Pricing and ordering policies for price-dependent demand in a supply chain of a single retailer and a single manufacturer

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This article discusses joint pricing and ordering policies for price-dependent demand in a supply chain consisting of a single retailer and a single manufacturer. The retailer places orders for products according to an EOQ policy and the manufacturer produces them on a lot-for-lot basis. Four mechanisms with differing levels of coordination are presented. Mathematical models are formulated and solution procedures are developed to determine the optimal retail prices and order quantities. Through extensive numerical experiments, we analyse and compare the behaviours and characteristics of the proposed mechanisms, and find that enhancing the level of coordination has important benefits for the supply chain.

Keywords: pricing and ordering policies; supply chain management; coordination mechanisms

1. Introduction

The economic order quantity (EOQ) model is a problem in Operations Management that has been extended to apply to newly emerging business environments by many researchers over several decades. Among those works, we are interested in price-inventory models for use in the study of interactions between pricing and ordering policies. Today, markets are very competitive and demand is sensitive to the price. Given that variations in demand significantly influence cost and revenue structures, firms contemplating a price adjustment must consider the resulting variation in demand. Furthermore, as firms compete in the global market, single firms in isolation are finding it increasingly difficult to survive and firm-to-firm competition is rapidly changing to supply-chain-to-supply-chain competition in the global market. This article discusses pricing and ordering policies under price-dependent demand in a supply chain, and examines the behaviours and efficiencies of the proposed coordination mechanisms.

Kunreuther and Richard (1971) initiated a price-inventory EOQ model for a linear price-demand function. Arcelus and Srinivasan (1985) presented an EOQ model maximising return on investment for a constant elasticity demand function. Abad (1988a,b) presented mathematical procedures to determine the optimal prices and order quantities for a linear price-demand function and a constant elasticity demand function when the supplier offers quantity discounts. Subsequently, Burwell, Dave, Fitzpatrick, and Roy (1991) extended Abad’s model by allowing for shortages. Rosenberg (1991) studied price-inventory models for a logarithmic concave demand function, and developed optimal solutions to the economic theory of the firm (ETF), profit and return on inventory investment (ROII) models. Lee (1993) proposed a geometric programming approach to find the retail price and the order quantity for a constant elasticity demand function. In a further extension of Abad’s model (1988a,b), Burwell, Dave, Fitzpatrick, and Roy (1997) incorporated freight discount into the model. Ray, Gerchak, and Jewkes (2005) analysed behaviours of a price-inventory model under two different pricing policies: independent decision and mark-up pricing. Dye (2007) developed a deterministic price-inventory model for deteriorating items with a time-dependent backlogging rate and Dye, Ouyang, and Hsieh (2007) extended this model to the case of maximising the net present value of profit. Tsao and Sheen (2007) considered a model with deteriorating items over a finite time horizon under conditions where the purchasing cost pattern is influenced by the lot size. However, all of the above studies examined price-inventory models at retailers only.

Increasing concerns about supply chain management have prompted examinations of coordination issues among supply chain members. Abad (1994)
studied the problem of coordination between a vendor and a buyer and showed that a lot-for-lot policy is optimal in the case where the vendor and buyer adopt equal inventory carrying cost rates. Parlar and Wang (1994) developed a model for a single supplier and a single buyer that uses a constant profit margin pricing policy. Weng (1995) presented an extensive discussion of the impacts of quantity discount policies on channel coordination in a system consisting of a supplier and a group of homogeneous buyers. His contribution was to analyse the role and control mechanism of quantity discounts in pricing and inventory policies in the context of channel coordination. Chen, Federgruen, and Zheng (2001) extended Weng’s discussion to the case of a single wholesaler and heterogeneous retailers. They showed that coordination can be achieved by a quantity discount and volume discount with a franchise fee. Boyaci and Gallego (2002) proposed a supply chain model to account for the impact of transfer prices and ownership of retail inventory with delayed payments and consignment for inventory. Ertek and Griffin (2002) analysed the impact of power structure on price, sensitivity of market price and profits in a two-stage supply chain. Abad and Jaggi (2003) considered a model in which a credit period is offered by the supplier and proposed a solution procedure based on the Pareto efficient solution for a cooperative game. Viswanathan and Wang (2003) introduced volume discounts and showed that simultaneously offering volume and quantity discounts can successfully coordinate the channel. Note that all of the above-mentioned studies assumed that the supplier has an infinite replenishment rate.

Under the assumption that the manufacturer has a finite production rate, Reyniers (2001) studied a vertical integration problem between a manufacturer and a retailer, and determined the optimal retail price and order quantity assuming that the manufacturer can set the production rate equal to the price-dependent demand rate. Banerjee (2005) dealt with the price decision problem and derived the optimal vendor price that minimises the system-wide cost assuming fixed final demand. Chen and Chen (2007) modelled a multi-product and multi-echelon supply chain in which the manufacturer produces perishable items.

The majority of papers discussing coordination mechanisms in supply chain management assume that members are willing to coordinate fully for the benefit of the supply chain. However, firms are primarily concerned with optimising their own profits, making it difficult to initiate full coordination and maintain for long periods of time. Instead, partial coordination systems can be implemented. Rosenberg (1991) introduced the ETF model, in which the retail price is determined first, and then the ordering policy is obtained based on the retail price. The vendor managed inventory (VMI) model is based on another partial coordination mechanism, in which the manufacturer takes full responsibility for maintaining the inventory at the retailer. The VMI model has attracted considerable interest from researchers and practitioners as a practical coordination strategy. Dong and Xu (2002) evaluated the influence of the VMI strategy in a supply chain consisting of a wholesaler and a retailer, and emphasised the importance of the VMI strategy as an effective alternative mechanism. Yu, Liang, and George (2006) proposed a Stackelberg game model in a VMI supply chain and Yu, Chu, and Chen (2009) extended the model to the case in which the manufacturer procures and stocks the raw material. Yao, Evers, and Dresner (2007) developed an analytical model that explores how supply chain parameters affect the cost saving to be realised from use of the VMI strategy. Gümüş, Jewkes, and Bookbinder (2008) analysed the consignment inventory and VMI in a supply channel under deterministic demand and provided some general conditions under which the strategies create benefits for each entity.

This article analyses the pricing and ordering policies under the conditions of price-dependent demand in a supply chain consisting of a single retailer and a single manufacturer. The retailer places orders based on the EOQ model, and the manufacturer produces the ordered quantity on a lot-for-lot basis. Although the lot-for-lot assumption is restrictive, it is easy to implement and widely used in practice, and provides insights for the proposed models. In Section 2, we present mathematical models depending on the level of coordination and develop procedures to obtain the optimal retail prices and order quantities for the proposed four coordination mechanisms. Extensive numerical experiments are executed and the behaviours of the proposed mechanisms are analysed and compared in Section 3. Conclusions are given in Section 4.

2. Models and analyses

Consider a supply chain consisting of a single retailer and a single manufacturer. Demand at the retailer is a decreasing function of the retail price, and is assumed to be deterministic and constant over time for a given retail price. The retailer places orders to the manufacturer based on a classical EOQ policy, and the manufacturer produces and delivers the ordered quantities according to a lot-for-lot policy. The parameters and decision variables necessary for
the mathematical models are as follows:

\[
\begin{align*}
R & \quad \text{production rate at the manufacturer} \\
c & \quad \text{unit production cost at the manufacturer} \\
S & \quad \text{setup cost at the manufacturer} \\
A & \quad \text{ordering cost at the retailer} \\
h_m & \quad \text{inventory holding cost per unit per year at the manufacturer} \\
h_r & \quad \text{inventory holding cost per unit per year at the retailer} \\
p & \quad \text{unit retail price at the retailer} \\
Q & \quad \text{order quantity at the retailer} \\
w & \quad \text{unit wholesale price at the manufacturer}
\end{align*}
\]

It is assumed that the demand function at the retailer is a linearly decreasing function of the retail price \(p\) and can be expressed as \(D(p) = a - bp\), \(a, b > 0\) where \(a\) and \(b\) are a market sizable factor and a price sensitiveness parameter, respectively. We also assume that \(R \geq D(0)\), which implies that the manufacturer has sufficient production capacity to meet the demand at the retailer. Figure 1 shows sample inventory trajectories at the retailer and the manufacturer for a fixed retail price.

The average profit functions of the retailer and the manufacturer are defined, respectively, by

\[
\begin{align*}
\Pi_R(p, Q) & = (p - w)D(p) - A \frac{Q}{R} D(p) - h_r Q \frac{A}{2}, \\
\Pi_M(w) & = (w - c)D(p) - S \frac{Q}{R} D(p) - h_m Q \frac{A}{2}.
\end{align*}
\]

The average inventory at the manufacturer is expressed as \(1/2 \{Q/R\} Q/D(p) = (Q/2) D(p)/R\) (Figure 1). In each of these equations, the first term represents the sales margin and the second and third terms are the average setup (ordering) cost and holding cost, respectively.

### 2.1. Full coordination

First, we consider a full coordination (FC) mechanism, in which the retailer and manufacturer are under single ownership. Then, the average joint profit function for the supply chain can be expressed by

\[
\Pi(p, Q) = \Pi_R(p, Q) + \Pi_M(w)
\]

\[
= \left(p - c - \frac{S + A}{Q} - \frac{h_m Q}{2R}\right) D(p) - \frac{h_r Q}{2}.
\]

Our objective is to determine the optimal retail price \(p^*\) and order quantity \(Q^*\) that maximise the average joint profit function in Equation (3).

First, we assume that \(Q\) is given. Substituting \(D(p) = a - bp\) into Equation (3), we find that \(\Pi(p, Q)\) is strictly concave in \(p\) since \(d^2 \Pi(p, Q)/dp^2 = -2b < 0\). Thus, the optimal retail price, obtained by solving \(d\Pi(p, Q)/dp = 0\), is

\[
p^*(Q) = \frac{1}{2} \left(\frac{h_m Q}{2R} + \left(\frac{a}{b} + c\right) + \left(\frac{S + A}{Q}\right)\right).
\]

It is obvious that \(p^*(Q)\) must satisfy \(D(p^*(Q)) = a - bp^* \geq 0\), which gives the following feasibility condition for \(Q\):

\[
f(Q) = \frac{h_m}{2R} Q^2 - \frac{(a - b - c)}{b} Q + \frac{(S + A)}{Q} \leq 0.
\]

Letting \(M\) be the discriminant of \(f(Q)\), we obtain \(M = (a/b - c)^2 - 2(S + A)h_m/R\). If \(M < 0\), the order quantity satisfying the feasibility condition in Equation (5) does not exist and \(D(p^*(Q)) = 0\) if \(M = 0\). Therefore, it is obvious that the proposed problem is not relevant when \(M \leq 0\). When \(M > 0\), \(f(Q) = 0\) has two positive solutions, \(Q_1\) and \(Q_2\) \((0 < Q_1 < Q_2)\), and the feasibility condition in Equation (5) is satisfied for \(Q\) values in the range \(Q_L \leq Q \leq Q_U\).

Substituting \(p^*(Q)\) into Equation (3), the average joint profit can be expressed as a function of \(Q\) only as follows:

\[
\Pi_1(Q) = \Pi(p^*(Q), Q) = \frac{b}{4} \left\{ \frac{h_m Q}{2R} - \left(\frac{a}{b} - c\right) \right\} + \frac{(S + A)}{Q} - \frac{h_r Q}{2}.
\]

The first and second derivatives of \(\Pi_1(Q)\) with respect to \(Q\) are, respectively, given by

\[
\begin{align*}
\frac{d}{dQ} \Pi_1(Q) & = \frac{b}{2} \left\{ \frac{h_m Q}{2R} - \left(\frac{a}{b} - c\right) + \frac{(S + A)}{Q}\right\} \\
& \times \left\{ \frac{h_m Q}{2R} - \left(\frac{S + A}{Q^2}\right) - \frac{h_r Q}{2}\right\} \\
& = \frac{1}{2Q^3} \left[ h_m b 4R Q^2 - \left( h_r + h_m \frac{(a - bc)}{2R}\right) Q^3 \right. \\
& + (a - bc)(S + A)Q - b(S + A)^2 \bigg],
\end{align*}
\]

\[
\begin{align*}
\frac{d^2}{dQ^2} \Pi_1(Q) & = \frac{b}{2} \left\{ \frac{h_m Q}{2R} - \left(\frac{S + A}{Q^2}\right) + \frac{2(S + A)}{Q^3}\right\} \\
& \times \left\{ \frac{h_m Q}{2R} - \left(\frac{a}{b} - c\right) + \left(\frac{S + A}{Q}\right)\right\}.
\end{align*}
\]
According to Descartes’ rule of signs, we know that $d\Pi_1(Q)/dQ = 0$ has either one or three positive roots since there are three sign changes in the coefficients of Equation (7). In addition, we see that

1. $\Pi_1(Q_L) = -h_rQ_L/2 < 0$ and $\Pi_1(Q_U) = -h_rQ_U/2 < 0$,
2. $d\Pi_1(Q_L)/dQ = d\Pi_1(Q_U)/dQ = -h_r/2 < 0$,
3. $d^2\Pi_1(Q)/dQ^2 > 0$ for $0 < Q \leq Q_L$ and $Q \geq Q_U$.

Hence, $d\Pi_1(Q)/dQ$ and $\Pi_1(Q)$ can have one of three shapes, as shown in Figure 2. When $d\Pi_1(Q)/dQ = 0$ has a single positive root, $\Pi_1(Q) = -h_rQ/2 < 0$ and $\Pi_1(Q)$ is decreasing in the feasible region ($Q_L \leq Q \leq Q_U$), and there is no feasible order quantity yielding a positive joint profit (Figure 2(a)). When $d\Pi_1(Q)/dQ = 0$ has three positive roots, which we denote $Q_1$, $Q_2$, and $Q_3$ ($Q_1 < Q_2 < Q_3$), $\Pi_1(Q)$ has one of two shapes where $Q_1$ and $Q_3$ are minimum points and $Q_2$ is a maximum point as shown in Figure 2(b) and (c). However, if $\Pi_1(Q_2) \leq 0$, there exists no feasible order quantity giving a positive joint profit (Figure 2(b)). Only when $Q_2$ satisfies $\Pi_1(Q_2) > 0$, a positive maximum average joint profit can be achieved at $Q^* = Q_2$ (Figure 2(c)). After $Q^*$ is obtained by the procedure outlined above, the optimal retail price can be determined using Equation (4). The detailed procedure for determining the optimal solution can be summarised as follows:

**Step 1:** Compute $M$ using Equation (5). If $M \leq 0$, stop. The problem is not relevant.

**Step 2:** Solve Equation (7) numerically. If it has three positive roots ($Q_1 < Q_2 < Q_3$) and $\Pi_1(Q_2) > 0$, $Q^* = Q_2$ is the optimal order quantity. Otherwise, the problem is not relevant since no feasible order quantity giving a positive average joint profit exists.

**Step 3:** Determine the optimal retail price $p^*(Q^*)$ using Equation (4).

### 2.2. Partial coordination

It is well known that supply chains with $FC$ among the supply chain members yield greater profits than those without $FC$. However, in the real world, $FC$ is not easy to implement, even within a single company. In this section, two partial coordination models are introduced and discussed. In the first model, which we call ‘PC1’, the supply chain is managed by a single ownership, however, the marketing department determines the retail price first, and then the purchasing department determines the ordering policy. This model is similar to the ETF model discussed by Rosenberg (1991) and is plausible when the sum of the direct cost covering setup and inventory carrying cost is relatively lower than the overall sales margin. The second model (PC2) is similar to the VMI model, in which the retailer and the manufacturer are independent of each other. The retailer provides full inventory- and market-related information to the manufacturer and the manufacturer takes full responsibility for maintaining the inventory.

Consider ‘PC1’ first. The sales margin and the average joint cost function of the supply chain are,
respectively, given by

\[ SM(p) = (p - c)D(p), \]

\[ TC(Q) = (S + A)D(p^*) + \frac{Q}{2} \left( h_r + h_m \frac{D(p^*)}{R} \right). \]  

First, the marketing department sets the optimal retail price \( p^* \) of a product to maximise the sales margin without considering the operating costs such as inventory and setup cost. Sequentially, the purchasing department determines the optimal order quantity \( Q^* \) to minimise the average joint cost and places orders to the manufacturing department. Since the sales margin, \( SM(p) \), is concave in \( p \), the optimal retail price can be obtained by \( p^* = (a + bc)/2b \) from the first-order condition, and the demand at the retailer is fixed at \( D(p^*) = (a - bc)/2 \). For a given demand \( D(p^*) \), the optimal order quantity that minimises the average joint cost function in Equation (10) is derived by solving \( dTC(Q)/dQ = 0 \) and is given by \( Q^* = \sqrt{2(S + A)}D(p^*)(h_r + h_mD(p^*)/R) \). The net profit for the supply chain is expressed by

\[ NP(p^*, Q^*) = (p^* - c)D(p^*) - \sqrt{2(S + A)}D(p^*)(h_r + h_mD(p^*)/R). \]

Next, the model ‘PC2’ is discussed. Letting \( w \) be the manufacturer’s wholesale price, the retailer’s profit function is given by

\[ \Pi_R(p) = (p - w)D(p). \]  

The optimal retail price \( p^* \) that maximises the retailer’s profit function in Equation (11) is obtained from \( d\Pi_R(p)/dp = 0 \) as \( p^*(w) = (a + bw)/2b \). Then the profit function at the manufacturer is arranged as in Equation (12).

\[ \Pi_M(w, Q) = \left( w - c - \frac{S + A}{Q} - \frac{h_mQ(w)}{2R} \right)D(p^*(w)) - \frac{h_rQ}{2}. \]  

Equation (12) has a similar structure to Equation (3). Hence, the optimal wholesale price \( w^* \) and order quantity \( Q^* \) that maximise \( \Pi_M(w, Q) \) can be derived by following the same procedure as in Section 2.1.

### 2.3. Non-coordination

Despite a rapid increase in interest in coordination issues in recent times, non-coordination (NC) still prevails in the real world. In other words, the retailer and the manufacturer act independently, each trying to maximise its own profit. We assume that the manufacturer has full information of the retailer. The retailer tries to maximise its profit function, which is given by

\[ \Pi_R(p, Q) = \left( p - w - \frac{A}{Q} \right)D(p) - \frac{h_rQ}{2}. \]  

Assuming that the wholesale price \( w \) is given, then the optimal retail price can be expressed as \( p^*(Q) = (a/b) + w + (A/Q)/2 \). Further, \( p^*(Q) \) must satisfy \( D(p^*(Q)) = a - lp^*(Q) \geq 0 \), which gives the feasible region for \( Q \) as \( Q \geq Q_1 = A/(a/b - w) \). Substituting \( p^*(Q) \) into Equation (13), the retailer’s profit function is

\[ \Pi_R(p^*(Q), Q) = \frac{b}{4} \left( \frac{a}{b} - w - \frac{A}{Q} \right)^2 - \frac{h_rQ}{2}. \]  

Since Equation (14) has a structure similar to that of \( \Pi_I(Q) \) in Equation (6), we can derive the optimal order quantity \( Q^* \) by following a procedure similar to that discussed in detail in Section 2.1. Next, the manufacturer’s profit function is arranged as a function of \( w \), that is

\[ \Pi_M(w) = \left( (w - c) - \frac{S}{Q^*(w)} - \frac{h_mQ^*(w)}{2R} \right)D(p^*(w)). \]  

The manufacturer’s objective is to determine the optimal wholesale price \( w^* \) that maximises its own profit in Equation (15). However, it is not easy to derive \( w^* \) analytically. We know that the wholesale price must exist in the range \( c < w < a/b \). Therefore, using a numerical approach, the optimal wholesale price can be obtained without much difficulty.

### 3. Numerical experiment

We propose four coordination mechanisms between a retailer and a manufacturer, denoted FC, PC1, PC2 and NC, and carry out numerical experiments to analyse and compare the behaviours of the proposed mechanisms. In the first two mechanisms, FC and PC1, the retailer and manufacturer belong to a single organisation and work together for the benefit of the whole supply chain. In the other two mechanisms, PC2 and NC, the retailer and manufacturer work independently to maximise their respective profits. The basic set of parameters used in the experiment is listed in Table 1, and the corresponding optimal solutions and profits for the four mechanisms are presented in Table 2.

Even though FC gives slightly more profit than PC1, the two mechanisms show similar behaviours. However, FC and PC1 generate much higher profits than PC2 and NC, in which the retailer and manufacturer independently pursue their own profits without considering the effects of their decisions...
on the other's profit. The markedly better performance of the mechanisms involving coordination can be explained in terms of the double marginalisation in the vertical integration. Table 2 shows that the optimal order quantity in NC is considerably smaller than in the other mechanisms. This arises because in NC the retailer determines the optimal order quantity based on its own setup cost and inventory holding cost, whereas in the other mechanisms the optimal order quantities are determined considering those direct costs jointly in the retailer as well as in the manufacturer.

Next, we conducted numerical experiments to examine the behaviours of the proposed coordination mechanisms under changes in the price sensitivity ($b$). Three values of the price sensitivity were used ($b = 20, 25, 30$) while fixing all other parameters to the values given in Table 1. The results are listed in Table 3, where percentage values represent the profits relative to the profit for the FC mechanism. For all four coordination mechanisms, profits decrease as the price sensitivity increases. However, the percentage differences in profits between the four coordination mechanisms increase as the price sensitivity increases. This implies that, when demand is sensitive to the retail price, coordination becomes more important for the benefit of the whole supply chain.

Tables 4 and 5 show the results of a sensitivity analysis of the setup cost and inventory holding cost in two coordination mechanisms, FC and PC1, respectively. Experiments are executed for three values of the setup cost ($S = 1000, 2000, 3000$) and three values of the inventory holding cost ($h_m = 2, 4, 6$). We see that, when the setup cost and/or the inventory holding cost are relatively low, the profits are similar for FC and PC1. However, as these costs increase, the differences in profits between the two mechanisms increase. This is partially due to the fact that the optimal retail price ($32.50$) in PC1 is independent of the setup cost and inventory holding cost, whereas the optimal retail price in FC is affected by these costs.

Tables 6 and 7 show the results of a similar sensitivity analysis for the other two coordination mechanisms, PC2 and NC. As the setup cost increases, we observe that (1) the proportion of the manufacturer's profit to the total profit decreases, and (2) the manufacturer's profit decreases more rapidly in NC than in PC2. This indicates that under these mechanisms, the manufacturer is affected more than the retailer by changes in the setup cost. One interesting observation is that the optimal order quantity in

### Table 1. Basic set of parameters.

<table>
<thead>
<tr>
<th>Production rate ($R$)</th>
<th>Unit production cost ($c$)</th>
<th>Setup cost ($S$)</th>
<th>Ordering cost ($A$)</th>
<th>Inventory holding cost</th>
<th>Demand function</th>
<th>Demand function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>$15$</td>
<td>$1000$</td>
<td>$300$</td>
<td>$2$</td>
<td>$3$</td>
<td>$1000$ $20$</td>
</tr>
</tbody>
</table>

### Table 2. Solutions for four coordination mechanisms for basic set of parameters.

<table>
<thead>
<tr>
<th></th>
<th>FC</th>
<th>PC1</th>
<th>PC2</th>
<th>NC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order quantity</td>
<td>476.84</td>
<td>495.93</td>
<td>348.53</td>
<td>176.16</td>
</tr>
<tr>
<td>Retail price ($)</td>
<td>34.10</td>
<td>32.50</td>
<td>42.27</td>
<td>42.24</td>
</tr>
<tr>
<td>Wholesale price ($)</td>
<td>–</td>
<td>–</td>
<td>34.54</td>
<td>32.78</td>
</tr>
<tr>
<td>Retailer’s profit ($)</td>
<td>–</td>
<td>–</td>
<td>1195.18</td>
<td>939.65</td>
</tr>
<tr>
<td>Manufacturer’s profit ($)</td>
<td>–</td>
<td>–</td>
<td>1867.56</td>
<td>1850.77</td>
</tr>
<tr>
<td>Total profit (% ratio to FC)</td>
<td>$4339.95 (100)</td>
<td>$4290.06 (98.85)</td>
<td>$3062.73 (70.57)</td>
<td>$2790.42 (64.30)</td>
</tr>
</tbody>
</table>

### Table 3. Sensitivity analysis for price sensitivity ($b$).

<table>
<thead>
<tr>
<th>$b$</th>
<th>Total profit (% ratio to FC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$4339.95 (100)$</td>
</tr>
<tr>
<td>25</td>
<td>$2257.59 (100)$</td>
</tr>
<tr>
<td>30</td>
<td>$1017.30 (100)$</td>
</tr>
</tbody>
</table>
Table 4. Sensitivity analysis for setup cost \((S)\) in \(FC\) and \(PC1\).

<table>
<thead>
<tr>
<th>(S = $1000)</th>
<th>(S = $2000)</th>
<th>(S = $3000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(FC)</td>
<td>(PC1)</td>
<td>(FC)</td>
</tr>
<tr>
<td>Order quantity</td>
<td>476.84</td>
<td>495.93</td>
</tr>
<tr>
<td>Retail price ($)</td>
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<td>32.50</td>
</tr>
<tr>
<td>Total profit</td>
<td>$4339.95</td>
<td>$4290.06</td>
</tr>
<tr>
<td>(% ratio to (FC))</td>
<td>(100)</td>
<td>(98.85)</td>
</tr>
</tbody>
</table>

Table 5. Sensitivity analysis for inventory holding cost \((hm)\) in \(FC\) and \(PC1\).

<table>
<thead>
<tr>
<th>(hm = $2)</th>
<th>(hm = $4)</th>
<th>(hm = $6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(FC)</td>
<td>(PC1)</td>
<td>(FC)</td>
</tr>
<tr>
<td>Order quantity</td>
<td>476.84</td>
<td>495.93</td>
</tr>
<tr>
<td>Retail price ($)</td>
<td>34.10</td>
<td>32.50</td>
</tr>
<tr>
<td>Total profit</td>
<td>$4339.95</td>
<td>$4290.06</td>
</tr>
<tr>
<td>(% ratio to (FC))</td>
<td>(100)</td>
<td>(98.85)</td>
</tr>
</tbody>
</table>

Note: \(hr = 1.5 \times hm\).

Table 6. Sensitivity analysis for setup cost \((S)\) in \(PC2\) and \(NC\).

<table>
<thead>
<tr>
<th>(S = $1000)</th>
<th>(S = $2000)</th>
<th>(S = $3000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(PC2)</td>
<td>(NC)</td>
<td>(PC2)</td>
</tr>
<tr>
<td>Order quantity</td>
<td>348.53</td>
<td>176.16</td>
</tr>
<tr>
<td>Retail price ($)</td>
<td>42.27</td>
<td>42.24</td>
</tr>
<tr>
<td>Wholesale price ($)</td>
<td>34.54</td>
<td>32.78</td>
</tr>
<tr>
<td>Retailer’s profit ($)</td>
<td>1137.18</td>
<td>939.65</td>
</tr>
<tr>
<td>Manufacturer’s profit</td>
<td>$1867.56</td>
<td>$1850.77</td>
</tr>
<tr>
<td>(% ratio to total profit)</td>
<td>(60.98)</td>
<td>(66.33)</td>
</tr>
<tr>
<td>Total profit ($)</td>
<td>3062.73</td>
<td>2790.42</td>
</tr>
</tbody>
</table>

Table 7. Sensitivity analysis for inventory holding cost \((hm)\) in \(PC2\) and \(NC\).

<table>
<thead>
<tr>
<th>(hm = $2)</th>
<th>(hm = $4)</th>
<th>(hm = $6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(PC2)</td>
<td>(NC)</td>
<td>(PC2)</td>
</tr>
<tr>
<td>Order quantity</td>
<td>348.53</td>
<td>176.16</td>
</tr>
<tr>
<td>Retail price ($)</td>
<td>42.27</td>
<td>42.24</td>
</tr>
<tr>
<td>Wholesale price ($)</td>
<td>34.54</td>
<td>32.78</td>
</tr>
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Note: \(hr = 1.5 \times hm\).
NC decreases as the setup cost increases. A higher setup cost incurs a higher wholesale price, which in turn incurs a higher retail price, thus leading to a demand reduction and hence a smaller optimal order quantity. As the inventory holding cost increases, the retailer receives increasingly more profits under PC2 than under NC, whereas the difference in manufacturer’s profits between the two mechanisms remains relatively unchanged. This shows that it is beneficial for the retailer to move from NC to PC2 when the inventory holding cost is relatively high.

4. Conclusions
This article discusses pricing and ordering policies for price-dependent demand under four different coordination mechanisms in a supply chain consisting of a single retailer and a single manufacturer. Mathematical models for the four coordination mechanisms are presented and procedures for determining the optimal solutions and the corresponding profit functions are discussed. Through extensive numerical experiments, the behaviours and characteristics of the proposed model are analysed and discussed. The proposed model can be extended to more generalised supply chain systems, such as multiple retailers, more complex inventory policies, different types of demand function and so on.

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References


