Development of a Model Reference Adaptive Controller of the Plantarflexion and Dorsiflexion Movements within the Sagittal Plane

Talon Garikayi, Stephen Matope, and Dawie van den Heever

Abstract—This paper presents a novel idea of implementing an adaptive servo system in which the desired performance of the foot kinematics is expressed in terms of a reference model, which gives the desired response to an electromyogram signal from the muscles so as to manipulate the behaviour of a robotic prosthetic limb. The foot kinematics was derived based on the scope of the design and a digital human lower limb was developed in Autodesk Inventor platform. The controller design was done based on a dc motor actuator. Empirical analysis of model reference adaptive system supported with parameter estimation techniques was developed. The control architecture development was simulated in Matlab and Simulink platform and the results were presented. The results showed that for small and precise angle movements a model reference adaptive control system was desirable, however mechanical inertia caused poor response to sudden small movements of the foot.

Keywords—Robotic, adaptive, dorsiflexion, electromyogram, plantarflexion, prosthesis,

I. INTRODUCTION

The human foot is one key element within the human lower limb anatomy and its functionality gives the extent to which a human is capable of performing various tasks. Due to increased civil wars, foot diseases and injuries at work, there has been a notable increase in the need for lower limb replacement [1]. Most commercially available human limbs are simply mechanical assistive devices which are viewed by many crippled people as mechanical extensions within the human body [2]. In some cases, the results of biological evolution have provided the optimal solution for the design engineer [3]. The development of a robotic prosthetic system that can enhance human physical capabilities and form part of the body will go a long way in improving community development. Although the idea of constructing such technologies is not new as seen by the successful designs such as the Deka Arm by FDA and the most recent i-Limb by the Scottish company Touch Bionics, great engineering and technological challenges still remain, even up to this period of time as permanent assistive devices are viewed by the physically challenged as separate, lifeless mechanisms and not intimate extension of the human body structurally, neurologically and dynamically. There is a need to have a good synergistic integration between the precision mechanical and electronic components. The unpredictable complex human foot kinematics has been a great challenge during the development of a suitable controller for the achievement of desired motions, hence the need for model reference adaptive control system. The paper unfolds as follows: section 2 presents the foot kinematics and the associated modelling procedure, section 3 presents the muscle signals that were used to provide the actuation signal, section 4 presents the controller design procedure, section 5 presents the model results and section 6 concludes the research.

II. MATHEMATICAL MODEL OF THE HUMAN FOOT

Motion at the ankle joint complex has been divided into that at the ankle and at the subtalar joints [4] [5]. Tuijthof et al [6] carried out a comprehensive study on 20 healthy subjects and found that computer tomography based stress-tests in-vivo in non-weight bearing conditions revealed that from maximal dorsi- to maximal plantar-flexion, the mean overall rotation is much higher at the ankle (63°) than at subtalar (4°) joint. Much smaller differences were observed in the complete natural range from maximal combined eversion-dorsiflexion to maximal combined inversion-plantarflexion (49° at the ankle, 30° at the subtalar). However, Lundgren et al [7] revealed that during the stance phase of walking, the joint rotations in the three anatomical planes were found to be on average about 15°, 8°, and 8° at the ankle joint, and about 7°, 10°, and 7° at the subtalar joint. Successful pioneered studies [8][9][10][11] revealed that combined motion at these two articulations was considered to be a rotation about a single or a double fixed axis.

According to Syrseloudis et al., [12] the human ankle has a complex multi-joint structure. The lower limb kinematics is...
Foot kinematics is greatly affected by bone sizes, relative positions and rotational axes, hence the rotation is further affected by shape of the mating surfaces. Finally ligaments, capsules and tendons also offer resistance to motion. The development of the foot kinematics is modelled as a serial manipulator such that shank link \( L_1 \) connected at center \( C_1 \) of the \( S_1S_2 \) section and the center \( C_2 \) of the \( S_3S_4 \) section. Lastly the lower foot-link is linked to the center \( C_3 \) of the \( S_5 \) which is on the fifth metatarsal. The main knee axis, defined by \( S_1 - S_2 \), is assumed to be fixed. The Upper Ankle Axis (UAA) and the Subtalar Axis (STA) are governed by the \( S_3 \) and \( S_6 \) axes respectively. Taking into consideration the three dimensional axes (XYZ) using the right hand rule, plantarflexion is a negative rotation and dorsiflexion is a positive rotation with respect to the UAJ (\( Z_2 \)), hence inversion is negative rotation and eversion is the positive rotation with respect to the SJT (\( Z_3 \)). Considering the Denavit-Hartenberg (DH) notation of 1955, the assigned relative frames \( O_i \) between the moving links are shown in Figure 1. \( T_{i+1}^{i} \) is the transformation matrix from \( O_{i+1} \) into \( O_i \) defined as follows:

\[
T_{i+1}^{i} = 
\begin{bmatrix}
    c\theta_i & sa_i & c\theta_i & a_i & 0 \\
    -ca_i & c\theta_i & sa_i & a_i & d_i \\
    0 & s\theta_i & c\theta_i & 0 & 0 \\
    0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Where
\[
c\theta_i = \cos\theta_i \quad \text{and} \quad -ca_i = -\cos a_i \sin \theta_i \quad \text{hence} \quad i = 1,2,3 \text{ and } 4
\]

The transformation matrix from the last into the first coordinate system is given by the relation:

\[
T_{1}^{4} = T_{1}^{2}T_{2}^{3}T_{3}^{4}
\]

The last coordinate system is that of the foot, system \( O_5X_1Y_1Z_4 \) on Figure 3. For a point \( S = [x \ y \ z \ 1]^T \) on this system, the above transformation into the first (shank) coordinate system can be expressed as \( S_{01} = T_{1}^{4}S \) (3)

These equations give a parametric formula of the D-H parameters in the movement of \( S \) with respect to the fixed coordinate system of the shank. The D-H parameters are defined as follows, by referring to Fig 3:

- \( \alpha_i \) is the twist angle between the \( Z_i \) and \( Z_{i+1} \) axes,
- \( a_i \) is the length of the common normal to the \( Z_i \) and \( Z_{i+1} \) axes,
- \( d_i \) is the offset between the common normals \( a_i \) and \( a_{i-1} \)
- \( \Theta_i \) is the rotation angle between \( X_1 \) and \( X_{i+1} \).

The independent variables of the model are the angles \( \Theta_2 \) (dorsiflexion / plantarflexion), and \( \Theta_3 \) (inversion / eversion), whereas \( \Theta_1 \) is constant. The zero configuration of the equivalent serial manipulator is taken to be when the foot is in the erect standing pose. The values of the variable angles about the zero configurations are taken to lie in the following ranges [16]:

\[-40^\circ \leq \Theta_2 \leq 25^\circ \quad \text{and} \quad -20^\circ \leq \Theta_3 \leq 20^\circ\]

This analysis concerns the right leg while movements of the left leg are assumed to be the mirror-image of the right leg [10]. The parameters \( a_5 \), \( a_6 \), \( d_6 \) depend on the foot anatomy and size. In Dul and Johnson [10] the transformation matrices expressed in Euler angles were estimated for a female subject. Standard instruments were used to measure the distances between the bony landmarks. After the calculation of several internal distances using the triangulation technique, the redundant distance method was used for the calculation of the transformation matrices between the foot and the talus, and between the talus and the shank frames. From these data, a kinematics model of the foot was based on homogeneous matrix transformations in Euler angles. Taking into account the previously mentioned motions, it might seem conceivable that a serial robot would be able to meet the requirements [13] [14]. The size of the serial chain must be quite small and actuators should be mounted on the joints. This makes the robotic foot irregular with insufficient stiffness capabilities. To overcome this problem, we propose the use of a single actuator at the foot ankle which is linked to all other space points of the
foot using links and hence a predictive model be used to estimate the position of each point in space at a given time. However, Syrseloudis et al., [12] proposed a 2-DOF parallel robot with a passive serial kinematics chain, which constraints the movements and a parallel chain, which provides the movements. The objective is to incorporate the advantages of parallel robots as they have rigidity, high manipulability and heavy loads handling.

III. ELECTROMYOGRAM (EMG) SIGNAL MODEL

Henneberg [15] suggested that when pathologic conditions arise in the motor system, whether in the spinal cord, the motor neurons, the muscle, or the neuromuscular junctions, the characteristics of the electrical signals in the muscle change. If properly detected, processed and transmitted signals in the muscles can be used to determine the principal movements of the limb. EMG is the registration and analysis of muscle signals. Until recently, electromyograms were recorded primarily for exploratory or diagnostic purposes; however, with the advancement of bioelectric technology, electromyograms also have become a fundamental tool in achieving artificial control of limb movement, i.e., functional electrical stimulation (FES) and rehabilitation. These signals were modelled as the input signals to the controller for position and speed control of the actuator.

IV. THE ADAPTIVE CONTROL FOR ANKLE ACTUATION

The solution to the so-called ‘adaptive control’ problem is akin to the elusive search for the ‘Holy Grail’ in the context of feedback control system design [16]. There is need for the designed controller to be somehow predictive in nature since the environment of which the robotic prostheses will be exposed will be too complex to deduce an effective mathematical model. For the lower limb design, the amount of body weight to be supported by the mechanical structure will be variable, hence designing the controller with a wide tolerance might result in instability of the whole system. We therefore propose the inclusion of predictive control algorithm in the controller. Pankaj et al [17] suggested that adaptive control involves modifying the control law used by the controller to cope with the fact that the parameters of the system being controlled change drastically due to change in environmental conditions or change in system itself [18]. Garikayi et al [19] explained adaptive control as a technique that is based on the fundamental characteristic of adaptation of living organisms. The main thrust in robotic prosthesis will be trying to make the human body adapt to the lifeless electromechanical structure which will be attached to it. The robotic manipulator shall be discussed in a general philosophy for designing ‘robust’ adaptive multivariable feedback control systems for linear time-invariant (LTI) designs that include both unmodelled dynamics and uncertain real parameters in the plant state-space description. The overall benefits of adaptive control algorithm is that it continuously and automatically measures the dynamic behaviour of the robotic prosthetic limb, compares it with the desired output and uses the difference to vary adjustable system parameters or to generate an actuating signal in such a way so that optimal performance can be maintained regardless of system changes such as weight, speed and temperature of the limb and human body. According to Wong [20] and Garikayi et al [19] this is achievable as they defined adaptive control as the control method used by a controller which must adapt to a controlled system with parameters which vary, or are initially uncertain.

The adjective ‘adaptive’ refers to the fact that the real parameter uncertainty and performance requirements require the implementation of feedback architecture with better performance and greater complexity than that of the best possible fixed non-adaptive controller.

Adaptive control does not need a priori information about the bounds on these uncertain or time varying parameters; it is concerned with control law changes as the parameters changes. Garikayi et al [19] stated that adaptive control is the attempt to “redesign” the controller while online, by looking at its performance and changing its dynamics in an automatic way. Lyapunov stability is used to derive these update laws and show convergence criterion (typically persistent excitation). Adaptive control is the ultimate feedback law that looks at the process and the performance of the controller and reshapes the controller closed loop behaviour autonomously. According to Cheng et al [21] the foundation of adaptive control is parameter estimation, thus projection mathematics and normalization are commonly used to improve the robustness of estimation algorithms. Common methods of estimation include recursive least squares and gradient descent. Both of these methods provide update laws which are used to modify estimates in real time (i.e., as the system operates).

Usually these recursive least squares and gradient descent methods (estimation methods) adapt the controllers to both the process statics and dynamics. In special cases the adaptation can be limited to the static behaviour alone, leading to adaptive control based on characteristic curves for the steady-states or to extreme value control, optimizing the steady state [21] [19]. Hence, there are several ways to apply adaptive control algorithms. Adaptive Control is made by combining online parameter estimator based on current measurements and control actions and control law that recalculates the controller based on those parameters. There are two main modalities of adaptive control: direct and indirect cases. Indirect adaptive control, which is one of two distinct approaches for the control of dynamical designs with unknown parameters, as it is commonly used, consists of two stages. In the first stage, the parameters of the plant are estimated dynamically online using input-output information [22] [23]. At every instant of time, assuming that the estimates represent the true values of the robotic prosthesis parameters, the control parameters are computed to achieve desired overall system characteristic.

In contrast with this, in direct adaptive control the control parameters are adjusted continuously based on the error between the output of the plant and the output of the reference
model. The latter results in the overall system being described by a set of non-linear differential equations, this in turn makes the formulation of the stability problems of such systems relatively straightforward. Duarte [24] concluded that while the two methods have continued to flourish for over two decades, very little is known about the precise relation between them.

V. DEVELOPMENT OF THE MODEL REFERENCE ADAPTIVE SYSTEM

The model is based on the derived transformations of the serial link as illustrated in Figure 4. The main goal is to achieve the desired plantarflexion and dorsiflexion motions with great ease under different time varying conditions.

The ankle actuation system uses a DC motor coupled to the serial link mechanism. The independent variables of the model are the angles Θ₂ (dorsiflexion / plantarflexion), and Θ₃ (inversion / eversion) which were termed Θ (position) during controller design. The torque (T) generated by a DC motor is proportional to the armature current (Iₐ) and the strength of the magnetic field (H). The required torque varies according to the control respective ankle motions. The model was developed with unconstrained movement of the foot angle so as to manipulate the links on the serial robotic prosthetic limb as shown in Figure 3. However, there is a need to improve the torque capacity of the system without increasing the actuator size, the strength of the magnetic field cannot be altered on real time basis therefore the magnetic flux density was pre-set.

If B is the magnetic flux density or magnetic induction and M is the magnetisation coefficient of the material, taking material quality as a constant, we can deduce that:

\[ B = \frac{\mu_0}{\mu} \mu_0 = \mu_0 - M \]  (4)

Where, \( B = \mu_0 (H + M) \) and \( \mu_0 \) magnetic permeability of free space

\[ B = \mu H \]  (5)

\[ \mu = \mu_0 - K \]  (6)

Where, \( K \) is the relative permeability of the material.

Assuming that magnetic field is constant, motor torque (T) is only proportional to current (I) by a constant, \( K \), therefore:

\[ T = K I \]  (7)

The back e.m.f (e) is proportional to the angular velocity of the shaft by a constant factor (Kₑ) thus,

\[ e = Kₑ \dot{\Theta} \]  (8)

Where \( \dot{\Theta} \) is the angular velocity. However, \( K_e = K_m \), therefore \( K \) will represent both the motor torque constant and back e.m.f constant.

\[ K = K_e = K_m \]  (9)

Taking into consideration Newton’s Second Law for the generation of acceleration as a result of unbalanced forces we have:

\[ J \ddot{\Theta} + b \dot{\Theta} = K \nu \]  (10)

According to Kirchoff’s Voltage Law, where summation of all voltages in the closed loop of the actuator is equal to zero:

\[ V_L + V_R = V - e \]  (11)

Applying the Laplace Transform so as to determine the Transfer Function of the actuator:

\[ Js^2 \theta(s) + bs \theta(s) = K I(s) \]  (13)

\[ (Js + b)s \theta(s) = K I(s) \]  (14)

\[ (Ls + R)I(s) = V(s) - K s \theta(s) \]  (15)

Deriving the open loop transfer function by eliminating the current (I) from the above equation, therefore the rotational speed is the output and the armature voltage is considered the input.

\[ G(s) = \frac{\theta(s)}{V(s)} = \frac{K}{(Js + b)(Ls + R) + K^2} \]  (16)

The position of the shaft, which in fact will be the orientation of the foot since the angle turned is directly proportional to the distance moved in either dorsiflexion or plantarflexion motion, is achieved by integrating the speed i.e. by dividing the \( G(s) \) by \( s \).

\[ \frac{\theta}{V(s)} = \frac{K}{s((Js + b)(Ls + R) + K^2)} \]  (17)

The main objective is to position the foot at a desired angle very precisely. Thus the steady state error should be equal to zero when given a commanded signal from the controller. Even a steady state error due to an external disturbance should be eliminated to zero. The DC motor used for this design was a GM9X34 supplied in the GEARS-IDS™ kit with the following specifications.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Voltage</td>
<td>E</td>
<td>12V</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>Kₜ</td>
<td>18.2 x 10⁻³ Nm/A</td>
</tr>
<tr>
<td>Back emf Constant</td>
<td>Kₑ</td>
<td>18.2 x 10⁻³ V/rad/s</td>
</tr>
<tr>
<td>Resistance</td>
<td>Rₜ</td>
<td>0.83Ω</td>
</tr>
<tr>
<td>Inductance</td>
<td>L</td>
<td>0.63mH</td>
</tr>
<tr>
<td>No-Load Current</td>
<td>Iₛ</td>
<td>0.33A</td>
</tr>
<tr>
<td>Peak Current (Stall)</td>
<td>Iₚ</td>
<td>14.5A</td>
</tr>
<tr>
<td>Moment of inertia of the motor</td>
<td>J</td>
<td>3.228 x 10⁻⁶ Kgm²</td>
</tr>
<tr>
<td>Motor viscous friction constant</td>
<td>b</td>
<td>3.5077 x 10⁻² Nms</td>
</tr>
</tbody>
</table>

Using the Matlab commands:

\[ s = tf('s') \]

\[ G_{motor} = K/(s(Js + b)(Ls + R) + K^2) \]

we yield the following plant model as a continuous transfer function:

\[ G_{motor} = \frac{0.0274}{8.87 \times 10^{-12} s^3 + 1.291 \times 10^{-5} s^2 + 0.0007648 s} \]  (18)
A closed loop Transfer function of the system was later generated using the Matlab command 
\[
\text{sys}_{-}cl = feedback(G_{motor})
\]
and the resultant transfer function was as follows:
\[
\text{sys}_{-}cl = \frac{0.018}{8.878 \times 10^{-15}s^3 + 1.291 \times 10^{-5}s^2 + 0.0007648s + 0.018}
\]
(19)

The closed loop transfer function obtained was then used to achieve the MRAC of the foot ankle controller design. Model Reference Adaptive Control Modeling is a method of designing a closed loop controller with parameters that can be updated to change the response of the system to match a desired model. There were many different methods for designing such a controller; however the researcher chose to use the MIT rule in continuous time. In the design of a MRAC using the MIT rule, the designers selected:
• The reference model,
• The controller structure
• And the tuning gains for the adjustment mechanism.

MRAC designing begins by defining the tracking error, e. This is simply the difference between the plant output and the reference model input:
\[
e = y_{plant} - y_{model}
\]
(20)

From this error a cost function of theta \( (J (\theta)) \) can be formed. \( J \) is given as a function of theta, with theta being the parameter that will be adapted inside the controller. The choice of this cost function will later determine how the parameters are updated. Below is the cost function displayed:
\[
J(\theta) = \frac{1}{2} e^2 (\theta)
\]
(21)

To find out how to update the parameter theta, an equation for the change in theta was formed such that if the goal was to minimize this cost related to the error, it was sensible to move in the direction of the negative gradient of \( J \). This change in \( J \) is assumed to be proportional to the change in theta. Thus, the derivative of \( J \) with respect to theta is equal to the negative change in \( J \). The result for the cost function chosen previously above is:
\[
\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta \theta}
\]
(22)
Thus
\[
\frac{\delta J}{\delta \theta} = -\gamma \frac{\delta e}{\delta \theta}
\]
(23)
Where \( \frac{\delta e}{\delta \theta} \) is the sensitivity derivative of the system.

This relationship between the change in theta and the cost function is known as the MIT rule. The MIT rule is central to adaptive nature of the controller. The sensitivity derivative is the partial derivative of the error with respect to theta. This determines how the parameter theta will be updated. A controller may contain several different parameters that require updating. Some may be acting on the input(s). Others may be acting on the output(s). The sensitivity derivative would need to be calculated for each of these parameters. The effect of the cost function selected resulted in all sensitivity derivatives being multiplied by the error.

The Adaptive Feed-forward Gain, \( G_m \) and \( G_p \) was used as the model and plant transfer functions as shown in Figure 2 above; \( k_o \) and \( k_p \) was used as the model and plant constants respectively. The input to output relationship was determined based on the transfer functions within the feed-forward adaptive control strategy. The constant \( k \) for this plant was unknown. However, a reference model was formed with a desired value of \( k \), and through adaptation of a feed-forward gain, the response of the plant can be made to match this model. The reference model was therefore chosen as the plant multiplied by a desired constant \( k_o \):
\[
\frac{y(s)}{u(s)} = kG(s)
\]
(24)

However using the same cost function in equation (2)
\[
\frac{d\theta}{dt} = -\gamma \frac{\delta e}{\delta \theta}
\]
(25)
The error in equation (1) is then restated in terms of the transfer functions multiplied by their inputs.
\[
e = y - y_m
\]
(26)
\[
e = kGU - G_mU_c
\]
(27)
\[
e = kGU \theta U_c - k_pG_mU_c
\]
(28)
As can be seen, this expression for the error contains the parameter theta \( (\theta) \) which is to be updated. To determine the update rule, the sensitivity derivative was calculated and restated in terms of the model output:
\[
\frac{\delta e}{\delta \theta} = kGU_c
\]
(29)
But
\[
kGU_c = \frac{k}{k_o}y_m
\]
(30)
Therefore
\[
\frac{\delta e}{\delta \theta} = \frac{k}{k_o}y_m
\]
(31)
Finally, the MIT rule was applied to give an expression for updating theta. The constants \( k \) and \( k_o \) were combined into gamma.
\[
\frac{d\theta}{dt} = -\gamma \frac{k}{k_o}y_m e = -\gamma y_m e
\]
(32)
To tune this system, the values of \( k_o \) and gamma were varied. The MIT rule by itself does not guarantee convergence or stability. An MRAC designed using the MIT rule was very
sensitive to the amplitudes of the signals. As a general rule, the value of \( \gamma \) was kept small. Tuning of \( \gamma \) was crucial to the adaptation rate and stability of the controller. It was then assumed that the controller has both an adaptive feedforward \( \Theta_1 \) and an adaptive feedback \( \Theta_2 \) gain as illustrated in Figure 3.

To derive expressions for the sensitivity derivatives associated with these parameters, the error function was restated to include \( \Theta_1 \) and \( \Theta_2 \). The equation for the error was first rewritten as the transfer function of the plant and model multiplied by their respective inputs. The input \( U_c \) is not a function of either of the adaptive parameters, and therefore can be ignored for now. However, the input \( U \) was rewritten using the feed forward and feedback gains. This was used to derive an equation for \( Y_{plant} \).

The EMG signal from the muscles will be filtered to such an extent that only the muscles signals (EMG) responsible for the sagittal plane movements will be harnessed. The signal will be mapped onto the desired range of positioning (\( \Theta_{min} - \Theta_{max} \)), where \( \Theta_{min} \) represents the minimum value during plantarflexion while \( \Theta_{max} \) represents the maximum value during dorsiflexion. The control signal within the range of 4-20mA will be used to actuate the ankle actuator. The output of the controller is the position and speed.

The adaptive controller for this system was done with respect to the following form:

\[
\begin{align*}
    u &= \theta_1 u_c - \theta_2 y_{plant} \\
    e &= y_{plant} - y_{model} = G_p u - G_m u_c \\
    y_{plant} &= G_p u = \left(\frac{0.018}{0.8s^2 + 1.291s^2 + 7.648s + 0.018}\right) (\theta_1 u_c - \theta_2 y_{plant}) \\
    \frac{\delta e}{\delta \theta_1} &= \frac{0.018 \theta_1}{0.8s^2 + 1.291s^2 + 7.648s + 0.018} u_c \\
    \frac{\delta e}{\delta \theta_2} &= \frac{0.018 \theta_2}{0.8s^2 + 1.291s^2 + 7.648s + 0.018} y_{plant}
\end{align*}
\]

The sensitivity derivatives obtained contain the parameters from the plant. The premise of design with MRAC assumes that the plant characteristics were not absolutely known. This seemingly places the design process at a dead end. However, the goal was to make the plant approach the model. If the model is close to the actual plant, the model characteristics can be substituted for the plant characteristics, giving the following sensitivity derivatives:

\[
\frac{\delta e}{\delta \theta_1} = \frac{0.018 \theta_1}{s^2 + a_{1m}s + a_{0m}} u_c
\]

Taking the derivative of the feed forward loop of the MRAC we have:

\[
\frac{\delta e}{\delta \theta_2} = -\frac{a_{1m}s + a_{0m}}{s^2 + a_{1m}s + a_{0m}} y_{plant}
\]

Then, applying the MIT rule, the update rules for each Theta was written. The block diagram for the system with the derived controller is shown in Figure 4.

![Image](image_url)

The plant input and output measurements were filtered before being multiplied by the error and gamma.

\[
\begin{align*}
    \frac{d\theta_1}{dt} &= -\gamma \frac{\delta e}{\delta \theta_1} e = -\gamma \left(\frac{a_{1m}s + a_{0m}}{s^2 + a_{1m}s + a_{0m}} u_c\right) e \\
    \frac{d\theta_2}{dt} &= -\gamma \frac{\delta e}{\delta \theta_2} e = \gamma \left(\frac{a_{1m}s + a_{0m}}{s^2 + a_{1m}s + a_{0m}} y_{plant}\right) e
\end{align*}
\]

The system was designed and tested for different desired positions including the maximum and minimum angle position of the sagittal plane.

VI. RESULTS

The desired MRAC system was developed and the results proved that for a single product such a robotic prosthetic leg the transient response of the system to dynamic changes is desirable as illustrated in Figure 5. The main goal for doing the research was to check the applicability of adaptive control algorithms in the control of the human ankle. The results reveal that it is possible to apply Model Reference adaptive control (MRAC) on the sagittal plane where the upward and downward movement of the foot angle is achieved; however, the system has a notable dead time on rapid response to a unit step input due to mechanical inertia. Further adjustments to the system were done with aid of a PID embedded algorithm within the Simulink block but this only resulted in an over-damped system i.e. an exponential decay with no oscillations hence increased transient response. The designed controller has a long rise time, hence to reduce the rise-time the actual
reference model was designed as a critically damped system which is characterized by a slight overshoot, very short rise time and no steady state error. Thus the authors recommend that the MRAC system be implemented if and only if the system is hybridized with an intelligent controller such as ANN or Fuzzy Logic.

![Fig. 5 Plant and Model outputs](image)

The reluctance of the system to start moving after receiving a command signal from the controller as illustrated in Figure 5 has caused the dead time of 3 seconds. Since inertia force always opposes the driving force the delay in response is acceptable, however it is recommended that the error value illustrated in Figure 5 be reduced from 6 seconds to 3 seconds since this anomaly signifies the responsiveness of the foot to sudden changes in desired angle and motion.

VII. CONCLUSION

The A model reference adaptive control system was developed and tested on its response to desired angles. The main objective was to determine whether a MRAC servo system is applicable for rapid response electromechanical systems such as ankle control of the lower limb. The system has a delay to response due to mechanical inertia within the electromechanical system. The developed system had no steady state error. To a greater extend the desired objectives were achieved; however there is need to develop a system to eliminate the delay due to inertia force thereby increasing the accuracy and resolution of the system.

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REFERENCES


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