Abstract—In this letter, we present a squaring method to simplify the decoding of orthogonal space–time block codes in a wireless system with an arbitrary number of transmit- and receive-antennas. Using this squaring method, a closed-form expression of signal-to-noise ratio after space–time decoding is also derived. It gives the same decoding performance as the maximum-likelihood ratio decoding while it shows much lower complexity.

Index Terms—Diversity, multipath channels, multiple antennas, space–time codes.

I. INTRODUCTION

RECENTLY, a new space–time coding has been proposed which combines signal processing at the receiver with coding techniques appropriate to multiple number of transmit- and receive-antennas wireless system [1]. Alamouti presents a simple transmit diversity scheme. Using two transmit- and one receive-antenna, the scheme provides the same diversity order as maximal-ratio receiver combining with one transmit- and two receive-antennas [2]. Based on Alamouti’s work, Tarokh et al. propose orthogonal space–time block codes (OSTBCs) combining the orthogonal coding method and this simple diversity technique [3]. Tarokh studies the encoding and decoding algorithms for various codes and provides simulation results demonstrating their performance [4]. In this letter, a squaring method to simplify the decoding of OSTBC is proposed, which is a generalization from real orthogonal designs in [3] to complex ones. In addition, a closed-form expression of signal-to-noise ratio (SNR) after OSTBC decoding is also derived.

II. SQUARING METHOD TO DECODE OSTBC

We consider a wireless communication system with \( n \) transmit- and \( m \) receive-antennas as shown in Fig. 1. The coefficient \( h_{i,j}(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m) \) in Fig. 1 is the path gain from transmit antenna \( i \) to receive antenna \( j \). We assume that these path gains are constant during a frame and vary from one frame to another (quasi-static flat fading).

Let \((x_1, x_2, \ldots, x_K)\) be the information sequence to be transmitted. A column orthogonal matrix

\[
G = \begin{pmatrix}
    c_1^1 & \cdots & c_1^2 & \cdots & c_1^K \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    c_p^1 & \cdots & c_p^2 & \cdots & c_p^K
\end{pmatrix}
\]

is used. At each time slot \( t(t = 1, 2, \ldots, p) \), signals \( c_1^1, c_2^2, \ldots, c_p^K \) are transmitted simultaneously from the \( n \) transmit antennas. Each signal \( c_i^j \) is a linear combination of the variables \( x_{11}, x_{22}, \ldots, x_{KK}, x_{11}^2, x_{22}^2, \ldots, x_{KK}^2 \). Noting the column orthogonal characteristic of the matrix \( G \) [3], we have

\[
G^* G = \left( \sum_{k=1}^{K} |x_k|^2 \right) I
\]

where the superscript “*” denotes transpose conjugate of the matrix and \( I \) is the \( K \times K \) identity matrix. Let

\[
\vec{x} = (x_1, x_2, \ldots, x_K, x_1^2, x_2^2, \ldots, x_K^2)^T
\]
be the signal vector, and let
\[ \tilde{h}_j = (h_{1,j}, h_{2,j}, \ldots, h_{m,j})^T \]
and
\[ \tilde{n}_j = (n_{1,j}^1, n_{2,j}^2, \ldots, n_{p,j}^p)^T \]
denote the channel vector, complex white Gaussian noise vector and received vector of the \( j \)th receive antenna, respectively. The superscript "T" means transpose of the vector and \( r_i^1 \) is the received signal from the \( j \)th antenna at time slot \( t \). Assume \( n_{i,j}^p \) has mean zero and variance \( N_0/2 \) per dimension. \( \tilde{r}_j \) can be expressed as
\[ \tilde{r}_j = H^j \cdot \tilde{x} + \tilde{n}_j \]
(2)
where \( H^j \) is defined as the channel matrix of the \( j \)th receive antenna with the following expression:
\[ H^j = \begin{pmatrix}
H_{1,1}^j & H_{1,2}^j & \cdots & H_{1,2K}^j \\
H_{2,1}^j & H_{2,2}^j & \cdots & H_{2,2K}^j \\
\vdots & \vdots & \ddots & \vdots \\
H_{p,1}^j & H_{p,2}^j & \cdots & H_{p,2K}^j
\end{pmatrix} \]
(3)

Let
\[ Y_j = (y_1^j, y_2^j, \ldots, y_m^j)^T = H^j \cdot \tilde{x}, \]
\[ j = 1, 2, \ldots, m \]
(4)
denote the received vector of the \( j \)th receive antenna in the absence of noise. Assuming perfect channel state information is available, the receiver computes the decision metric [4]
\[ D = \sum_{i=1}^{n} \sum_{j=1}^{m} \left| r_i^j - \frac{n}{i} h_i, j t \right|^2 \]
over all code words
\[ c_1^1, c_1^2, \ldots, c_1^p, c_2^1, c_2^2, \ldots, c_2^p, \ldots, c_p^1, c_p^2, \ldots, c_p^p \]
and decides in favor of the code word that minimizes the sum \( D \).

Assuming that transmission at the baseband employs a signal constellation \( A \) with \( Q \) elements and noting the relationship between the matrix \( G \) and the code word, we must compute \( Q^K \) metrics for maximum-likelihood decoding using (5), while we need only compute \( K \cdot Q \) metrics using the method in [3], [4] without decoding performance loss. Only several examples of OSTBC are illustrated in [3] and [4]. Using the squaring method, we generalize the method in [3] and [4] and simplify the decoding of the OSTBC.

From (5), (2), and (1), the matrix form of (5) can be written as
\[ D = \sum_{j=1}^{m} \left[ R_j - G \cdot \tilde{h}_j \right]^* \left[ R_j - G \cdot \tilde{h}_j \right] \]
\[ = \sum_{j=1}^{m} R_j^* R_j - \sum_{j=1}^{m} R_j^* H^j \tilde{x} - \sum_{j=1}^{m} \tilde{x}^* H^j R_j \]
\[ + \left( \sum_{k=1}^{K} |x_k|^2 \right) \cdot \left( \sum_{i=1}^{n} |h_i, j|^2 \right) \]
(6)
Noting that all the coefficients of the cross terms with respect to \( x_k, x_{k'} (k \neq k') \) are zero in (6), we only consider the terms comprising of \( x_k \), \( x_k^* \) and \( |r_i|^2 \). Thus, the minimization of (6) is equivalent to minimizing all these \( K \) parts separately
\[ D_k = -\sum_{j=1}^{m} \left( r_i^1 H_{1,k}^j + r_i^2 H_{2,k}^j + \cdots + r_i^p H_{p,k}^j \right) x_k \]
\[ -\sum_{j=1}^{m} \left( r_i^1 H_{1,k+k}^j + r_i^2 H_{2,k+k}^j + \cdots + r_i^p H_{p,k+k}^j \right) x_k \]
\[ -\sum_{j=1}^{m} \left( r_i^1 H_{1,k+k}^j + r_i^2 H_{2,k+k}^j + \cdots + r_i^p H_{p,k+k}^j \right) x_k \]
\[ -\sum_{j=1}^{m} \left( r_i^1 H_{1,k+k}^j + r_i^2 H_{2,k+k}^j + \cdots + r_i^p H_{p,k+k}^j \right) x_k^* \]
\[ + \left( \sum_{j=1}^{m} \sum_{i=1}^{n} |h_i, j|^2 \right) \cdot |x_k|^2, \]
\[ k = 1, 2, \ldots, K. \]
(7)
Let
\[ B = \sum_{j=1}^{m} \left( r_i^1 H_{1,k}^j + r_i^2 H_{2,k}^j + \cdots + r_i^p H_{p,k}^j \right) \]
\[ + \sum_{j=1}^{m} \left( r_i^1 H_{1,k+k}^j + r_i^2 H_{2,k+k}^j + \cdots + r_i^p H_{p,k+k}^j \right) \]
(8)
\[ C = \sum_{j=1}^{m} \sum_{i=1}^{n} |h_i, j|^2 \]
(9)
where \( B \) is complex and \( C \) is positive. Substitution of (8) and (9) into (7) yields
\[ D_k = -B x_k - B^* x_k^* + C |x_k|^2. \]
(10)
Multiplying the right-hand side of (10) by the real positive value \( C \) and adding the positive real value \( |B|^2 \), the following decision metric is obtained:
\[ D_k = |B|^2 - C B x_k - C B^* x_k^* + C |x_k|^2 \]
\[ = |B - C x_k|^2, \]
\[ k = 1, 2, \ldots, K. \]
(11)
Hence, we have proved that \( B \) may be used as the combing scheme, i.e.,
\[ \tilde{x}_k = B. \]
(12)
The maximum-likelihood detection amounts to minimizing the following decision metrics:
\[ D_k = \left| \tilde{x}_k - \sum_{j=1}^{m} \sum_{i=1}^{n} |h_i, j|^2 x_k \right|^2, \]
\[ k = 1, 2, \ldots, K. \]
(13)

III. PERFORMANCE ESTIMATE

Summarizing the descriptions in Section II, one can obtain \( K \) combining schemes which are \( K \) independent output branches at the receiver using squaring method if perfect channel state information is available. It is difficult to estimate the decoding performance for the decoding method directly using (5). Since the decoding performance results of the methods using (5) and
In this section, we prove that all the output branches have the same SNR. Noting (12), (4), and (2), we have

\[ \tilde{x}_k = \sum_{j=1}^{m} \left( r^+_j H^j_{1,k} + r^+_j H^j_{2,k} + \cdots + r^+_j H^j_{p,k} \right) + \sum_{j=1}^{m} \left( y^+_j H^j_{1,k+k} + y^+_j H^j_{2,k+k} + \cdots + y^+_j H^j_{p,k+k} \right) \]

\[ = \sum_{j=1}^{m} \left( y^+_j H^j_{1,k} + y^+_j H^j_{2,k} + \cdots + y^+_j H^j_{p,k} \right) + \sum_{j=1}^{m} \left( y^+_j H^j_{1,k+k} + y^+_j H^j_{2,k+k} + \cdots + y^+_j H^j_{p,k+k} \right) \]

\[ + \sum_{j=1}^{m} \left( n^+_j H^j_{1,k} + n^+_j H^j_{2,k} + \cdots + n^+_j H^j_{p,k} \right) + \sum_{j=1}^{m} \left( n^+_j H^j_{1,k+k} + n^+_j H^j_{2,k+k} + \cdots + n^+_j H^j_{p,k+k} \right). \]

(14)

In the absence of noise, the decision metric is zero when the correct constellation point is selected at the receiver. As a result, the signal term of (14) should be

\[ y_k = \sum_{j=1}^{m} \left( y^+_j H^j_{1,k} + y^+_j H^j_{2,k} + \cdots + y^+_j H^j_{p,k} \right) + \sum_{j=1}^{m} \left( y^+_j H^j_{1,k+k} + y^+_j H^j_{2,k+k} + \cdots + y^+_j H^j_{p,k+k} \right) \]

\[ = \left( \sum_{j=1}^{m} \sum_{i=1}^{n} |h_{i,j}|^2 \right) x_k. \]

(15)

Assume \( E_s = E[x_k^2] = \cdots = E[x_K^2] \), where \( E[\cdot] \) denotes expectation. We express the average signal power of the \( k \)-th output branch at the receiver as

\[ E_R = E[y_k^2] = \left( \sum_{j=1}^{m} \sum_{i=1}^{n} |h_{i,j}|^2 \right)^2 E_s. \]

(16)

Considering the influence of noise, we write the noise term in (14) separately

\[ \tau_k = \sum_{j=1}^{m} \left( r^+_j H^j_{1,k} + r^+_j H^j_{2,k} + \cdots + r^+_j H^j_{p,k} \right) + \sum_{j=1}^{m} \left( r^+_j H^j_{1,k+k} + r^+_j H^j_{2,k+k} + \cdots + r^+_j H^j_{p,k+k} \right) \]

\[ + \sum_{j=1}^{m} \left( n^+_j H^j_{1,k} + n^+_j H^j_{2,k} + \cdots + n^+_j H^j_{p,k} \right) + \sum_{j=1}^{m} \left( n^+_j H^j_{1,k+k} + n^+_j H^j_{2,k+k} + \cdots + n^+_j H^j_{p,k+k} \right). \]

(17)

For an arbitrary given information sequence \((x_1, x_2, \ldots, x_K)\), we obtain from (4) that

\[ Y_j Y_j^* = x^* H^j x = \bar{Y}_j \bar{G}^* \bar{G} \bar{h}_j \]

\[ = \left( \sum_{j=1}^{n} |h_{i,j}|^2 \right) \left( \sum_{k=1}^{K} |x_k|^2 \right). \]

(18)

Expanding \( \bar{Y}_j \bar{Y}_j^* \) and comparing it with (18) yields the coefficient of \( |x_k|^2 \)

\[ \sum_{i=1}^{n} \left| H_{i,k} \right|^2 + \left| H_{i,k+k} \right|^2 \right) \sum_{i=1}^{n} |h_{i,j}|^2. \]

(19)

From (17) and (19), the output noise power of the branch \( k \) can be written as

\[ P_N = E[|y_k|^2] = E\left[ |y_{k,k}|^2 \right] \]

\[ = \sum_{j=1}^{m} \sum_{i=1}^{n} \left| H_{i,k} \right|^2 + \left| H_{i,k+k} \right|^2 N_0 \]

\[ = \sum_{j=1}^{m} \sum_{i=1}^{n} |h_{i,j}|^2 N_0. \]

(20)

From (16) and (20), all the output branches give the same SNR

\[ SNR_{\text{avg}} = E_R/P_N = \sum_{j=1}^{m} \sum_{i=1}^{n} |h_{i,j}|^2 E_s/N_0. \]

(21)

Assume the total energy of a block is limited to \( E_{\text{tot}} = K \cdot E_0 \) (\( E_0 \) is the transmitting energy for each source symbol). Using orthogonal matrix \( G \) to transmit the sequence \((x_1, x_2, \ldots, x_K)\), the total energy can also expressed as

\[ E_{\text{tot}} = \sum_{i=1}^{K} \sum_{k=1}^{n} |x_k|^2 = \sum_{i=1}^{K} \left( \sum_{k=1}^{n} |x_k|^2 \right) \]

\[ = \sum_{i=1}^{K} |x_i|^2 = n \cdot K \cdot E_s. \]

(22)

Thus,

\[ E_s = E_0/n. \]

(23)

Substitution of (23) into (16) yields

\[ E_R = \left( \sum_{j=1}^{m} \sum_{i=1}^{n} |h_{i,j}|^2 \right)^2 \frac{E_0}{n}. \]

(24)

Assume the constellation at the receiver satisfies

\[ E_R = \mu \cdot d_R^2 \]

(25)

where \( d_R \) is the minimum distance of the constellation, and \( \mu \) is a constant depending on different constellations. Using minimum distance sphere bound, the instant symbol error rate bound is given by

\[ P_{e,\text{inst}}(l_{i,j}) \leq \exp \left( -\frac{d_R^2}{4P_N} \right) = \exp \left( -\frac{E_R}{4\mu P_N} \right) \]

\[ = \exp \left( \frac{n \sum_{i=1}^{m} |h_{i,j}|^2}{4 \mu \cdot n \cdot N_0} \right). \]

(26)
We assume \( h_{i,j} \) are independent samples of zero-mean complex Gaussian random variables having variance 0.5 per dimension. Thus \( |h_{i,j}| \) are independent Rayleigh distributions with pdf

\[
p(h_{i,j}) = 2|h_{i,j}| \exp(-|h_{i,j}|^2).
\]

(27)

Thus, the average symbol error rate bound is given by

\[
P_e \leq \left( \frac{1}{1 + \frac{E_0}{4\mu N_0 n}} \right)^{nm}.
\]

(28)

From (28), one can conclude that the symbol error rate decreases while the number of transmit- or receiver-antennas increases. When \( m \) is given, we have

\[
P_e \leq \exp\left(-\frac{E_0}{4\mu N_0 m}\right) \quad (n \to \infty).
\]

(29)

This means that the increasing number of transmit antennas does not give an infinite advantage.

IV. CONCLUSION

We propose a general linear maximum-likelihood OSTBC scheme. The resulting decoding scheme can be applied to generalized complex orthogonal designs with arbitrary number of transmit- and receive-antennas, and possesses a very low computational complexity. In addition, a closed-form expression of SNR after ST decoding is also derived.

ACKNOWLEDGMENT

The authors thank the three anonymous reviewers whose comments have greatly enhanced the quality of this letter and Dr. J. Zhao and Dr. X. Tong for helpful discussions.

REFERENCES


