Abstract—In this paper, we present a control design for the tele-operation of a miniature unmanned aerial vehicle known as an X4-flyer. A simple dynamic nonlinear model for the vehicle, valid for quasi-stationary flight conditions, is derived as a basis for the control design. An attitude control based on information issued from an Inertial Measurement Unit is designed. In order to control the vehicle altitude, an adaptive controller avoiding the ground effects and based on measurements issued from an ultrasonic low cost sensor is designed. In order to compute the altitude velocity, an estimator based on the proposed modelling is used. At the end of the paper, experimental results are presented.

I. INTRODUCTION

The interest for controlling the small scale remote control hovering system suited for stationary or quasi-stationary flight increases more and more particularly in military field or for the surveillance and inspection in dangerous or awkward environments [1]. For example, one can imagine the use of such vehicles in order to explore a contaminated area before a human intervention. In this paper, we are interested in the remote control problem of an unmanned aerial vehicle known as the “X4-flyer” (fig. 1) which is a four rotors vertical take off and landing (VTOL) vehicle capable to perform stationary or quasi stationary flights. The mechanical simplicity of the “X4-flyer” compared to a classical helicopter allows to have a reliable and commercially exploitable system. However, the remote control of such a vehicle is very difficult for a neophyte and need several hours of training for being able to success a stationary flight. In this paper, we are particulary interested in the design of a controller which stabilizes the attitude and altitude of the vehicle from an Inertial Measurement Unit (IMU) and a telemetric sensor, and allows to a neophyte to control the X4-flyer by sending intuitive orders to move in 3D-space. Consequently, we suggest to control the mini rotor craft in controlling its attitude in order to generate accelerations. To stabilize the vehicle in altitude, an altitude controller based on ultrasonic measurements is designed. The main problem in designing such a controller is comes from the unknown variations of the global thrust coefficient of the propellers. Indeed, this thrust coefficient depends on several aerodynamical parameters and the distance between the ground and the vehicle [2]. To accurately estimate the complex aerodynamic effects involved in the thrust coefficient it requires a computational effort which is beyond the limits of any computer that is likely to be embedded on such a vehicle. Basic analysis of the aerodynamics of the rotor blades is used to justify the proposed approach. An adaptive control design is used to estimate the global thrust coefficient on-line while control regulation is achieved. More precisely we propose,

Fig. 1: The X4-flyer

II. MODELING

The X4-flyer is an omnidirectional VTOL (vertical take off and landing) vehicle ideally suited for stationary and quasi-stationary flight conditions. The control of an X4-flyer is
achieved by the differential control of the thrust generated by each propeller. Up/down motion is controlled by collectively increasing or decreasing the thrust of all four motors. The thrust difference between the forward and the rear rotor (resp. the left/right rotor) creates a pitch torque inducing translation forward/rear (resp. left/right) motion. To finish, let’s see the yaw control. When a propeller turns, it has to overcome air resistance. In the X4-flyer, both sets of front-rear and left-right motors turn in opposite directions (cf Figure 2). Moreover, the reactive torque is essentially a function of the propeller rotational velocity. Consequently, controlling the X4-flyer yaw control. When a propeller turns, it has to overcome air resistance. In the X4-flyer, both sets of front-rear and left-right (resp. body fixed frame $E_a$) which is attached to the center of mass of the vehicle.

The position of the center of mass of the vehicle with respect to the inertial frame $\mathcal{R}_i$ is denoted $x$. Let $v$ (resp. $\Omega$) denotes the linear (resp. angular) velocity of the center of mass expressed in the inertial frame $\mathcal{R}_i$ (resp. body fixed frame $\mathcal{R}_a$). Define the attitude $R$ of the body fixed frame with respect to the inertial frame by means of Euler angles $\xi = (\text{yaw}(\phi), \text{pitch}(\theta))$ and roll $(\psi))$.

$$ R = \begin{pmatrix} c_\phi c_\theta & s_\phi s_\theta c_\phi - c_\psi s_\phi & c_\phi s_\theta c_\phi + s_\psi s_\phi \\ c_\phi s_\theta & s_\phi s_\theta s_\phi + c_\psi c_\phi & s_\phi s_\theta c_\phi - c_\psi s_\phi \\ -s_\phi & s_\phi c_\phi & c_\phi c_\theta \end{pmatrix} \quad (1) $$

where the following shorthand notations for trigonometric functions are used: $c_\alpha = \cos(\alpha)$, $s_\alpha = \sin(\alpha)$, $t_\alpha = \tan(\alpha)$. According to [4], the evolution of $R$ is given by

$$ \dot{R} = Rsk(\Omega) \quad (2) $$

where $sk(\Omega)$ is an antisymmetric matrix such that $sk(\Omega)v$ is the vector cross product between $\Omega$ and $v$. Let $\Gamma \in \mathcal{R}_a$ be the control torques derived from differential thrusts provided by the rotors on the airframe in order to modify the attitude of the vehicle. Let $T$ be the global thrust generated by the propellers rotation. Newton’s equations of motion yield

$$ \dot{m\dot{v}} = -TR_e + mge_3 \quad (3) $$

where $m$ represents the mass of the airframe, $I$ its inertia matrix expressed in $\mathcal{R}_a$ supposed diagonal and $g$ the gravity.

The lift $f_i$ generated by the rotor $i$ turning at the speed $\omega_i$ in free air may be expressed as [3]

$$ f_i = -b\omega_i^2 E_3 \quad (4) $$

where $b$ is the global thrust coefficient. It is a positive parameter depending on air density, rotor blade collective pitch and geometric blade characteristics. This parameter can be considered as constant when the X4-flyer is in quasi-hover conditions and far from the ground. Note that, the ground proximity cancels the air’s kinetic energy communicated by the rotors at the ground contact and change this energy into a pressure energy. These ground effects increase the thrust coefficient $b$ (cf [5] page 57). For each rotor, the variations of the thrust coefficient $b$ is mainly a function of the distance $h$ between the ground and the rotor and the diameter $D$ of this one. The variation of $b$ in function of the altitude $h$ is given in Table (3). The reactive torque due to the rotor drag generated by a rotor in free air may be modeled as

$$ \tau_i = -\kappa \omega_i |\omega_i| E_3 \quad (5) $$

Where the positive constant $\kappa$ depends on air density, rotor blade collective pitch and geometric blade characteristics. With the above consideration and considering that propellers have a symmetrical disposition around the gravity center $G$ with $d$ as offset of each propeller from the center of mass, we can deduce the expression of the control torques $\Gamma$ and the expression of the global thrust $T$ generated by the rotation of the four motors:

$$ \begin{pmatrix} T \\ \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} = \begin{pmatrix} b & b & b & b \\ 0 & db & 0 & -db \\ db & 0 & -db & 0 \\ -\kappa & \kappa & -\kappa & \kappa \end{pmatrix} \begin{pmatrix} E_3 \\ E_2 \\ E_1 \\ E_0 \end{pmatrix} \quad (6) $$

Due to the flexibility of the propellers, gyroscopic effects generated by the propellers rotation are not transmitted to the body frame but the generated blink of the fans modify a little the rotation plan of the propellers. As both sets of front-rear...
and left-right motors turn in opposite directions and due to the symmetry of the body frame, these effects can be considered as negligible.

Consequently, the dynamics model of the "X4-flyer" is

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= g\varepsilon_3 - \frac{1}{m}TRe_3 \\
\dot{R} &= Rsk(\Omega) \\
\dot{\Omega} &= -\Omega \times \Omega + \Gamma
\end{align*}
\]

and \( T \) and \( \Gamma \) can be deduced from \( \omega \) via (6).

### III. Control of the Rotational Dynamics

In this section, we would like to control the rotor-craft attitude dynamics from the IMU measurement which supply a rotational matrix \( R \) defined by (1). The desired attitude supplied by the operator is noted \( R_d \). Define a first error term \( \delta = R_d - R \). The purpose now is to insures the convergence of the error term (\( \delta = \Omega - \Omega_d \)) to zero. Define the error term \( \delta \) as

\[
\delta = \Omega - \Omega_d
\]

In order to stabilise \( \delta \) to zero, let define a new storage function defined as follows:

\[
W = \frac{1}{2}\text{tr}(I_3 - \delta) + \frac{1}{2}\delta^2
\]

Tacking the time derivative of (21),

\[
\dot{W} = -\frac{1}{2}k_2||\delta||_F^2 + \frac{1}{2}\text{tr}\left(\text{sk}(\delta)\frac{d}{dt}(\text{vex}(\delta))\right)
\]

Deriving \( k_2 > 0 \) is a positive gain and "vex" represents the inverse operator of "sk". The purpose now is to insure the convergence of the error term (\( \Omega - \Omega_d \)) to zero. It yields

\[
\dot{W} = -\frac{1}{2}k_2||\delta||_F^2 + \frac{1}{2}\text{tr}\left(\text{sk}(\delta)\frac{d}{dt}(\text{vex}(\delta))\right)
\]

Noting that

\[
\delta = \Omega - \Omega_d
\]

is algebraic (Eq. 10). In order to control the rotational dynamics, we consider \( \Gamma \) as

\[
\Gamma = I\left[\Omega_d - k_2 \frac{d}{dt}(\text{vex}(\delta))\right] + k_3 \Omega
\]

At this step of our study, note that the relation between \( \Omega \) and \( \Gamma \) is algebraic (Eq. 10). In order to control the rotational dynamics, we consider \( \Gamma \) as

\[
\Gamma = I\left[\Omega_d - k_2 \frac{d}{dt}(\text{vex}(\delta))\right] + k_3 \Omega
\]

with \( k_3 \geq 0 \). Using the above controller in time derivative of the storage function, it yields

\[
\dot{W} = -\frac{1}{2}k_2||\delta||_F^2 - k_3\delta^2
\]
insuring exponential convergence of $I_3 - \dot{R}$ to zero. Indeed, if denote by $(\theta, a)$ the angle-axis coordinates of $R$ ($\dot{R} = \exp (6\theta k(u))$, one has [4]:

$$\frac{1}{2} ||I_3 - \dot{R}||^2 = (1 - \cos(\theta))^2 = 2 \sin^2(\theta)$$

and

$$\frac{1}{2} ||\pi R\dot{R}||^2 = 2 \sin^2(\theta)$$

Consequently, it is possible to choose $k_2$ and $k_3$ such that $R$ converges exponentially to $R^d$.

From the expressions of the control torque $\Gamma$ (25) we use the relation (6) to obtain the desired speed for each rotor.

IV. ALTITUDE CONTROL

In this section, we propose to design an adaptable altitude control allowing the stabilization of the X4-flyer for any distance with the ground. This control is inspired from [2]. From (6), (7) and (8), we can extract the altitude dynamics as follows:

$$\dot{z} = v_z$$

(27)

$$\dot{v}_z = \frac{1}{m} T e_3^T R e_3 + g = -b\varpi + g$$

(28)

where $z$ is the vehicle altitude, $v_z$ the altitude velocity, $b$ is the global thrust coefficient and

$$\varpi = e_3^T R e_3 \sum_{i=1}^{i=4} \omega_i^2$$

(29)

Assume that $e_3^T R e_3 > 0$ insuring that the vehicle pitch and roll remains smaller than $\frac{\pi}{2}$. This assumption can be insured by the attitude controller. Let define $u$ as a new control input that we want to generate.

$$\varpi = \rho u \quad \text{and} \quad \dot{\rho} = \frac{1}{b}$$

(30)

where $b$ is an estimate of the thrust coefficient $b$, $\dot{\rho}$ represents the estimate of $\rho$. The idea of the control consists in estimating on-line the thrust coefficient $b$ in order to apply the new control input $u$ to stabilize the altitude vehicle. Developing (30), it yields

$$b\varpi = b\rho u = (1 - b(\rho - \dot{\rho}))u$$

(31)

$$= (1 - b\tilde{\rho})u$$

(32)

where $\tilde{\rho} = \rho - \dot{\rho}$. Thus from (27-28) the altitude dynamics can be written as follows:

$$\dot{z} = v_z$$

(33)

$$\dot{v}_z = \frac{1}{m}(u - b\rho u) + g$$

(34)

In order to insure the convergence of the altitude $z$ to a desired constant altitude $z_d$, let define the error function $\delta_1 = z - z_d$ which the time derivative is $\dot{\delta}_1 = v_z$. It yields,

$$\dot{\delta}_1 = v_z$$

(35)

$$\dot{\delta}_1 = -k_1 \delta_1 + k_1 \delta_2$$

(36)

where $\delta_2 = \delta_1 + \frac{4 \rho}{ka}$. Tacking the time derivative of $\delta_2$.

$$\dot{\delta}_2 = -k_1 \delta_1 + k_1 \delta_2 - \frac{1}{m k_1} (u - b\rho u) + \frac{1}{k_1} q$$

(37)

and tacking $u$ as follows,

$$u = mg + mk_1(k_1 + k_2)\delta_2,$$

(38)

it yields,

$$\dot{\delta}_2 = -k_1 \delta_1 - k_2 \delta_2 + \frac{1}{m k_1} b\rho u$$

(39)

Let define the following Lyapunov function

$$V = \frac{1}{2} \delta_1^2 + \frac{1}{2} \delta_2^2 + \frac{1}{2k_3} b\rho^2$$

(40)

Tacking the time derivative of this one and introducing (36) and (37), it yields

$$\dot{V} = -k_1 \delta_1^2 - k_2 \delta_2^2 + \frac{1}{m k_1} \delta_2 b\rho u + \frac{1}{k_3} b\rho \dot{\rho}$$

(41)

Assume that the dynamics of $\rho$ is negligible with respect to the dynamics of the altitude controller ($\dot{\rho} = -\rho$), it yields

$$\dot{V} = -k_1 \delta_1^2 - k_2 \delta_2^2 + b\rho(\frac{1}{m k_1} \delta_2 u - \frac{1}{k_3} \dot{\rho})$$

(42)

Tacking $\dot{\rho}$ as follows

$$\dot{\rho} = k_3 \frac{1}{mk_1} \delta_2 u$$

(43)

with $k_1$, $k_2$ and $k_3$ strictly positif, it yields

$$\dot{V} = -k_1 \delta_1^2 - k_2 \delta_2^2$$

(44)

From (44), one can deduce the convergence of $\delta_1$ and $\delta_2$ to zero. From LaSalle principle’s, one can conclude the convergence of $\rho$ to zero. Indeed, as $\delta_1$ and $\delta_2$ converge to zero. By continuity, $\delta_1$ and $\delta_2$ converge to zero too. Consequently, (39) insures the convergence of $\rho u$ to zero. From (38), the control $u$ converges to $mg$ which is different from zero and therefore, $\rho$ converges to zero.

V. ALTITUDE VELOCITY ESTIMATION

As the altitude sensor return a noised value, the computation of the altitude velocity cannot be obtained by a classical Euler approximation of the sensor signal. Consequently, we have to design an estimator of the velocity $v_z$ which can easily be embedded with the others algorithms. Let the following estimator issued from equations (27) and (28) with $k_1$ and $k_2$ strictly positive,

$$\dot{\tilde{z}} = \tilde{v}_z + k_1 z - \dot{\tilde{z}}$$

(45)

$$\dot{\tilde{v}}_z = g - \frac{b\varpi}{m} + k_2(z - \dot{\tilde{z}})$$

(46)

where $\varpi$ is defined by (29) and where $\tilde{z}$ and $\tilde{v}_z$ represent respectively the estimated altitude and its velocity. Let define two error terms

$$\tilde{z} = z - \tilde{z} \quad \text{and} \quad \tilde{v}_z = v_z - \tilde{v}_z$$

(47)
where $z$ represents the measurement issued from the sensor. The error term evolution is describe by

$$
\dot{e} = \begin{pmatrix} -k_1 & 1 \\ -k_2 & 0 \end{pmatrix} e + \begin{pmatrix} 0 \\ -\dot{b} \end{pmatrix} \omega \tag{48}
$$

where $e$ represents the vector $\begin{pmatrix} \tilde{z} \\ \tilde{\nu}_z \end{pmatrix}$ and $\dot{b} = b - \dot{b}$. Let $P$ be a positive definite matrix such that $A^T P + PA = -I_3$ where $I_3$ is the identity matrix. The expression of $P$ is given by

$$
P = \begin{pmatrix} \frac{1+k_2}{2k_b} & -\frac{1}{2} & \frac{1}{2} k_2 \\ -\frac{1}{2} & \frac{1}{2} + k_2 + k_1^2 \end{pmatrix} \tag{49}
$$

Let $V$ be the Lyapunov function defined as follows

$$
V = \frac{1}{2} \dot{e}^T P \dot{e} + \frac{1}{2k_b} \tilde{b}^2 \tag{50}
$$

with $k_b$ strictly positive. Taking the time derivative of $V$ and assuming that the dynamics of $\dot{b}$ is negligible with respect to the dynamics of the altitude controller, it yields

$$
\dot{V} = \frac{1}{2} \dot{e}^T [A^T P + PA] \dot{e} + \tilde{b} \omega^T (0 \quad -1) P e + \frac{1}{k_b} \dot{b} \hat{\omega} \tag{51}
$$

Tacking $\dot{\tilde{b}}$ as follows

$$
\dot{\tilde{b}} = -k_3 \omega^T (0 \quad -1) P e, \tag{52}
$$

It yields $\dot{V} = -\frac{1}{2} \|\dot{e}\|^2$. Using the argument of the persistent excitations ($\omega \neq 0$) one can conclude the exponential stability of the estimator (more concise proof will be presented in the final version of the paper).

VI. EXPERIMENTAL RESULTS

In this section, experimental results issued from an implementation of the above algorithms on the experimental X4-flyer made by the CEA (fig. 1) are presented.

A. Prototype description

The considered prototype is an X4-flyer equipped with a set of five electronic boards (fig. 4a).

The first board integrates the motors controller which regulate the rotation speed of the four propellers. The second board integrates an Inertial Measurement Unit (IMU) constituted of 3 low cost MEMS accelerometers, which give the gravity components in the body frame, 3 angular rate sensors and 2 magnetometers. However the magnetic field induced by the four motors makes the magnetometers measurement too noisy. Consequently, the magnetometers are not used and the angular rate sensor is integrated around the $E_3$ axis in order to measure the yaw of the vehicle. On the third board, a Digital Signal Processing (DSP) cadenced at 150 MIPS is embedded and performs all the computations to stabilize the system from the IMU data. A complementary filter embedded on the DSP, which is not explained in this paper, estimates from the accelerometers and the angular rate sensors the vehicle attitude. A proximeter card allows the link with different proximity sensors like ultrasonic or infra red sensors (used for the altitude control). The last board supply a serial wireless communication between the operator’s joystick and the vehicle. A Lithium-Polymer battery allows the power supply during nearly 10 min. The weight of the prototype is about 500g. Each board has been designed by the CEA.

The measurement used for the altitude control is obtained from a low cost ultrasonic sensor (SRF10) which has a range of 10cm to 3 meters (fig. 4b). As this sensor has to be embedded on the hovering system, its weight is very important. Although this sensor has a low weight, one drawback is that the measurement issued from this sensor is sometimes unreliable.

During a flight, a ground deported software allow us to check flight parameters via the serial wireless transmission and to plot the parameters evolution of the flight. The picture (4) shows the prototype during a flight.

B. Attitude control

In our experimentation, the inputs of the considered system are pitch and yaw angles given by the operator’s joystick position. The global thrust $T$ is generated by the altitude control designed in section V. The algorithm loop time is 6ms. Control gains have been chosen in order to conciliate a good stability of the system for stationary flight and a quick
reaction of it when the operator want to move the vehicle. With these gains adjustments, the hovering vehicle has a good stationary and quasi-stationary flight and it can be piloted by a unskilled people. Figure (5) represents the results of the attitude control algorithm during a vertical flight. This figure shows the components of the gravity in the body fixed frame during the flight. The solid plot represents the measured gravity components by the sensors and the dashed plot the desired the desired attitude issued from the operator joystick. It shown that even if the attitude measurements are disturbed, the X4-flyer remains stable and follows the desired attitude.

C. Altitude control

This control is added to the above attitude control. During the experimentations, the altitude velocity is supplied by the estimator presented in section 5. In order to prove the interest of this velocity estimator, we have represented the velocity estimated compared to the velocity computed using Euler approximation (fig. 7). It is shown that the velocity issued from the estimator is more regular and less disturbed. However, some false measurements from the ultrasonic sensor can disturb estimation of the altitude and its velocity.

Figure 6 shows the altitude evolution respect to the desired one, the different corrections made by the controller and the evolution of the global thrust coefficient. We note that perturbations due to a bad measurement from the ultrasonic sensor can destabilize the system. Note that during this experimentation, the evolution of the altitude after 30s (when no sensor perturbations happen) is very good. However, generally the system evolution seems to be good in spite of the bad quality of the sensor.

D. Conclusion about the experimentations

During the flight, the operator can easily operate the rotorcraft as he wants and when no action on the joystick happens, the vehicle remains stable in hover. Moreover, automatic take off and landing have been tested with success.

VII. CONCLUSION

In this article, in order to allow an easy remote control of an unmanned aerial vehicle known as an X4-flyer, we have designed an attitude control based on a modeling of the rotor-craft valid for stationary and quasi stationary flight. For the altitude stabilization, an adaptive controller has been developed based on ultrasonic measurement in order to limit ground effects. This controller allows automatic take off and landing proceedings. These control laws gives good results in flight and assure that the rotor-craft remains stable and easily remote controlled by unskilled people. The altitude velocity estimator used for the altitude control gives good results in spite of the bad quality measurements of the sensor.

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