed that increased slope had no impact on the probability of occurrence, whereas expert II believed the response increased; both opinions are reflected in the combined prior (Figure 2). For method C, the prior 95% CIs are broader than those obtained for the other two elicitation methods.

The posterior means and 95% CIs of the coefficient for slope, for the two experts from the three elicitation methods, are summarised in Table 5. The corresponding posterior distributions are displayed in Figure 2. As expected, these posterior distributions reflect the expert priors, but it is of interest to observe the effect of the different priors in light of the data.

For comparison, the posterior means and 95% CIs for the model with non-informative priors are displayed in Table 6, and Figure 2 shows the corresponding posterior distributions. The posterior distributions from the priors of experts I and II for slope obtained from methods A and C are similar to those with non-informative priors (Figure 2), but less diffuse. However, for method B the posterior distributions, from the priors of experts I and II, are concentrated around zero, whereas the posterior distributions from the model with non-informative priors are concentrated around one. This indicates that both experts' opinions are similar to the observed data for methods A and C.

The means and 95% CIs of aspect elicited from experts I and II using methods A, B and C are displayed in Table 4. Figure 3 shows the non-informative priors, the expert informed priors (of both experts from the three elicitation methods) and the posterior distributions of aspect generated from each of these priors.

Each method elicited different characteristics of aspect, therefore comparison of the prior and posterior distributions for the aspect coefficient is difficult. However, the prior distributions of both experts acquired using all three methods reflect the belief that the probability of presence is higher at sites with northerly aspects (Table 4 and Figure 3). The CIs of aspect for expert II are smaller in all three methods compared to expert I (Table 4), indicating that expert II was more confident in their opinion about the impact of aspect on the occurrence of rock-wallabies under all three methods.

Table 5 summarises the posterior means and 95% CIs of the regression coefficient for aspect arising from the two experts under the three elicitation methods; and Figure 3 displays the posterior distributions from these priors. The posterior means from expert informed priors developed from methods A and C are similar to the respective expert informed priors, particularly for expert II. As expected, the posterior distributions are tighter than the priors, particularly for method C. However, for method B there is a shift in the prior and posterior means (Figure 3).

The posterior means and 95% CIs for the model with non-informative priors are summarised in Table 6 and Figure 2. The posterior distributions from the priors of experts I and II for aspect obtained

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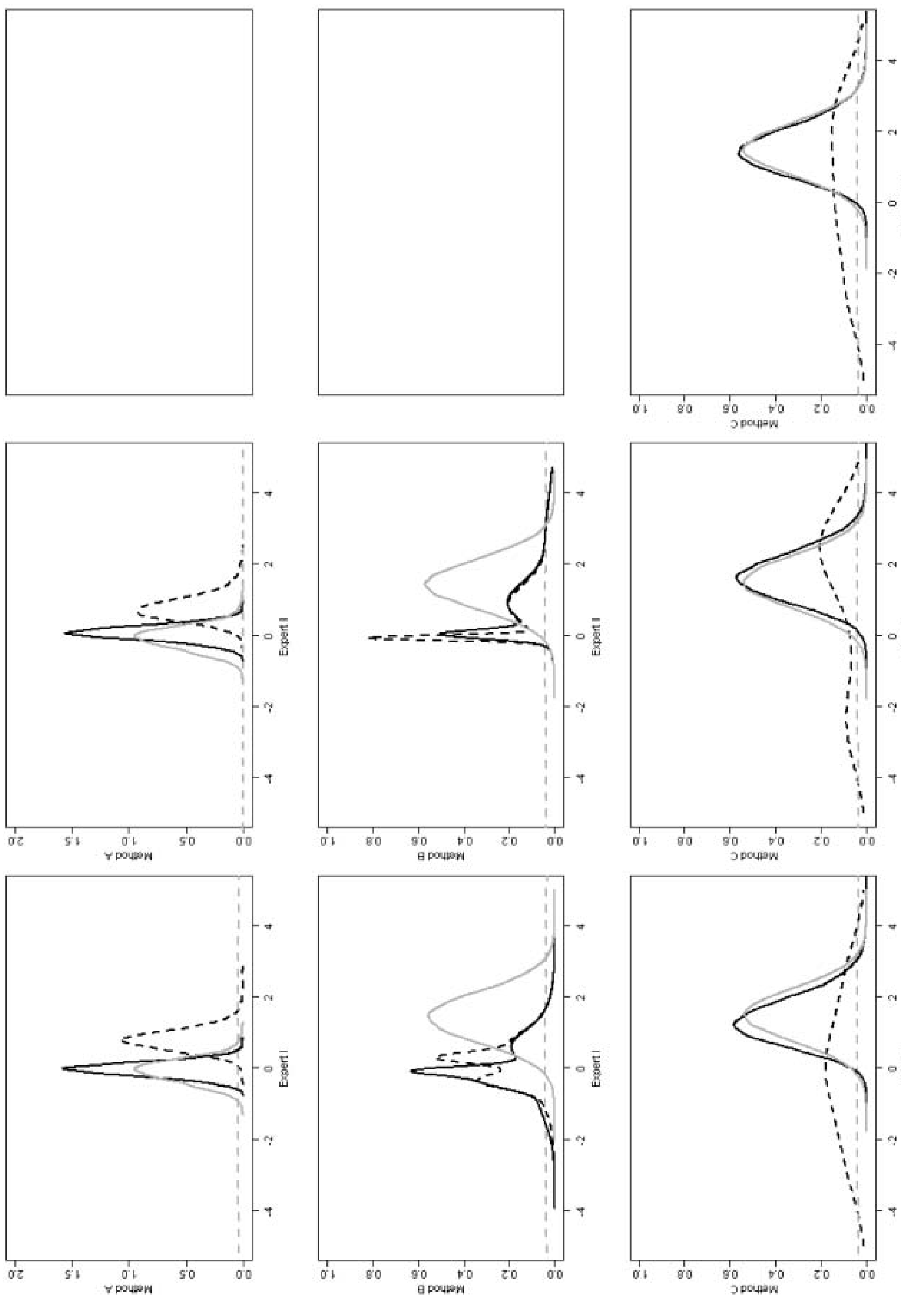

Figure 2. Prior (dashed line) and posterior (solid line) plots of the slope parameter in model with experts I or II or both expert's informed priors (black) obtained from the three elicitation methods or weakly informative priors (grey)

Table 5. Summary of the standardised posterior means and 95% credible intervals (CI) of the two experts from the three elicitation methods for slope and aspect coefficients

	Method A		Method B		Method C		
	Expert I	Expert II	Expert I	Expert II	Expert I	Expert II	Both expert
Slope	−0.02	0.11	0.07	1.08	1.38	1.68	1.50
	(−0.74, 0.71)	(−0.67, 0.90)	(−3.72, 4.51)	(−0.65, 4.59)	(0.15, 2.86)	(0.38, 3.13)	(0.21, 3.00)
Aspect	0.78	−0.15	1.81	−0.53	1.43	1.55	1.49
	(−0.25, 1.81)	(−0.96, 0.66)	(−0.63, 4.25)	(−2.41, 1.35)	(−0.26, 3.01)	(−0.07, 3.10)	(−0.16, 3.06)

Table 6. Summary of the standardised posterior, model with non-informative priors, means and 95% credible intervals (CI) from the three elicitation methods for slope and aspect coefficients

	Method A	Method B	Method C
Slope	−0.02	1.45	1.51
	(−1.27, 1.23)	(0.003, 1.40)	(0.21, 3.10)
Aspect	−0.11	0.82	1.31
	(−1.69, 1.48)	(0.01, 0.86)	(−0.41, 3.07)

from methods C are similar to those with non-informative priors. Likewise the posterior distributions from expert II acquired using method A are similar to those with non-informative priors, indicating that expert I's opinion is similar to the observed data for method C and the opinion of expert II concurs with the data for methods A and C.

4. DISCUSSION

This paper compares three elicitation methods for the logistic regression model: a geographically assisted P-method (Denham and Mengersen, 2007), a graphically assisted predictive PV-method (Kynn, 2005) and a questionnaire delivered that elicits a simplified version of the expert's opinion directly (O'Leary *et al*., 2008). The opinions of two experts were elicited using these three methods with the intention of modelling the habitat suitability of the threatened Australian brush-tailed rock-wallaby. These three approaches differ in the type of elicitation, the prior model, the elicitation tool and requirement of a facilitator.

The results indicate that the method of elicitation can indeed influence an expert-based prior. There were distinct similarities and dissimilarities in the elicited prior means of the slope coefficient between the three elicitation methods and the two experts. In particular, the prior distributions for the slope coefficient were similar for expert II using all three methods. The dissimilarities in opinion between the two experts on slope may be due to their differences in knowledge. Specifically, expert I has more knowledge of the species in New South Wales, and slope may affect the species differently in this state. However, the three methods evoked different priors for expert I. Comparison of the prior and posterior distributions for the aspect coefficient was difficult because each method elicited different characteristics of this variable. However, all three methods identified that the opinion of both experts is that sites with northern aspects have the highest probability of presence. In addition, even though the two rock-wallaby experts have very different research experiences and knowledge, the results acquired in this trial show that their opinions on aspect and slope were comparable.

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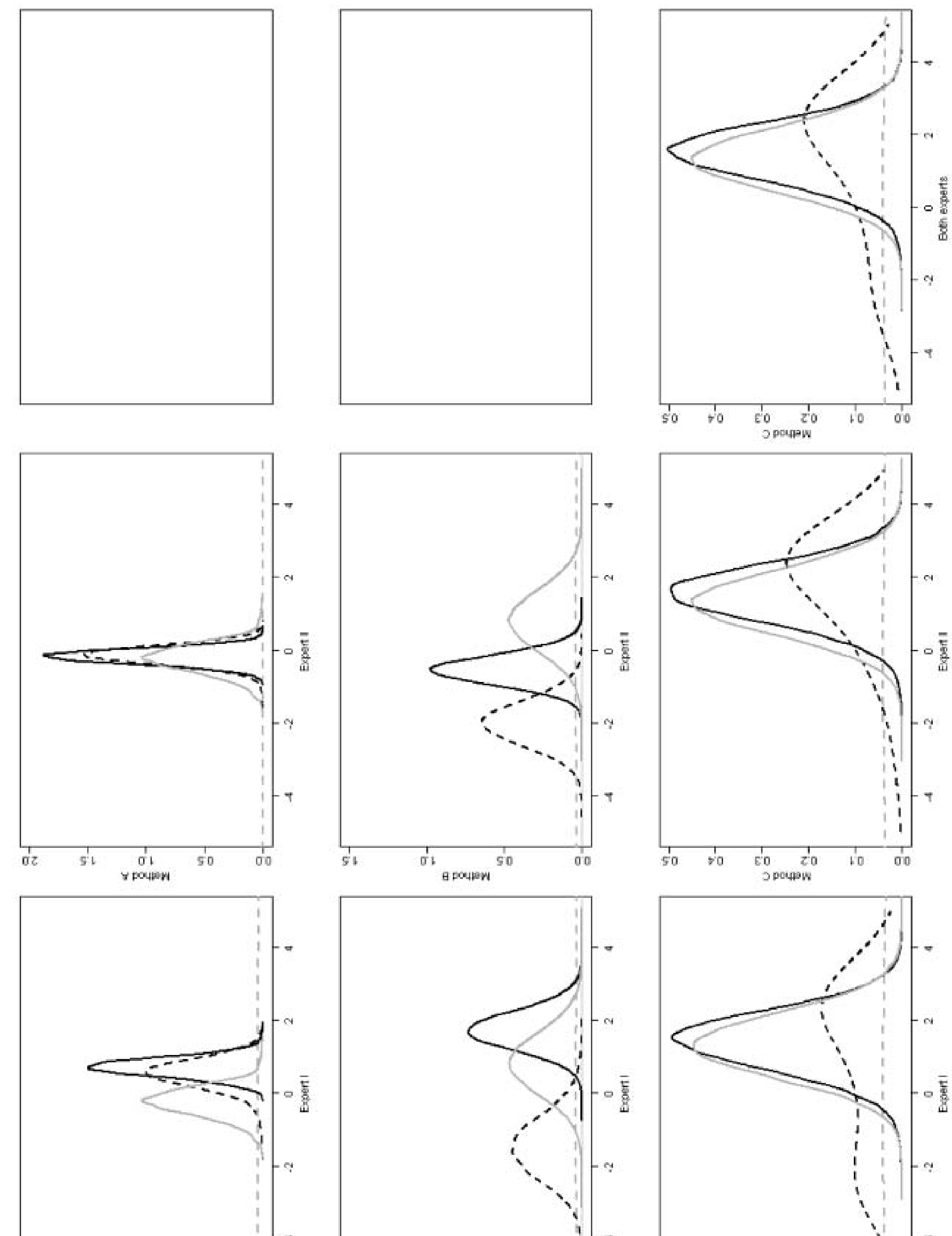

Figure 3. Prior (dashed line) and posterior (solid line) plots of the aspect parameter in model with experts I or II or both expert's informed priors (black) obtained from the three elicitation methods or weakly informative priors (grey)

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Selection of an elicitation method is determined by several issues. These include the expert's knowledge of statistics, their mapping skills, time available, access to experts and funding. The chosen elicitation method should balance the expert's knowledge of statistics and mapping with the output required. If there is limited time to carry out the elicitation, limited funding available, experts are in another city or remote areas and/or the expert has limited statistical knowledge then an approach such as method C should be selected. If more time and funding is available then methods A and/or B should be selected, provided that the expert is given sufficient training. Selection between methods A and B may be determined by whether GIS data is relevant (eg landscape scale habitat modelling), the expert's mapping and statistical skills and the facilitator's access to mapping resources and software. If the expert has limited knowledge of the model being applied and some geographical skills then an approach such as method A should be applied.

The observed dataset of the rock-wallaby case study used to compare the three elicitation methods comprised only 50 observed sites. A frequentist analysis of this dataset (Murray, 2002) was inconclusive as most coefficients were not significantly different from zero, although there was some indication (large coefficients but large standard errors) that the factors for slope, aspect and rock type may be useful for predicting the probability of site occupancy by the rock-wallaby. This case study is an example of a situation arising often in ecology, where the data are limited, possibly biased and possibly non-representative (Austin and Meyers, 1996; Thuiller *et al*., 2004). Collecting data on the rock-wallaby is difficult as it is mostly located in remote areas, being rocky, steep gorges which are therefore very difficult to access. Not surprisingly the observed dataset derived from fieldwork contained limited presence and absence data, therefore possibly missing important features that were relevant to modelling the species' habitat preferences. This trial reveals that expert knowledge can be important when modelling rare event data, such as rare and threatened species, since experts can provide additional information that may not be represented in the dataset.

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REFERENCES

Al-Awadhi AS, Garthwaite PH. 2006. Quantifying expert opinion for modelling fauna habitat distributions. *Computational Statistics* **21**(1): 121–140.

Al-Awadhi S, Garthwaite PH. 2001. Prior distribution assessment for a multivariate normal distribution: an experimental study. *Journal of Applied Ecology* **28**(1): 5–23.

Albert JH, Chib S. 1993. Bayesian analysis of binary and polychotocmous response data. *Journal of the American Statistical Association* **88**: 669–679.

Alpert M, Raiffa H. 1982. A progress report on the training of probability assessors. In *Judgment under Uncertainty: Heuristics and Biases*, Kahneman D, Slovic P, Tversky A (eds). Cambridge University Press: Cambridge; 294–305.

Austin MP. 2002. Spatial prediction of species distribution: an interface between ecological theory and statistical modelling. *Ecological Modelling* **157**: 101–118.

Austin MP, Meyers JA. 1996. Current approaches to modelling the environmental niche of eucalypts: implications for management of forest biodiversity. *Forest Ecology and Management* **85**: 95–106.

Bedrick EJ, Christensen R, Johnson W. 1996. A new perspective on priors for generalized linear models. *Journal of the American Statistical Association* **91**: 1450–1460.

Besag JE, Green PJ, Higdon DM, Mengersen KL. 1995. Bayesian computation and stochastic systems (with discussion). *Statistical Science* **10**: 3–66.

Borsuk ME. 2004. Predictive assessment of fish health and fish kills in the Neuse river estuary using elicited expert judgment. *Human and Ecological Risk Assessment* **10**(2): 415–434.
Brotons W, Thuiller W, Araujo MB, Hirzel AH. 2004. Presence-absence versus presence-only modelling methods for predicting bird habitat suitability. *Ecography* **27**: 437–448.
Carter K, Goldizen A. 2003. Habitat choice and vigilance behaviour of brush-tailed rock-wallabies (*Petrogale penicillata*) within their nocturnal foraging ranges. *Wildlife Research* **30**: 355–364.
Chen MH, Dey DK. 2003. Variable selection for multivariate logistic regression models. *Journal of Statistical Planning and Inference* **111**: 37–55.
Chen MH, Ibrahim JG, Shao QM, Weiss RE. 2003. Prior elicitation for model selection and estimation in generalized linear mixed models. *Journal of Statistical Planning and Inference* **111**: 57–76.
Chi MTHC, Glaser R, Farr MJ (eds). 1988. *The Nature of Expertise*. Lawrence Erlbaum Associates: New Jersey.
Low Choy SJ, O'Leary R, Mengersen K. 2008. Elicitation by design in ecology: using expert opinion to inform priors for Bayesian statistical models. *Ecology* (in press).
Clemen R, Reilly T. 2001. *Making Hard Decisions with Decision Tools*. Pacific Grove, CA: Duxberry.
Clemen RT, Winkler RL. 1999. Combining probability distributions from experts in risk analysis. *Risk Analysis* **19**(2): 187–203.
DeGroot MH. 1974. Reaching a consensus. *Journal of the American Statistical Association* **69**: 118–121.
DeGroot MH. 1988. A Bayesian view of assessing uncertainty and comparing expert opinion. *Journal of Statistical Planning and Inference* **20**: 295–306.
Denham R, Mengersen K. 2007. Geographically assisted expert elicitation for species' distribution models. *Bayesian Analysis* **2**(1): 99–136.
Eldridge MDB. 1997. Rock-wallaby conservation: essential data and management priorities. Proceedings of the 1994 National rock-wallaby Symposium workshops. *Australian Mammalogy* **19**: 325–330.
Engler R, Guisan A, Rechsteiner L. 2004. An improved approach for predicting the distribution of rare and endangered species from occurence and pseudo-absence data. *Journal of Applied Ecology* **41**: 263–274.
Garthwaite PH. 1998. Quantifying expert opinion for modelling habitat distributions. *Technical Report*, Sustainable Forest Management Technical Report, Department of Natural Resources Queensland.
Garthwaite PH, Al-Awadhi AS. 2006. Quantifying opinion about a logistic regression using interactive graphics. *Technical Report*, Department of Statistics Technical Report, Open University.
Garthwaite PH, Kadane J, O'Hagan A. 2005. Statistical methods for eliciting probability distributions. *Journal of the American Statistical Association* **100**(470): 680–700.
Gill J, Walker L. 2005. Elicited priors for Bayesian model specifications in political science research. *The Journal of Politics* **67**(3): 841–872.
Graham C, Ferrier S, Huettman F, Moritz C, Peterson A. 2004. New developments in museum-based informatics and applications in biodiversity analysis. *Trends Ecology and Evolution* **19**: 497–503.
Guisan A, Zimmermann NE. 2000. Predictive habitat distribution models in ecology. *Ecological Modelling* **135**: 147–186.
Hogarth RM. 1975. Cognitive processes and the assessment of subjective probability (with discussion). *Journal of the American Statistical Association* **70**: 271–294.
Hosmer DW, Lemeshow S. 1989. *Applied Logistic Regression*. Wiley: New York, USA.
Ibrahim JG, Ryan LM, Chen MH. 1998. Using historical controls to adjust for covariates in trend tests for binary data. *Journal of the American Statistical Association* **93**: 1282–1293.
Jenkinson D. 2005. The elicitation of probabilities—A review of the statistical literature. *Technical Report*, Bayesian Elicitation of Expert's Probabilities (BEEP) Working Paper.
Kadane JB, Dickey JM, Winkler RL, Smith WS, Peters SC. 1980. Interactive elicitation of opinion for a normal linear model. *Journal of the American Statistical Association* **75**: 845–854.
Kadane J, Wolfson L. 1998. Experiences in elicitation. *The Statistician* **47**: 3–19.
Kahneman D, Tversky A. 1972. Subjective probability: a judgment of representativeness. *Cognitive Psychology* **3**: 430–454.
Kuhnert PM, Martin TG, Mengersen K, Possingham HP. 2005. Assessing the impacts of grazing levels on bird density in woodland habitat: a Bayesian approach using expert opinion. *Environmetrics* **16**(7): 717–747.
Kynn M. 2005. Eliciting expert knowledge for Bayesian logistic regression in species habitat modelling in natural resources. *PhD Thesis*, Queensland University of Technology.
Kynn M. 2008. The 'heuristics and biases' bias in expert elicitation. *Journal of the Royal Statistical Society, Series A* **171**: 239–264.
Manel S, Dias J, Ormerod S. 1999. Comparing discriminant analysis, neural networks and logistic regression for predicting species distributions: a case study with a Himalayan river bird. *Ecological Modelling* **120**: 337–347.
Manel S, Williams H, Ormerod S. 2001. Evaluating presence-absence models in ecology: the need to account for prevalence. *Journal Of Applied Ecology* **38**: 921–931.
Martin TG, Kuhnert PM, Mengersen K, Possingham HP. 2005. The power of expert opinion in ecological models: a Bayesian approach examining the impact of livestock grazing on birds. *Ecological Applications* **15**(1): 266–280.
Maynes GM, Sharman GB. 1983. Brush-tailed rock-wallaby. In *The Complete Book of Australian Mammals*, pp. 211–212. Strahan R (ed.). Angus and Robertson: Sydney.

DOI: 10.1002/env

Morgan MG, Henrion M. 1990. *Uncertainty: A Guide to Dealing With Uncertainty in Quantitative Risk and Policy Analysis*. Cambridge University Press: New York.
Moritz C, Richardson KS, Ferrier S, Monteith G, Stanisic J, Williams SE, Whiffin T. 2001. Biogeographical concordance and efficiency of taxon indicators for establishing conservation priority in a tropical rainforest biota. *Proceedings of the Royal Society B-Biological Sciences* **268**(1479): 1875–1881.
Mullin TM. 1986. Understanding and supporting the process of probabilistic estimation. *PhD Thesis*, Carnegie Mellon University.
Murray JV 2002. Analysis and habitat modelling of the brush-tailed rock-wallaby (*Petrogale penicillata*) in southest Queensland. In School of Animal Studies. *Technical Report*, University of Queensland.
Murray J, Low Choy SJ, McAlpine C, Possingham H, Goldizen A. 2008. The importance of ecological scale for wildlife conservation in naturally fragmented environments: A case study of the brush-tailed rock-wallaby (*Petrogale penicillata*). *Biological Conservation* **141**: 7–22.
O'Hagan A. 1998. Eliciting expert beliefs in substantial practical applications. *The Statistician* **47**: 21–35.
O'Hagan A, Buck CE, Daneshkhah A, Eiser JR, Garthwaite PH, Jenkinson DJ, Oakley JE, Rakow T. 2006. *Uncertain Judgements: Eliciting Expert Probabilities*. Wiley: United Kingdom.
O'Leary R, Mengersen K, Low Choy SJ. 2008. A mixture model approach to representing simple expert information as priors in logistic regression. Technical report, School of Mathematical Sciences, Queensland University of Technology.
Paruccini M. 1994. *Applying Multicriteria Aid for Decision to Environmental Management*. Kluwer: Dordrecht.
Pearce JL, Cherry K, Drielsma M, Ferrier S, Whish G. 2001. Incorporating expert opinion and fine-scale vegetation mapping into statistical models of faunal distribution. *Journal of Applied Ecology* **38**: 412–424.
Pearce J, Ferrier S, Scotts D. 2001. An evaluation of the predictive performance of distributional models for flora and fauna in north-east New South Wales. *Journal of Environmental Management* **62**: 171–184.
Radeloff VC, Pidgeon AM, Hostert P. 1998. Habitat and population modelling of roe deer using an interactive geographic information system. *Ecological Modelling* **114**: 287–304.
Rushton S, Ormerod S, Kerby G. 2004. New paradigms for modelling species distributions? *Journal of Applied Ecology* **41**: 193–200.
Sachdev M, Sun JL, Tsiatis A, Nelson C, Mark D, Jollis J. 2004. The prognostic importance of comorbidity for mortality in patients with stable coronary artery disease. *Journal of the American College Cardiology* **43**(4): 576–582.
Scott J, Heglund P, Morrison M, Haufler J, Raphael M, Wall W, Samson F. 2002. *Predicting Species Occurrences: Issues of Accuracy and Scale.* Island Press: Washington.
Shanteau J. 1989. Psychological characteristics and strategies of expert decision makers. In B. Rohrmann, L. Beach, C. Vlek, S. Watson (Eds.), *Advances in Decision Research*, pp. 203–215. Amsterdam: North Holland.
Shepherd GG, Kirkwood CW. 1994. Managing the judgmental probability elicitation process: a case study of analyst/manager interaction. *IEEE Transactions on Engineering Management* **41**: 414–425.
Spetzler CS, Staël von Holstein CS. 1975. Probability encoding in decision analysis. *Management Science* **22**(3): 340–358.
Spiegelhalter DJ, Thomas A, Best NG, Gilks WR, Lunn D. 2003. *BUGS: Bayesian inference using Gibbs sampling*. MRC Biostatistics Unit, Cambridge, England. www.mrc-bsu.cam.ac.uk/bugs/
Thuiller W, Brotons L, Araujo MB, Lavorel S. 2004. Effects of restricting environmental range of data to project current and future species distributions. *Ecography* **27**(2): 165–172.
Tversky A, Kahneman D. 1974. Judgment under uncertainty: heuristics and biases. *Science* **185**: 1124–1131.
Walls L, Quigley J. 2001. Building prior distributions to support Bayesian reliability growth modelling using expert judgement. *Reliability Engineering and System Safety* **74**: 117–128.
Walpole, RE, Myers RH. 1993. *Probability and Statistics for Engineers and Scientists* (5th ed.). New York: Macmillan.
Winkler R. 1967. The assessment of prior distributions in Bayesian analysis. *Journal of the American Statistical Association* **62**: 776–800.
Yamada K, Elith J, McCarthy M, Zerger A. 2003. Eliciting and integrating expert knowledge for wildlife habitat modelling. *Ecological Modelling* **165**: 251–264.
Yu D. 2002. Modeling and measuring the effects of imprecision in accident management. *Annals of Nuclear Energy* **29**: 821–833.

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