A New Command Governor Architecture for Transient Response Shaping

by

Tansel Yucelen
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
(770) 331-8496
FAX: (404) 894-2760
tansel@gatech.edu

Eric Johnson
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0150
(404) 385-2519
FAX: (404) 894-2760
eric.johnson@ae.gatech.edu

Abstract

In this paper, we develop a control framework for stabilization and command following of nonlinear uncertain dynamical systems. The proposed methodology consists of a new command governor architecture and an adaptive controller. The command governor is a dynamical system which adjusts the trajectory of a given command to follow an ideal reference system capturing a desired closed-loop dynamical system behavior in transient time. Specifically, we show that the controlled nonlinear uncertain dynamical system can approach the ideal reference system by choosing the design parameter of the command governor. In addition, an adaptive element is used to asymptotically assure that the error between the controlled nonlinear uncertain dynamical system and the ideal reference system is reduced in long term. Therefore, the proposed methodology not only has closed-loop transient and steady state performance guarantees but can also shape the transient response by adjusting the trajectory of the given command with the command governor. We highlight that there exists a trade-off between the adaptive controller’s learning rate and the command governor’s design parameter. This key feature of our framework allows rapid suppression of system uncertainties without resorting to a high learning rate in the adaptive controller. Furthermore, we discuss the robustness properties of the proposed approach with respect to high-frequency dynamical system content such as measurement noise and/or unmodeled dynamics. A numerical example is provided to demonstrate the efficacy of the proposed architecture.

Key Words: Nonlinear uncertain dynamical systems; stabilization and command following; command governor; adaptive control; transient and steady state performance guarantees; rapid suppression of system uncertainties

Running Title: Command Governor Architecture
1. Introduction

Mathematical models are critical in capturing and studying the physical phenomena that undergo spatial and temporal evolution arising in most science and engineering applications. These models are usually derived using the first-principles of physics and fundamental physical laws. However, due to system complexity, time-variations, nonlinearities, disturbances, and measurement noise, models are based on simplifying approximations, resulting in system uncertainties.

Fixed-gain robust control methods can be used for dynamical systems when mathematical models do not adequately capture the physical phenomena because of these system uncertainties [1–3]. However, such control methods require the knowledge of characterized bounds resulting from system uncertainty parameterization, which may not be trivial to determine from a practical standpoint. Furthermore, in the face of high uncertainty levels, these methods may fail to satisfy a given system performance requirement. On the other hand, adaptive control methods are able to deal with high uncertainty levels and require less modeling information than do robust control methods. These facts make adaptive control theory a candidate for many science and engineering applications.

Adaptive control approaches can be classified as either direct or indirect [4–6]. Direct adaptive controllers adapt feedback gains in response to system variations without requiring a parameter estimation algorithm. This property distinguishes them from indirect adaptive controllers that employ an estimation algorithm to approximate unknown system parameters and adapt controller gains. The control framework of this paper builds on a well-known class of direct adaptive controllers, specifically, model reference adaptive controllers.

Whitaker et al. [7,8] originally proposed the model reference adaptive control concept. In particular, model reference adaptive control schemes have three major components, namely, an ideal reference system (model), an update law, and a controller. The ideal reference system captures a desired closed-loop dynamical system behavior for which its output (resp. state) is compared with the output (resp. state) of the uncertain dynamical system. This comparison results in an error signal used to drive the update law online. Then, the controller
adapts feedback gains to minimize this error signal using the information received from the update law. It is of practical importance to note that the output (resp. state) of the uncertain dynamical system can be far different from the output (resp. state) of the ideal reference system in transient time, even though this scheme guarantees that the distance between the uncertain dynamical system and the ideal reference system vanishes asymptotically (in long term, i.e., steady state). Therefore, a high learning rate can be used in the update law to yield fast adaptation to rapidly suppress the system uncertainties in transient time.

While numerous applications have used adaptive control, the necessity of high-gain feedback for achieving fast adaptation can be a serious limitation of adaptive controllers [9–11]. Specifically, in certain applications fast adaptation is required to achieve stringent stabilization or command following performance specifications in the face of large system uncertainties and abrupt changes in system dynamics. In such situations, high-gain adaptive control is necessary for rapidly reducing the mismatch between the uncertain dynamical system and the ideal reference system. However, update laws with high learning rates are not robust against high-frequency dynamical system content. That is, update laws with high learning rates possibly yield to control signals with high levels of measurement noise content and can excite unmodeled dynamics, resulting in system instability for practical applications [12]. Hence, a critical trade-off between system stability and control adaptation rate exists in most adaptive control approaches, with some notable exceptions [13–15].

The authors in [13] present a high-gain adaptive control approach for fast adaptation predicated on an optimal control problem. The authors in [14] use a low-pass filter that effectively subverts high-frequency oscillations attributable to fast adaptation, and their approach has guaranteed transient and steady state performance. More recently, the authors in [15] present a high-gain adaptive control approach that allows fast adaptation without hindering system robustness. Even though the high-gain adaptive control methodologies documented in [14] and [15] are promising, they require the knowledge of a conservative upper bound on the unknown constant gain appearing in their uncertainty parameterization. While this conservative upper bound may be available for some applications, the actual upper
bound may change and exceed its conservative estimate, for example, when an aircraft undergoes a sudden change in dynamics as a result of reconfiguration, deployment of a payload, docking, or structural damage [16]. In such circumstances, the performance of these adaptive controllers may be poor, because tuning them online with a new conservative upper bound is not possible. Furthermore, the performance of these adaptive controllers in the face of high uncertainty levels may not be satisfactory as well, since both controllers converge to a standard adaptive controller as the upper bound on the unknown constant gain becomes arbitrarily large (see, for example, Section 2.1.2 of [14] and Section 4 of [15]).

This paper presents a control framework for stabilization and command following of nonlinear uncertain dynamical systems. Our approach differs from the previous adaptive control approaches as our objective is to achieve rapid suppression of system uncertainties without using a high-gain adaptive controller but by altering a given command using a new command governor architecture. Specifically, the command governor is a dynamical system that adjusts the trajectory of a given command to follow an ideal reference system in transient time (e.g., learning phase). We show that the controlled nonlinear uncertain dynamical system can approach the ideal reference system by choosing the design parameter of the command governor. In addition, an adaptive element is used to asymptotically assure that the error between the controlled nonlinear uncertain dynamical system and the ideal reference system is reduced in long term. Therefore, the proposed methodology not only has closed-loop transient and steady state performance guarantees but can also shape the transient response by adjusting the trajectory of the given command with the command governor. We highlight that there exists a trade-off between the adaptive controller’s learning rate and the command governor’s design parameter. This key feature of our framework allows rapid suppression of system uncertainties without resorting to a high learning rate in the adaptive controller. Furthermore, we discuss the robustness properties of the proposed approach with respect to high-frequency dynamical system content such as measurement noise and/or unmodeled dynamics. Although this paper presents the command governor architecture in the context of a particular adaptive control formulation, this architecture can
be used in a complimentary way with many other approaches to adaptive control.

The organization of the paper is as follows. Section 2 presents the notation used throughout the paper and Section 3 provides the problem formulation. The command governor architecture is introduced in Section 4. The stability of the closed-loop dynamical system highlighting the transient and steady state performance guarantees is analyzed in Section 5. Section 6 shows that the command governor shapes the transient response of the controlled nonlinear dynamical system. Section 7 discusses the robustness issues of our framework with respect to high-frequency dynamical system content. Section 8 presents a numerical example to demonstrate the efficacy of the proposed architecture. Finally, conclusions are summarized in Section 9.

2. Notation

The notation used in this paper is fairly standard. Specifically, \( \mathbb{R} \) denotes the set of real numbers, \( \mathbb{R}^n \) denotes the set of \( n \times 1 \) real column vectors, \( \mathbb{R}^{n \times m} \) denotes the set of \( n \times m \) real matrices, \( \mathbb{R}_+ \) (resp., \( \mathbb{R}_+^+ \)) denotes the set of positive (resp., nonnegative-definite) real numbers, \( \mathbb{R}_+^{n \times n} \) (resp., \( \mathbb{R}_+^{n \times n}^+ \)) denotes the set of \( n \times n \) positive-definite (resp., nonnegative-definite) real matrices, \( \mathbb{S}^{n \times n} \) denotes the set of \( n \times n \) symmetric real matrices, \( \mathbb{D}^{n \times n} \) denotes the \( n \times n \) real matrices with diagonal scalar entries, \( (\cdot)^T \) denotes transpose, \( (\cdot)^{-1} \) denotes inverse, \( (\cdot)^+ \) denotes the Moore-Penrose generalized inverse, and “\( \triangleq \)” denotes equality by definition. In addition, we write \( \lambda_{\min}(A) \) (resp., \( \lambda_{\max}(A) \)) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix \( A \), \( \sigma_{\min}(A) \) (resp., \( \sigma_{\max}(A) \)) for the minimum (resp., maximum) singular value of the Hermitian matrix \( A \), \( \det(A) \) for the determinant of the Hermitian matrix \( A \), \( \text{tr}(\cdot) \) for the trace operator, \( \text{vec}(\cdot) \) for the column stacking operator, \( A^L \) for the left inverse \( (A^T A)^+ A^T \) of \( A \in \mathbb{R}^{n \times m} \), \( P_A \) for the projection matrix \( A A^L \) of \( A \in \mathbb{R}^{n \times m} \), \( \| \cdot \|_2 \) for the Euclidian norm, \( \| \cdot \|_\infty \) for the infinity norm, and \( \| \cdot \|_F \) for the

\[ ^1 \text{In the context of handling state and control constraints of dynamical systems, several command governor approaches are studied in the literature (see, for example, [17–19] and references therein). Even though the command governor architecture of this paper alters a given command similar to those approaches, our objective is to address the poor transient performance phenomenon of adaptive controllers as applied to nonlinear uncertain dynamical systems, and hence, the proposed architecture significantly differs from the existing command governor approaches.} \]
Frobenius matrix norm. Furthermore, for a signal \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) defined for all \( t \geq 0 \), the truncated \( L_\infty \) norm and the \( L_\infty \) norm [20] are defined as \( \|x_r(t)\|_{L_\infty} \triangleq \max_{1 \leq i \leq n}(\sup_{0 \leq t \leq r}|x_i(t)|) \) and \( \|x(t)\|_{L_\infty} \triangleq \max_{1 \leq i \leq n}(\sup_{t \geq 0}|x_i(t)|) \), respectively.

3. Problem Formulation

Consider the nonlinear uncertain dynamical system given by

\[
\dot{x}(t) = Ax(t) + Bu(t) + D\delta(x(t)), \quad x(0) = x_0, \quad t \in \mathbb{R}_+,
\]

where \( x(t) \in \mathbb{R}^n \) is the state vector available for feedback, \( u(t) \in \mathbb{R}^m \) is the control input, \( \delta : \mathbb{R}^n \to \mathbb{R}^m \) is an uncertainty, \( A \in \mathbb{R}^{n \times n} \) is a known system matrix, \( B \in \mathbb{R}^{n \times m} \) is an unknown control input matrix, \( D \in \mathbb{R}^{n \times m} \) is a known uncertainty input matrix, and the pair \((A, B)\) is controllable.

**Assumption 3.1.** The uncertainty in (1) is parameterized as

\[
\delta(x) = W^T\sigma(x), \quad x \in \mathbb{R}^n,
\]

where \( W \in \mathbb{R}^{s \times m} \) is an unknown weight matrix and \( \sigma : \mathbb{R}^n \to \mathbb{R}^s \) is a known basis function of the form \( \sigma(x) = [\sigma_1(x), \sigma_2(x), \ldots, \sigma_s(x)]^T \).

**Assumption 3.2.** The unknown control input matrix is parameterized as

\[
B = DA, \quad (3)
\]

where \( \det(D^TD) \neq 0 \) and \( \Lambda \in \mathbb{R}_+^{m \times m} \cap \mathbb{D}^{m \times m} \) is an unknown control effectiveness matrix.

Next, consider the ideal reference system\(^2\) (model) capturing a desired closed-loop dynamical system performance given by

\[
\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(0) = x_0, \quad t \in \mathbb{R}_+,
\]

where \( x_r(t) \in \mathbb{R}^n \) is the reference state vector, \( c(t) \in \mathbb{R}^m \) is a given uniformly continuous bounded command\(^3\), \( A_r \in \mathbb{R}^{n \times n} \) is the Hurwitz reference system matrix, and \( B_r \in \mathbb{R}^{n \times m} \) is the command input matrix.

---

\(^2\)For the ease of exposition, we consider that both (1) and (4) have the same initial conditions. Since the state vector of (1) is available for feedback and our results can be readily applied to the other case, this is not restrictive.

\(^3\)For the stabilization problem, \( c(t) \equiv 0 \).
Our first objective is to construct a feedback control law $u(t)$ such that the state vector $x(t)$ asymptotically follows the reference state vector $x_r(t)$ subject to Assumptions 3.1 and 3.2. For this purpose, consider

$$u(t) = u_n(t) + u_a(t), \quad (5)$$

where $u_n(t) \in \mathbb{R}^m$ is the nominal feedback control law and $u_a(t) \in \mathbb{R}^m$ is the adaptive feedback control law. Furthermore, let the nominal feedback control law be given by

$$u_n(t) = K_1x(t) + K_2c(t), \quad (6)$$

where $K_1 \in \mathbb{R}^{m \times n}$ and $K_2 \in \mathbb{R}^{m \times m}$ are the nominal feedback gain and the nominal feedforward gain, respectively, such that $A_r = A + DK_1$, $B_r = DK_2$, and $\det(K_2) \neq 0$ hold. Using (5) and (6) in (1) along with Assumptions 3.1 and 3.2 yields

$$\dot{x}(t) = A_r x(t) + B_r c(t) + DA \left[ u_a(t) + W_\sigma^T \sigma(x(t)) + W_{u_n}^T u_n(t) \right], \quad (7)$$

where $W_\sigma \triangleq W \Lambda^{-1} \in \mathbb{R}^{s \times m}$ and $W_{u_n} \triangleq [I - \Lambda^{-1}] \in \mathbb{D}^{m \times m}$.

Next, let the adaptive feedback control law be given by

$$u_a(t) = -\hat{W}_\sigma^T(t) \sigma(x(t)) - \hat{W}_{u_n}^T(t) u_n(t), \quad (8)$$

where $\hat{W}_\sigma(t) \in \mathbb{R}^{s \times m}$ and $\hat{W}_{u_n}(t) \in \mathbb{R}^{m \times m}$ are the estimates of $W_\sigma$ and $W_{u_n}$, respectively, satisfying the update laws

$$\dot{\hat{W}}_\sigma(t) = \Gamma_\sigma \sigma(x(t)) e^T(t) PD, \quad \dot{\hat{W}}_{u_n}(0) = 0, \quad t \in \mathbb{R}_+, \quad (9)$$

$$\dot{\hat{W}}_\sigma(t) = \Gamma_u u_n(t) e^T(t) PD, \quad \dot{\hat{W}}_{u_n}(0) = 0, \quad t \in \mathbb{R}_+, \quad (10)$$

where $\Gamma_\sigma \in \mathbb{R}_+^{s \times s} \cap \mathbb{S}^{s \times s}$ and $\Gamma_u \in \mathbb{R}_+^{m \times m} \cap \mathbb{S}^{m \times m}$ are the learning rates,

$$e(t) \triangleq x(t) - x_r(t), \quad (11)$$

is the system error, and $P \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ is a solution of the Lyapunov equation

$$0 = A_r^T P + PA_r + R, \quad (12)$$

$^4$Since $A_r$ is Hurwitz, it follows from converse Lyapunov theory [21] that there exists a unique $P$ satisfying (12) for a given $R$.\[6\]
where $R \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ can be viewed as an additional learning rate.

Now, using (8) in (7) yields

$$
\dot{x}(t) = A_x x(t) + B_x c(t) - D \Lambda \left[ \hat{W}^T(t) \sigma(x(t)) + \hat{W}^T_{un}(t) u_n(t) \right],
$$

and the system error dynamics is given by using (4) and (13) as

$$
\dot{e}(t) = A_e e(t) - D \Lambda \left[ \hat{W}^T_{s}(t) \sigma(x(t)) + \hat{W}^T_{un}(t) u_n(t) \right], \quad e(0) = 0, \quad t \in \mathbb{R}_+,
$$

where $\hat{W}_s(t) \triangleq \hat{W}_s(t) - W_s \in \mathbb{R}^{s \times m}$ and $\hat{W}_{un}(t) \triangleq \hat{W}_{un}(t) - W_{un} \in \mathbb{R}^{m \times m}$.

The update laws given by (9) and (10) can be derived using Lyapunov analysis by considering the Lyapunov function candidate $V(e, \hat{W}_s, \hat{W}_{un}) = e^T P e + \text{tr} \left( \hat{W}_s \Lambda \hat{W}_s^T \right) + \text{tr} \left( \hat{W}_{un} \Lambda \hat{W}_{un}^T \right)$. Note that $V(0, 0, 0) = 0$ and $V(e, \hat{W}_s, \hat{W}_{un}) > 0$ for all $(e, \hat{W}_s, \hat{W}_{un}) \neq (0, 0, 0)$. In addition, $V(e, \hat{W}_s, \hat{W}_{un})$ is radially unbounded. Now, differentiating $V(e, \hat{W}_s, \hat{W}_{un})$ yields

$$
\dot{V}(e(t), \hat{W}_s(t), \hat{W}_{un}(t)) = -e^T(t) Re(t) - 2e^T(t) P \Lambda \hat{W}^T_s(t) \sigma(x(t)) - 2e^T(t) P \Lambda \hat{W}^T_{un}(t) u_n(t) + 2 \text{tr} \left( \hat{W}^T_s(t) \Gamma^{-1}_s \hat{W}_s(t) \Lambda \right) + 2 \text{tr} \left( \hat{W}^T_{un}(t) \Gamma^{-1}_{un} \hat{W}_{un}(t) \Lambda \right),
$$

where using (9) and (10) results in

$$
\dot{V}(e(t), \hat{W}_s(t), \hat{W}_{un}(t)) = -e^T(t) Re(t) \leq 0,
$$

which guarantees that the system error $e(t)$ and the weight errors $\hat{W}_s(t)$ and $\hat{W}_{un}(t)$ are Lyapunov stable, and hence, are bounded for all $t \in \mathbb{R}_+$. Since $\sigma(x(t))$ and $u_n(t)$ are bounded for all $t \in \mathbb{R}_+$, it follows from (14) that $\dot{e}(t)$ is bounded, and hence, $\dot{V}(e(t), \hat{W}_s(t), \hat{W}_{un}(t))$ is bounded for all $t \in \mathbb{R}_+$.

Now, it follows from Barbalat’s lemma [20] that $\lim_{t \to \infty} \dot{V}(e(t), \hat{W}_s(t), \hat{W}_{un}(t)) = 0$, which consequently shows that $e(t) \to 0$ as $t \to \infty$.

Note that $x(t)$ can be far different from $x_r(t)$ in transient time, even though the above analysis shows that the state vector $x(t)$ converges to the reference state vector $x_r(t)$ asymptotically (in steady state). As discussed, since it is not desirable to use high learning rates in (9) and (10) for achieving fast adaptation in transient time due to practical limitations, our second objective is to achieve rapid suppression of system uncertainties by altering the given command using a command governor architecture.

**Remark 3.1.** For the case when the nonlinear uncertain dynamical system given by (1) includes bounded exogenous disturbances, measurement noise, and/or the uncertainty in (1)
cannot be perfectly parameterized, then Assumption 3.1 can be relaxed by considering
\[
\delta(t, x) = W(t)^T \sigma(x) + \varepsilon(t, x), \quad x \in \mathcal{D}_x,
\]
where \( W(t) \in \mathbb{R}^{s \times m} \) is an unknown time-varying weight matrix satisfying \( \| W(t) \|_F \leq w, w \in \mathbb{R}_+ \), and \( \| \dot{W}(t) \|_F \leq \dot{w}, \dot{w} \in \mathbb{R}_+ \), \( \sigma : \mathcal{D}_x \to \mathbb{R}^s \) is a known basis function of the form \( \sigma(x) = [1, \sigma_1(x), \sigma_2(x), \ldots, \sigma_{s-1}(x)]^T \), \( \varepsilon : \mathbb{R}_+ \times \mathcal{D}_x \to \mathbb{R}^m \) is the system modeling error satisfying \( \| \varepsilon(t, x) \|_2 \leq \epsilon, \epsilon \in \mathbb{R}_+ \), and \( \mathcal{D}_x \) is a compact subset of \( \mathbb{R}^n \). In this case, the update laws given by (9) and (10) can be replaced by
\[
\dot{\hat{W}}_\sigma(t) = \Gamma_\sigma \text{Proj}[\hat{W}_\sigma(t), \sigma(x(t))e^T(t)PD], \quad \hat{W}_\sigma(0) = \hat{W}_{\sigma 0}, \quad t \in \mathbb{R}_+,
\]
\[
\dot{\hat{W}}_{u_n}(t) = \Gamma_{u_n} \text{Proj}[\hat{W}_{u_n}(t), u_n(t)e^T(t)PD], \quad \hat{W}_{u_n}(0) = \hat{W}_{u_n 0}, \quad t \in \mathbb{R}_+,
\]
with \( \Gamma_\sigma = \gamma_\sigma I_s, \gamma_\sigma \in \mathbb{R}_+ \), and \( \Gamma_{u_n} = \gamma_{u_n} I_m, \gamma_{u_n} \in \mathbb{R}_+ \), to guarantee the uniform boundedness\(^5\) of the system error \( e(t) \) and the weight errors \( \hat{W}_\sigma(t) \) and \( \hat{W}_{u_n}(t) \), where Proj denotes the projection operator [22].

4. **Command Governor Architecture**

This section introduces a new command governor architecture for the adaptive control problem described in Section 3. Specifically, let the command \( c(t) \) used in (4) and (6) be given by
\[
c(t) = c_d(t) + Gg(t),
\]
where \( c_d(t) \in \mathbb{R}^m \) is the given uniformly continuous bounded command\(^6\) and \( Gg(t) \in \mathbb{R}^m \) is the command governor signal with \( G \in \mathbb{R}^{m \times n} \) being the matrix defined by
\[
G \triangleq K_2^{-1} D^T = K_2^{-1}(D^T D)^{-1} D^T,
\]
and \( g(t) \in \mathbb{R}^n \) being the command governor output generated by
\[
\dot{f}(t) = -\lambda f(t) + \lambda e(t), \quad f(0) = 0, \quad t \in \mathbb{R}_+,
\]
\[
g(t) = \lambda f(t) + (A_t - \lambda I_n) e(t),
\]
\(^5\)Even though this case implies a change in our first objective, our second objective remains the same.
\(^6\)For the stabilization problem, \( c_d(t) \equiv 0 \).
where $f(t) \in \mathbb{R}^n$ is the command governor state vector and $\lambda \in \mathbb{R}_+$ is the command governor gain.

A block diagram showing the command governor based adaptive control framework is given in Figure 4.1.

**Remark 4.1.** In Section 3, $c(t)$ was the given bounded external command used for command following purposes. In this section and the rest of the paper, however, $c_d(t)$ denotes this given bounded external command.

**Remark 4.2.** The addition of the command governor signal $Gg(t)$ to the given command $c_d(t)$ in (18) does not change the system error dynamics given by (14), because $c(t)$ is used in both (4) and (6). Therefore, the update laws (9) and (10) for $\hat{W}_\sigma(t)$ and $\hat{W}_{u_n}(t)$ remain
the same, respectively. In this case, however, (4) and (13) have the form
\[\dot{x}_r(t) = A_r x_r(t) + B_r c_d(t) + P_D g(t),\] (22)
\[\dot{x}(t) = A_r x(t) + B_r c_d(t) + P_D g(t) - D\Lambda [\tilde{W}_\sigma^T(t)\sigma(x(t)) + \tilde{W}_{u_n}^T(t)u_n(t)],\] (23)
respectively, where \(P_D = DD^L = D(D^TD)^{-1}D^T\). Even though this implies the modification of the ideal reference system with the signal \(P_D g(t)\), as we see later, it is possible to suppress the effect of \(D\Lambda [\tilde{W}_\sigma^T(t)\sigma(x(t)) + \tilde{W}_{u_n}^T(t)u_n(t)]\) in (23) through \(P_D g(t)\) by properly choosing the command governor gain \(\lambda\). This key feature will allow (23) to approximate the ideal reference system (i.e., (4) with \(c(t)\) is replaced by \(c_d(t)\)) in transient time.

Remark 4.3. The analysis presented in Section 3 relies on the boundedness of the command \(c(t)\). In this section, however, we cannot \textit{a priori} assume the boundedness of the command \(c(t)\) given by (18), since it includes the command governor signal \(Gg(t)\). Therefore, the analysis presented in Section 3 needs to be extended, due to the fact that the command governor architecture (20) and (21) is introduced to the adaptive control formulation.

5. Stability Analysis

This section analyzes the stability of the command governor based adaptive control framework. For this purpose, the system error dynamics and the weight error dynamics can be given by, respectively, (14),
\[\dot{\tilde{W}}_{\sigma}(t) = \Gamma_\sigma \sigma(x(t))e^T(t)PD, \quad \tilde{W}_{\sigma}(0) = \tilde{W}_{\sigma_0}, \quad t \in \mathbb{R}_+;\] (24)
\[\dot{\tilde{W}}_{u_n}(t) = \Gamma_{u_n} u_n(t)e^T(t)PD, \quad \tilde{W}_{u_n}(0) = \tilde{W}_{u_n_0}, \quad t \in \mathbb{R}_+;\] (25)
where \(\tilde{W}_{\sigma_0} \triangleq \tilde{W}_{\sigma} - W_{\sigma}\) and \(\tilde{W}_{u_n_0} \triangleq \tilde{W}_{u_n} - W_{u_n}\). For the following theorem, we assume that the choice of \(R\) in (12) satisfies \(R = R_0 + \gamma \lambda I_n\), where \(R_0 \in \mathbb{R}^{n \times n}_+ \cap S^{n \times n}_+\) and \(\gamma \in \mathbb{R}_+\) is an arbitrary constant that can be chosen to be sufficiently small. Therefore, this assumption is technical and does not place restrictions on the selection of \(R\).

\footnote{The stability analysis of this section highlights the transient and steady state performance guarantees of the proposed framework. The next section will show that the command governor shapes the transient response of the controlled nonlinear dynamical system (1).}
Theorem 5.1. Consider the nonlinear uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the reference system given by (4) with the command given by (18), the feedback control law given by (5) along with (6), (8), (9), and (10), and the command governor given by (20) and (21). Then, the solution \( (e(t), \tilde{W}_\sigma(t), \tilde{W}_{u_n}(t), f(t)) \) of the closed-loop dynamical system given by (14), (20), (24), and (25) is Lyapunov stable for all \( (0, \tilde{W}_{\sigma_0}, \tilde{W}_{u_n_0}, 0) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^{m \times m} \times \mathbb{R}^n \) and \( t \in \mathbb{R}_+ \), and \( \lim_{t \to \infty} e(t) = 0 \), \( \lim_{t \to \infty} f(t) = 0 \), \( \lim_{t \to \infty} g(t) = 0 \), and \( \lim_{t \to \infty} (c(t) - c_d(t)) = 0 \). For \( t \in \mathbb{R}_+ \), in addition, the system error dynamics, the weight error dynamics, and the command governor dynamics satisfy the transient performance bounds given by, respectively,

\[
\|e(t)\|_{\infty} \leq \frac{\sqrt{\epsilon_\gamma}}{\lambda_{\min}(P)},
\]

\[
\|\text{vec}(\tilde{W}_\sigma(t))\|_{\infty} \leq \frac{\epsilon_\gamma}{(\lambda_{\min}(\Lambda)\lambda_{\min}(\Gamma_{\sigma}^{-1}))},
\]

\[
\|\text{vec}(\tilde{W}_{u_n}(t))\|_{\infty} \leq \frac{\epsilon_\gamma}{(\lambda_{\min}(\Lambda)\lambda_{\min}(\Gamma_{u_n}^{-1}))},
\]

\[
\|f(t)\|_{\infty} \leq \frac{\sqrt{\epsilon_\gamma}}{\gamma},
\]

where

\[
\epsilon_\gamma \triangleq \|\Gamma_{\sigma}^{-1}\|_F\|\tilde{W}_{\sigma_0}\Lambda\tilde{W}^T_{\sigma_0}\|_F + \|\Gamma_{u_n}^{-1}\|_F\|\tilde{W}_{u_n_0}\Lambda\tilde{W}^T_{u_n_0}\|_F.
\]

Proof. To show Lyapunov stability of the closed-loop dynamical system (14), (20), (24), and (25), consider the Lyapunov function candidate

\[
\mathcal{V}(e, \tilde{W}_\sigma, \tilde{W}_{u_n}, f) = e^T Pe + \text{tr} (\tilde{W}_\sigma \Lambda^\frac{1}{2})^T \Gamma_{\sigma}^{-1} (\tilde{W}_\sigma \Lambda^\frac{1}{2}) + \text{tr} (\tilde{W}_{u_n} \Lambda^\frac{1}{2})^T \Gamma_{u_n}^{-1} (\tilde{W}_{u_n} \Lambda^\frac{1}{2}) + \gamma f^T f,
\]

and note that \( \mathcal{V}(0, 0, 0, 0) = 0 \). Since \( P \in \mathbb{R}^{n \times n}_+ \cap S^{n \times n}_+, \Gamma_\sigma \in \mathbb{R}^{s \times s}_+ \cap S^{s \times s}_+, \Gamma_{u_n} \in \mathbb{R}^{m \times m}_+ \cap S^{m \times m}_+, \Lambda \in \mathbb{R}^{m \times m}_+ \cap I^{m \times m}, \) and \( \gamma \in \mathbb{R}_+ \), \( \mathcal{V}(e, \tilde{W}_\sigma, \tilde{W}_{u_n}, f) > 0 \) for all \( (e, \tilde{W}_\sigma, \tilde{W}_{u_n}, f) \neq (0, 0, 0, 0) \). In addition, \( \mathcal{V}(e, \tilde{W}_\sigma, \tilde{W}_{u_n}, f) \) is radially unbounded. Differentiating (31) along the closed-loop trajectories of (14), (20), (24), and (25) yields

\[
\dot{\mathcal{V}}(e(t), \tilde{W}_\sigma(t), \tilde{W}_{u_n}(t), f(t)) = -e^T(t)Re(t) - 2\gamma \lambda f^T(t)f(t) + 2\gamma \lambda f^T(t)e(t).
\]
Since $R = R_0 + \gamma \lambda I_n$ holds, then (32) can be rewritten as

$$
\dot{V}(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)) = -e^T(t) R_0 e(t) - \gamma \lambda f^T(t) f(t) - \gamma \lambda (e^T(t) e(t)
- 2 f^T(t) e(t) + f^T(t) f(t)),
$$

which yields

$$
\dot{V}(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)) = -e^T(t) R_0 e(t) - \gamma \lambda f^T(t) f(t) - \gamma \lambda \|e(t) - f(t)\|_2^2
\leq -e^T(t) R_0 e(t) - \gamma \lambda f^T(t) f(t) \leq 0, \quad t \in \mathbb{R}_+.
$$

Hence, the closed-loop dynamical system given by (14), (20), (24), and (25) is Lyapunov stable for all $(0, \tilde{W}_\sigma_0, \tilde{W}_u_0, 0) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^{m \times m} \times \mathbb{R}^n$ and $t \in \mathbb{R}_+$. Now, since $\sigma(x(t))$ and $u_0(t)$ are bounded for all $t \in \mathbb{R}_+$, it follows from (14) that $\dot{e}(t)$ is bounded. Furthermore, since $f(t)$ is also bounded, then $\dot{V}(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)))$ is bounded for all $t \in \mathbb{R}_+$. Now, it follows from Barbalat’s lemma [20] that $\lim_{t \to \infty} \dot{V}(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)) = 0$, which consequently shows that $\lim_{t \to \infty} e(t) = 0$ and $\lim_{t \to \infty} f(t) = 0$. As a direct consequence, $\lim_{t \to \infty} g(t) = 0$ and $\lim_{t \to \infty} (c(t) - c_d(t)) = 0$ hold.

Finally, since $\dot{V}(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)) \leq 0$ for $t \in \mathbb{R}_+$, this implies that

$$
V(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)) \leq V(0, \tilde{W}_\sigma_0, \tilde{W}_u_0, 0).
$$

Using the inequalities

$$
V(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(n), f(t)) \geq \lambda_{\min}(P) \|e(t)\|_2^2,
$$

$$
V(0, \tilde{W}_\sigma_0, \tilde{W}_u_0, 0) \leq \|\Gamma^{-1}_\sigma\|_F \|\tilde{W}_\sigma_0 \tilde{W}_{\sigma_0}^T\|_F + \|\Gamma^{-1}_{u_0}\|_F \|\tilde{W}_{u_0} \tilde{W}_{u_0}^T\|_F,
$$

in (35) results in

$$
\|e(t)\|_2 \leq \sqrt{\epsilon_V / \lambda_{\min}(P)}.
$$

Since $\| \cdot \|_{\infty} \leq \| \cdot \|_2$, and this bound is uniform, then (38) yields

$$
\|e_r(t)\|_{\infty} \leq \sqrt{\epsilon_V / \lambda_{\min}(P)},
$$

(39)
and hence, (26) is a direct consequence of (39) due to the fact that (39) holds uniformly in $\tau$. Similarly, using the inequalities
\[
V(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(t), f(t)) \geq \lambda_{\min}(\Lambda)\lambda_{\min}(\Gamma^{-1})\|\tilde{W}_\sigma(t)\|_F^2, \\
V(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(t), f(t)) \geq \lambda_{\min}(\Lambda)\lambda_{\min}(\Gamma^{-1})\|\tilde{W}_u(t)\|_F^2, \\
V(e(t), \tilde{W}_\sigma(t), \tilde{W}_u(t), f(t)) \geq \gamma\|f(t)\|_F^2,
\]
and repeating the above analysis yields (27), (28), and (29). This completes the proof.

Remark 5.1. Theorem 5.1 shows that $\lim_{t \to \infty} g(t) = 0$, and as a direct consequence, the modified reference system in (22) asymptotically converges to the ideal reference system given by
\[
\dot{x}_r(t) = A_rx_r(t) + B_rc_d(t).
\]
Therefore, our first objective is fulfilled, since the distance between the controlled nonlinear uncertain dynamical system (1) and the ideal reference system (40) vanishes in steady-state.

Remark 5.2. According to (26) and (30), if we use high learning rates in (9) and (10), then the distance between $x(t)$ and $x_r(t)$ can be made arbitrarily small in transient time. However, as we see in the next section, it is possible to follow the ideal reference system (40) in transient time by properly choosing the command governor gain $\lambda$, and hence, using high learning rates to shape the transient response is not the only option.

Remark 5.3. Theorem 5.1 highlights not only the stability but also the transient and steady state performance guarantees of the closed-loop dynamical system given by (14), (20), (24), and (25). Furthermore, a similar result can be obtained for the case when the nonlinear uncertain dynamical system given by (1) includes bounded exogenous disturbances, measurement noise, and/or the uncertainty in (1) cannot be perfectly parameterized by using the update laws (16) and (17) in Remark 3.1 instead of (9) and (10) (see, for example, Theorem 4.1 of [15]).

6. Shaping the Transient Response

In this section, we show that the controlled nonlinear uncertain dynamical system (1) approximates the ideal reference system (40) in transient time. We also highlight that there exists a trade-off between the command governor gain $\lambda$ and the adaptive controller’s learning
rates $\Gamma_\sigma$ and $\Gamma_u$. For the following proposition, presenting the main result of this section, let $s$ and $\mathcal{L}_s\{\cdot\}$ be the Laplace variable and the Laplace transform operator, respectively. Furthermore, for a given signal $\theta(t)$, $\mathcal{L}_s\{\theta(t)\} = \theta(s) \triangleq \frac{k}{s+k} \theta(s)$ and $\mathcal{L}_s\{\theta_{lf}(t)\} = \theta_{lf}(s)$ denote the first-order low-pass filtered $\theta(t)$ and the first-order high-pass filtered $\theta(t)$, respectively, such that $\theta(t) = \theta_{lf}(t) + \theta_{hf}(t)$, where $k \in \mathbb{R}_+$. Note that if $\theta(\cdot) \in \mathcal{L}_\infty$, then $\theta_{lf}(\cdot) \in \mathcal{L}_\infty$ and $\theta_{hf}(\cdot) \in \mathcal{L}_\infty$.

**Proposition 6.1.** Consider the nonlinear uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the reference system given by (4) with the command given by (18), the feedback control law given by (5) along with (6), (8), (9), and (10), and the command governor given by (20) and (21). Then, (1) satisfies

$$
\dot{x}(t) = A_r x(t) + \tilde{B}_c \tilde{c}_d(t),
$$

(41)

where $\tilde{B}_c \triangleq [B_c, P_B]$ and $\tilde{c}_d(\cdot) \triangleq [c_d^T(\cdot), \dot{e}_{hf}(\cdot)]^T \in \mathcal{L}_\infty$ with $\mathcal{L}_s\{\dot{e}(t)\} \triangleq s L_s\{\dot{e}(t)\}$. If, in addition, the command governor gain $\lambda$ is sufficiently large, then (41) approximates the ideal reference system (40) in the limit as $\lambda \to \infty$.

**Proof.** The command governor system given by (20) and (21) can be written in Laplace domain as

$$
\mathcal{G}_{e \to g}(s) = \lambda(s + \lambda)^{-1} I_n + (A_r - \lambda I_n),
$$

(42)

where $\mathcal{G}_{e \to g}(s) \triangleq g(s)/e(s)$. Since (42) can be rewritten as

$$
\mathcal{G}_{e \to g}(s) = \frac{s^2}{\lambda} I_n + \frac{s + \lambda}{\lambda} (A_r - \lambda I_n) = \frac{(s + \lambda) A_r - \lambda s I_n}{s + \lambda} = A_r - \frac{s}{\lambda s + 1} I_n,
$$

(43)

then (43) gives

$$
\mathcal{L}_s\{g(t)\} = A_r e(s) - \frac{s}{\lambda s + 1} e(s).
$$

(44)

Next, we can write (23) as

$$
\dot{x}(t) = A_r x(t) + B_c c_d(t) + P_B g(t) - D A \left[ \bar{W}^T(t) \sigma(x(t)) + \bar{W}^T_{u_n}(t)u_n(t) \right]
\nonumber
\nonumber
= A_r x(t) + B_c c_d(t) + D \left( (D^T D)^{-1} D^T g(t) - \Lambda \left[ \bar{W}^T_{\sigma}(t) \sigma(x(t)) + \bar{W}^T_{u_n}(t)u_n(t) \right] \right).
$$

(45)

---

8This can be shown as follows. Suppose, ad absurdum, $\theta_{hf}(\cdot) \notin \mathcal{L}_\infty$. Since $\theta(\cdot) \in \mathcal{L}_\infty$ and $\frac{k}{s+k}$ is an asymptotically stable transfer function, then $\theta_{lf}(\cdot) \in \mathcal{L}_\infty$. Since $\|\theta_{hf}(t)\|_2 \leq \|\theta(t)\|_2 + \|\theta_{hf}(t)\|_2$, then $\theta_{hf}(\cdot) \in \mathcal{L}_\infty$, which is a contradiction.
Furthermore, \((14)\) can be rewritten as
\[
-D\Lambda[\tilde{W}_\sigma^T(t)\sigma(x(t)) + \tilde{W}_{un}^T(t)u_n(t)] = \dot{e}(t) - A_re(t)
\]
\[
\Rightarrow -\Lambda[\tilde{W}_\sigma^T(t)\sigma(x(t)) + \tilde{W}_{un}^T(t)u_n(t)] = (D^TD)^{-1}D^T(\dot{e}(t) - A_re(t)).
\]

where \(\dot{\scriptscriptstyle e}\) is in \(L_\infty\) as a direct consequence of Theorem 5.1. Using (46) in (45) yields
\[
\dot{x}(t) = A_rx(t) + B_ic_d(t) + D((D^TD)^{-1}D^Tg(t) + (D^TD)^{-1}D^T(\dot{e}(t) - A_re(t)))
\]
\[
= A_rx(t) + B_ic_d(t) + P_D\left(g(t) + (\dot{\scriptscriptstyle e}(t) - A_re(t))\right).
\]

Finally, using (44) in (47) gives
\[
\dot{x}(t) = A_rx(t) + B_ic_d(t) + P_D\dot{\scriptscriptstyle e}_{hf}(t),
\]

where (48) can be rearranged as (41) leading to the result, where \(c_d(\cdot)\) is in \(L_\infty\) and \(\dot{\scriptscriptstyle e}_{hf}(\cdot)\) is in \(L_\infty\), and hence, \(\tilde{c}_d(\cdot)\) is in \(L_\infty\). If, in addition, the command governor gain \(\lambda\) is sufficiently large, then \(\dot{\scriptscriptstyle e}_{hf}(t) \approx 0\), and therefore, (48) approximates the ideal reference system (40) in the limit as \(\lambda \to \infty\). This completes the proof.

Remark 6.1. Since Proposition 6.1 shows that the controlled nonlinear uncertain dynamical system (1) approximates the ideal reference system (40) in transient time for a (sufficiently) large command governor gain \(\lambda\), then the learning rates \(\Gamma_\sigma\) and \(\Gamma_{un}\) for (9) and (10) can be chosen to be (sufficiently) small. In fact, if \(\lambda = \infty^9\), then the command governor perfectly shapes the transient response of (1), such that (1) becomes exactly (40), and hence, \(\Gamma_\sigma\) and \(\Gamma_{un}\) can be chosen to be zero. Furthermore, it should be clear from (26) and (30) for the \(\lambda = 0\) case that \(\Gamma_\sigma\) and \(\Gamma_{un}\) needs to be chosen to be sufficiently large in order to guarantee that the distance between \(x(t)\) and \(x_r(t)\) is sufficiently small in transient time. Therefore, there exists a trade-off between the command governor gain \(\lambda\) and the learning rates \(\Gamma_\sigma\) and \(\Gamma_{un}\). As discussed, this key feature allows rapid suppression of system uncertainties without resorting to high learning rates in the adaptive controller, and hence, our second objective is fulfilled.

---

\(^9\)Since \(\lambda = \infty\) is not a valid case, we need the adaptive feedback control law with (sufficiently) small learning rates for stability.
Remark 6.2. The proposed command governor can be also an effective approach for disturbance rejection problem\textsuperscript{10}. To see this, consider

\[
\dot{x}(t) = Ax(t) + Bu(t) + D[\delta(x(t)) + d(t)].
\]

where \(d(t) \in \mathbb{R}^m\) is a bounded exogenous disturbance. In this case, (45) and (46) becomes, respectively,

\[
\dot{x}(t) = A_r x(t) + B_r c_d(t) + D\left((D^T D)^{-1} D^T g(t) - \Lambda \left[\tilde{W}^T(t) \sigma(x(t)) + \tilde{W}^T u_n(t) u_n(t) - \tilde{d}(t)\right]\right),
\]

\[
-\Lambda \left[\tilde{W}^T(t) \sigma(x(t)) + \tilde{W}^T u_n(t) u_n(t) - \tilde{d}(t)\right] = (D^T D)^{-1} D^T (\dot{e}(t) - A_r e(t)),
\]

where \(\tilde{d}(t) \triangleq \Lambda^{-1} d(t),\) and hence, (50) and (51) along with (44) yields approximately to (40) for a (sufficiently) large \(\lambda.\) That is, the proposed command governor based adaptive control architecture does not require any additional feedback signal for disturbance rejection problem in contrast to many other adaptive controllers (see, for example, [15] and references therein). For this case, note that, there is a need to replace the update laws given by (9) and (10) with (16) and (17), respectively, as noted in Remarks 3.1 and 5.3.

7. Robust Command Governor Based Adaptive Control

For the case when the nonlinear uncertain dynamical system (1) contains high-frequency dynamical system content, it should be noted from (44) that a (sufficiently) large command governor gain \(\lambda\) can amplify this content at the output of the command governor, and hence, this amplification can adversely affect the robustness properties of our framework. For this purpose, this section robustifies the proposed command governor architecture to make the output less sensitive to possible high-frequency dynamical system content. Specifically, let the command \(c(t)\) be given by

\[
c(t) = c_d(t) + Gg_t(t),
\]

where \(Gg_t(t)\) is the modified command governor signal with \(G \in \mathbb{R}^{m \times n}\) being the matrix defined by (19) and \(g_t(t) \in \mathbb{R}^n\) being the modified command governor output generated

\textsuperscript{10}The discussion given in this remark directly applies to the case discussed in Remark 3.1. That is, unlike standard adaptive controllers, the proposed framework makes (1) close to (40) as \(\lambda\) increased even in the case of uniform boundedness.
through the low-pass filter
\[
\dot{g}_f(t) = -\eta g_f(t) + \eta g(t), \quad g_f(0) = 0, \quad t \in \mathbb{R}_+,
\] (53)
where \( \eta \in \mathbb{R}_+ \) is the command governor filter gain and \( g(t) \) satisfies (20) and (21).

**Remark 7.1.** In Laplace domain, the low-pass filter given by (53) can be written as
\[
G_{g \rightarrow g_f}(s) = \frac{1}{\frac{1}{\eta} s + 1},
\] (54)
where \( G_{g \rightarrow g_f}(s) \triangleq g_f(s)/g(s) \). It should be noted that the selection of the low-pass filter (54) that filters out the high-frequency dynamical system content contained in \( g(t) \) is not unique.

Once again, for the following theorem, we assume that the choice of \( R \) in (12) satisfies
\[
R = R_0 + \gamma \lambda I_n
\]
where \( R_0 \in \mathbb{R}^{n \times n} \cap S^{n \times n} \) and \( \gamma \in \mathbb{R}_+ \) is an arbitrary constant.

**Theorem 7.1.** Consider the nonlinear uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the reference system given by (4) with the command given by (52), the feedback control law given by (5) along with (6), (8), (9), and (10), and the command governor given by (20), (21), and (53). Then, the solution \((e(t), \tilde{W}_\sigma(t), \tilde{W}_{uu}(t), f(t), g_f(t))\) of the closed-loop dynamical system given by (14), (20), (24), (25), and (53) is Lyapunov stable for all \( (e(t), \tilde{W}_\sigma(t), \tilde{W}_{uu}(t), f(t), g_f(t)) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^{m \times m} \times \mathbb{R}^n \times \mathbb{R}^n \) and \( t \in \mathbb{R}_+ \), and \( \lim_{t \to \infty} e(t) = 0, \lim_{t \to \infty} f(t) = 0, \lim_{t \to \infty} g(t) = 0, \lim_{t \to \infty} g_f(t) = 0, \) and \( \lim_{t \to \infty} (c(t) - c_d(t)) = 0 \). For \( t \in \mathbb{R}_+ \), in addition, the system error dynamics, the weight error dynamics, the command governor dynamics, and the low-pass filter dynamics satisfy the transient performance bounds given by (26), (27), (28), (29), and
\[
\|g_f(t)\|_{\infty} \leq \sqrt{\epsilon_V/\xi},
\] (55)
where \( \epsilon_V \) is given by (30) and \( \xi \in \mathbb{R}_+ \) is chosen such that \( \min\left\{ \frac{\lambda_{\min}(R_0)}{\eta \|A_t - \lambda I_n\|_F^2}, \frac{\gamma}{\eta \lambda} \right\} > \xi \) holds.

**Proof.** To show Lyapunov stability of the closed-loop dynamical system (14), (20), (24), (25), and (53), consider the Lyapunov function candidate
\[
\mathcal{V}(e, \tilde{W}_\sigma, \tilde{W}_{uu}, f, g_f) = e^T P e + \text{tr} \left( \tilde{W}_\sigma \Lambda_{\tilde{\sigma}}^{\frac{1}{2}} \Gamma_{\sigma}^{-1}(\tilde{W}_\sigma \Lambda_{\tilde{\sigma}}^{\frac{1}{2}}) \right) + \text{tr} \left( \tilde{W}_{uu} \Lambda_{\tilde{\sigma}}^{\frac{1}{2}} \Gamma_{\sigma}^{-1}(\tilde{W}_{uu} \Lambda_{\tilde{\sigma}}^{\frac{1}{2}}) \right)
+ \gamma f^T f + \xi g_f^T g_f,
\] (56)

\[\text{11}\]The command governor filter rate \( \eta \) needs to be chosen as a low gain. Because, in this case, (53) behaves as an efficient low-pass filter.
and note that \( \mathcal{V}(0, 0, 0, 0, 0) = 0 \). Since \( P \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}_+^{n \times n}, \Gamma_\sigma \in \mathbb{R}_+^{s \times s} \cap \mathbb{S}_+^{s \times s}, \Gamma_u \in \mathbb{R}_+^{m \times m} \cap \mathbb{S}_+^{m \times m}, \Lambda \in \mathbb{R}_+^{m \times m} \cap \mathbb{D}_+^{m \times m}, \gamma \in \mathbb{R}_+, \text{ and } \xi \in \mathbb{R}_+, \mathcal{V}(e, \bar{W}_\sigma, \bar{W}_{u}, f, g_t) > 0 \) for all \( (e, \bar{W}_\sigma, \bar{W}_{u,}, f, g_t) \neq (0, 0, 0, 0, 0) \). In addition, \( \mathcal{V}(e, \bar{W}_\sigma, \bar{W}_{u,}, f, g_t) \) is radially unbounded. Differentiating (56) along the closed-loop trajectories of (14), (20), (24), (25), and (53) yields

\[
\dot{\mathcal{V}}(e(t), \bar{W}_\sigma(t), \bar{W}_{u}(t), f(t), g_t(t)) \leq -e^T(t)R_0e(t) - \gamma \lambda f^T(t)f(t) - 2\xi \eta g^T(t)g_t(t) +2\xi \eta g^T(t)g(t) = -e^T(t)R_0e(t) - \gamma \lambda f^T(t)f(t) - 2\xi \eta g^T(t)g_t(t) +2\xi \eta g^T(t)[\lambda f(t) + (A_t - \lambda I)n(t)] \\
\leq -\lambda_{\min}(R_0)||e(t)||_2^2 - \gamma \lambda ||f(t)||_2^2 - 2\xi \eta ||g_t(t)||_2^2 + 2\xi \eta \lambda ||g_t(t)||_2||f(t)||_2 + 2\xi \eta \varphi ||g_t(t)||_2||e(t)||_2, \tag{57}
\]

where \( \varphi \triangleq ||A_t - \lambda I||_F \). Consider 2\( xy \leq \mu x^2 + \frac{1}{\mu} y^2 \) that follows from Young’s inequality [23] applied to scalars \( x \in \mathbb{R} \) and \( y \in \mathbb{R} \), where \( \mu \in \mathbb{R}_+ \). Using this fact for the last two terms in (57) gives

\[
2\xi \eta \lambda ||g_t(t)||_2||f(t)||_2 \leq \xi \eta \lambda \mu_1 ||g_t(t)||_2^2 + \frac{1}{\mu_1} \xi \eta \lambda ||f(t)||_2^2, \tag{58}
\]

\[
2\xi \eta \varphi ||g_t(t)||_2||e(t)||_2 \leq \xi \eta \varphi \mu_2 ||g_t(t)||_2^2 + \frac{1}{\mu_2} \xi \eta \varphi ||e(t)||_2^2. \tag{59}
\]

Now, setting \( \mu_1 = \lambda^{-1} \) and \( \mu_2 = \varphi^{-1} \), and using (58) and (59) in (57) yields

\[
\dot{\mathcal{V}}(e(t), \bar{W}_\sigma(t), \bar{W}_{u}(t), f(t), g_t(t)) \leq -\alpha_1 ||e(t)||_2^2 - \lambda \alpha_2 ||f(t)||_2^2, \tag{60}
\]

where \( \alpha_1 \triangleq \lambda_{\min}(R_0) - \xi \eta \varphi^2 \in \mathbb{R}_+ \) and \( \alpha_2 \triangleq \gamma - \xi \eta \lambda \in \mathbb{R}_+ \) since \( \xi \in \mathbb{R}_+ \) is chosen such that \( \min\left\{ \frac{\lambda_{\min}(R_0)}{\xi||A_t - \lambda I||_F^2}, \frac{2\lambda}{\xi} \right\} > \xi \). Therefore, \( \dot{\mathcal{V}}(e(t), \bar{W}_\sigma(t), \bar{W}_{u}(t), f(t), g_t(t)) \leq 0 \) for all \( t \in \mathbb{R}_+ \). Hence, the closed-loop dynamical system given by (14), (20), (24), (25), and (53) is Lyapunov stable for all \( (0, \bar{W}_{\sigma}, \bar{W}_{u,0}, 0, 0) \in \mathbb{R}^n \times \mathbb{R}^{s \times s} \times \mathbb{R}^{m \times m} \times \mathbb{R}^n \times \mathbb{R}^n \text{ and } t \in \mathbb{R}_+ \). Now, since \( \sigma(x(t)) \) and \( u_a(t) \) are bounded for all \( t \in \mathbb{R}_+ \), it follows from (14) that \( \dot{e}(t) \) is bounded. Furthermore, since \( \dot{f}(t) \) is also bounded, then \( \dot{\mathcal{V}}(e(t), \bar{W}_\sigma(t), \bar{W}_{u}(t)) \) is bounded for all \( t \in \mathbb{R}_+ \). Now, it follows from Barbalat’s lemma [20] that \( \lim_{t \to \infty} \dot{\mathcal{V}}(e(t), \bar{W}_\sigma(t), \bar{W}_{u}(t), f(t), g_t(t)) = 0 \), which consequently shows that \( \lim_{t \to \infty} e(t) = 0 \) and \( \lim_{t \to \infty} f(t) = 0 \). As a direct consequence,
\[ \lim_{t \to \infty} g(t) = 0, \lim_{t \to \infty} g(t) = 0, \text{ and } \lim_{t \to \infty} (c(t) - c_d(t)) = 0 \] hold. The rest of the proof follows similar to the proof of Theorem 5.1, and hence, is omitted.

Next, we revisit the case when the command governor gain \( \lambda \) is (sufficiently) large and show that the controlled nonlinear dynamical system (1) approximates a Hurwitz linear time-invariant dynamical system having the form of the ideal reference system (40) modified by an error term which asymptotically goes to zero.

**Proposition 7.1.** Consider the nonlinear uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the reference system given by (4) with the command given by (52), the feedback control law given by (5) along with (6), (8), (9), and (10), and the command governor given by (20), (21), and (53). Then, (1) satisfies

\[ \dot{x}(t) = A_r x(t) + \tilde{B}_r \tilde{c}_d(t) + P_D (g(t) - g(t)), \]  

where \( \tilde{B}_r \triangleq [B_r, P_D] \), \( \tilde{c}_d(\cdot) \triangleq [c_d^T(\cdot), \dot{e}_h^T(\cdot)]^T \in \mathcal{L}_\infty \), and \( g(t) - g(t) \in \mathcal{L}_\infty \), with \( \mathcal{L}_s \{ \dot{e}_h(t) \} \triangleq \frac{s}{s+\lambda} \mathcal{L}_s \{ \dot{e}(t) \} \). If, in addition, the command governor gain \( \lambda \) is sufficiently large, then (61) approximates the ideal reference system (40) modified by the term \( P_D (g(t) - g(t)) \) which satisfies \( \lim_{t \to \infty} P_D (g(t) - g(t)) = 0 \).

**Proof.** The proof is identical to the proof of Proposition 6.1 and, hence, is omitted.

**Remark 7.2.** Proposition 7.1 shows that the controlled nonlinear uncertain dynamical system (1) approximates the ideal reference system (40) modified by the term \( P_D (g(t) - g(t)) \) having the property \( \lim_{t \to \infty} P_D (g(t) - g(t)) = 0 \) for a (sufficiently) large command governor gain \( \lambda \). That is, if \( g(t) - g(t) \) is close to zero, then (1) still approximates the ideal reference system (40).

Even though, a necessary and sufficient condition for \( g(t) - g(t) \) to stay close to zero is a large command governor filter rate \( \eta \), this can adversely affect the robustness properties of our framework as discussed earlier\(^{12}\). An alternative way to achieve this objective is to use the state predictor approach introduced in [24]. Specifically, since \( g(t) - g(t) \) goes to zero depending on how fast \( e(t) \) goes to zero\(^{13}\), one can use the state predictor for shaping

\(^{12}\)Because, \( g(t) \approx g(t) \) for a large command governor filter rate \( \eta \).

\(^{13}\)Using (44) and (54), this can be shown by writing \( \mathcal{L}_s \{ g(t) - g(t) \} = (\frac{\lambda s^2}{s^2 + (\lambda + \eta)s + \lambda \eta} - \frac{s}{s + \eta} A_r) e(s) \).
the transient error system dynamics in order to suppress \( e(t) \) faster. For this purpose, we replace (4) with the state predictor given by

\[
\dot{x}_r(t) = A_r x_r(t) + B_r c(t) + \kappa P e(t), \quad x_r(0) = x_0, \quad t \in \mathbb{R}_+,
\]

(62)

where \( \kappa \in \mathbb{R}_+ \) and \( P \in \mathbb{R}_+^{n \times n} \cap S^{n \times n} \) is a solution of the Lyapunov equation given by (12).

Now, by using (13) and (62), the system error dynamics can be given by

\[
\dot{e}(t) = \tilde{A}_r e(t) - DA \tilde{W}_\sigma^T(t) \sigma(x(t)) + \tilde{W}_u^T(t) u_n(t), \quad e(0) = 0, \quad t \in \mathbb{R}_+,
\]

(63)

where \( \tilde{A}_r \triangleq A_r - \kappa P \in \mathbb{R}^{n \times n} \) is Hurwitz\(^{14}\).

**Remark 7.3.** In [24], the transient error system performance for (63) is analyzed using singular perturbation theory [20]. In particular, by forming the boundary-layer system capturing the transient time response, it is shown that \( e(t) \) goes to zero faster as the value of \( \kappa \) increased (see Section V of [24] for details). Therefore, this implies that \( g_f(t) - g(t) \) goes to zero faster depending on the value of \( \kappa \).

For the case when (4) replaced with the state predictor given by (62), the command governor architecture is given by (20), (52), (53), and

\[
g(t) = \lambda f(t) + (\tilde{A}_r - \lambda I_n) e(t),
\]

(64)

where \( e(t) = x(t) - x_r(t) \) with \( x_r(t) \) being the state vector of (62). The following theorem analyzes the stability in this case.

**Theorem 7.2.** Consider the nonlinear uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the state predictor given by (62) with the command given by (52), the feedback control law given by (5) along with (6), (8), (9), and (10), and the command governor given by (20), (53), and (64). Then, the solution \((e(t), \tilde{W}_\sigma(t), \tilde{W}_{u_n}(t), f(t), g_f(t))\) of the closed-loop dynamical system given by (20), (24), (25), (53), and (63) is Lyapunov stable for all \((0, \tilde{W}_{\sigma_0}, \tilde{W}_{u_n_0}, 0, 0) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^{m \times m} \times \mathbb{R}^n \times \mathbb{R}^n \) and \( t \in \mathbb{R}_+ \), and \( \lim_{t \to \infty} e(t) = 0 \), \( \lim_{t \to \infty} f(t) = 0 \), \( \lim_{t \to \infty} g(t) = 0 \), \( \lim_{t \to \infty} g_f(t) = 0 \), and \( \lim_{t \to \infty} (c(t) -

---

\(^{14}\)To see this, write (12) as \(-2\kappa P^2 = (A_r - \kappa P)^T P + P(A_r - \kappa P) + R\) and note that \( \kappa P^2 \in \mathbb{R}_+^{n \times n} \cap S^{n \times n} \). Then, as a direct consequence, \( 0 \geq \tilde{A}_r^T P + P \tilde{A}_r + R \), which implies that \( \tilde{A}_r \) is Hurwitz.
\( c_d(t) = 0 \). For \( t \in \mathbb{R}_+ \), in addition, the system error state vector, the weight update error dynamics, the command governor dynamics, and the low pass filter dynamics satisfy the transient performance bounds given by (26), (27), (28), (29), (55) where \( \epsilon_V \) is given by (30) and \( \xi \in \mathbb{R}_+ \) is chosen such that \( \min \left\{ \frac{\lambda_{\min}(R_0)}{\eta \|A_r - \lambda I\|_F}, \frac{\gamma}{\eta \lambda} \right\} > \xi \) holds.

**Proof.** To show Lyapunov stability of the closed-loop dynamical system (20), (24), (25), (53), and (63), consider the Lyapunov function candidate given by (56). Differentiating (31) along the closed-loop trajectories of (20), (24), (25), (53), and (63) yields

\[
\dot{V}(e(t), \tilde{W}_g(t), \tilde{W}_u(t), f(t), g_l(t)) \leq -\alpha_1 \|e(t)\|_2^2 - 2\kappa \|Pe(t)\|_2^2 - \lambda \alpha_2 \|f(t)\|_2^2,
\]

where the rest of the proof follows similar to the proofs of Theorems 5.1 and 7.1, and hence, is omitted.

**Remark 7.4.** Analogous to Proposition 7.1, it can be identically shown that the controlled nonlinear uncertain dynamical system (1) approximates the ideal reference system (40) modified by the term \( P_D(g_f(t) - g(t)) \) having the property \( \lim_{t \to \infty} P_D(g_f(t) - g(t)) = 0 \) for a (sufficiently) large command governor gain \( \lambda \). However, as discussed in Remark 7.3, \( g_f(t) - g(t) \) goes to zero faster as \( \kappa \) increased for the case when state predictor (62) is employed instead of (4).

**Remark 7.5.** The high-frequency dynamical system content discussed throughout this section can be also due to measurement noise and/or unmodeled dynamics. If this is the case and the update laws given by (9) and (10) are replaced with (16) and (17), respectively, then the framework proposed in this section robustifies the controlled nonlinear uncertain dynamical system with respect to possible measurement noise and/or unmodeled dynamics.

A block diagram showing the robust command governor based adaptive control framework is given in Figure 7.1.

8. **Illustrative Numerical Example**

Consider a nonlinear controlled wing rock aircraft dynamics model given by

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
u(t) + \delta(x(t))
\end{bmatrix},
\begin{bmatrix}
x_1(0) \\
x_2(0)
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix},
\quad t \in \mathbb{R}_+,
\] (65)

where \( x_1 \) represents the roll angle in radians and \( x_2 \) represents the roll rate in radians per second. In (65), \( \delta(x) \) representing an uncertainty of the form \( \delta(x) = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 |x_1| x_2 + \)
\[ \alpha_4 |x_2| x_2^3 + \alpha_5 x_1^3, \] where \( \alpha_i, i = 1, \ldots, 5, \) are unknown parameters that are derived from the aircraft aerodynamic coefficients [25]. For our numerical example, we set \( \alpha_1 = 0.1414, \) \( \alpha_2 = 0.5504, \) \( \alpha_3 = -0.0624, \) \( \alpha_4 = 0.0095, \) and \( \alpha_5 = 0.0215. \) We choose \( K_1 = [-0.16, -0.57] \) and \( K_2 = 0.16 \) for the nominal controller design that yields to a reference system with a natural frequency of \( \omega_n = 0.40 \) rad/s and a damping ratio \( \zeta = 0.707. \) For the standard adaptive controller design (i.e., \( c(t) = c_d(t) \)), \( \sigma(x) = [x_1, x_2, |x_1| x_2, |x_2| x_2, x_1^3]^T \) is chosen as the basis function, and we set \( R = I_2, \) \( \Gamma_\sigma = 25I_2, \) and \( \Gamma_{u_n} = 0, \) since the control effectiveness matrix \( \Lambda \) is known in this case. For the proposed robust command governor based adaptive control architecture (Figure 7.1), we use the same basis function and \( R. \) Furthermore, we set \( \Gamma_\sigma = 0.01I_2, \) \( \Gamma_{u_n} = 0, \) \( \lambda = 50, \) \( \eta = 2.5, \) and \( \kappa = 1. \) Here, our aim is to track a given
filtered square-wave roll angle command $c_d(t)$.

Figure 8.1 shows the nominal control performance under ideal conditions (i.e., $\delta(x(t)) \equiv 0$). Note that the nominal control performance under adverse conditions (i.e., $\delta(x(t)) \neq 0$) is unstable, and hence, we need to use adaptive control laws. Figures 8.2 – 8.4 show the performance of the standard adaptive controller under adverse conditions, where a satisfactory closed-loop dynamical system response could not be achieved. In fact, the control histories of these figures include high-frequency oscillations, where the frequency of these oscillations gets higher as the magnitude of the command increased. Figures 8.5 – 8.7 show the performance of the proposed robust command governor based adaptive controller under adverse conditions, where for each command amplitude, we can able to obtain a satisfactory closed-loop dynamical system response which is clearly superior as compared to the standard adaptive controller. It is of importance to note that for a command with low amplitude, the adaptive control history is close to zero. However, for a command with high amplitude, the adaptive control history is dominant when it is compared with the nominal control history. This result is expected since the proposed command governor shapes the transient system response. In all of these figures, in addition, the controlled nonlinear dynamical system response behaves almost linearly (i.e., predictable), as expected from our theory.

Next, we insert an input time-delay of 0.3 seconds in order to test the robustness of the proposed architecture against unmodeled dynamics. Figures 8.8 and 8.9 present results for the commands with moderate and high amplitudes, respectively. These responses are almost identical to the equivalent responses given in Figures 8.6 and 8.7 for the same command histories but without an input time-delay. Note that the achieved time-delay margin of the standard adaptive controller is 0.15 seconds and 0.01 seconds for the commands with moderate and high amplitudes, respectively. In our future research, we will investigate possible guaranteed robustness properties of the proposed framework against input time-delays given this trend. Finally, we added measurement noise to the system, and the performance of the proposed architecture under adverse conditions is given in Figure 8.10. In this figure, the effect of the measurement noise on the control histories is acceptable.
Figure 8.1: Nominal control performance for $x_1(t)$, $c_d(t)$, and $u_n(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with moderate amplitude.

Figure 8.2: Standard adaptive control performance for $x_1(t)$, $c_d(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with low amplitude.

Figure 8.3: Standard adaptive control performance for $x_1(t)$, $c_d(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with moderate amplitude.

Figure 8.4: Standard adaptive control performance for $x_1(t)$, $c_d(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with high amplitude.

9. Conclusion

Although adaptive control theory offers mathematical tools to achieve system performance without excessive reliance on dynamical system models, its applications to safety critical systems are limited due to poor transient performance. Motivating from this standpoint, the contribution of this paper is to introduce a new command governor architecture in
Specifically, we first analyze the transient and steady state performance guarantees of our adaptive control design process for adjusting the trajectory of a given command in order to follow an ideal reference system in transient time, and hence, the proposed command governor based adaptive control performance for a command with high amplitude.

Figure 8.5: Proposed robust command governor based adaptive control performance for $x_1(t)$, $c_d(t)$, $c(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with low amplitude.

Figure 8.6: Proposed robust command governor based adaptive control performance for $x_1(t)$, $c_d(t)$, $c(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with moderate amplitude.

Figure 8.7: Proposed robust command governor based adaptive control performance for $x_1(t)$, $c_d(t)$, $c(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with high amplitude.

Figure 8.8: Proposed robust command governor based adaptive control performance for $x_1(t)$, $c_d(t)$, $c(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with moderate amplitude under an input time-delay of 0.3 seconds.

the adaptive control design process for adjusting the trajectory of a given command in order to follow an ideal reference system in transient time, and hence, the proposed command governor-based adaptive controller addresses the poor transient performance phenomenon. Specifically, we first analyze the transient and steady state performance guarantees of our framework, and then show that the controlled nonlinear uncertain dynamical system approx-
Figure 8.9: Proposed robust command governor based adaptive control performance for $x_1(t)$, $c_d(t)$, $c(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with high amplitude under an input time-delay of 0.3 seconds.

Figure 8.10: Proposed robust command governor based adaptive control performance for $x_1(t)$, $c_d(t)$, $c(t)$, $u_n(t)$, and $u_a(t)$ in degrees, and $x_2(t)$ in degrees per second for a command with moderate amplitude under measurement noise.

imates the ideal reference system in transient time by choosing a high command governor gain. Since the high command governor gain may adversely affect the robustness against high-frequency dynamical system content, we further robustified the proposed framework by employing a low-pass filter and a state predictor. The illustrative example indicates that the presented theory and its numerical results are compatible. Future research will include extensions to uncertain dynamical systems with limited state informations and state and control constraints, and applications to physical systems.
References


