Near-Optimum Detection for Distributed Space-Time Block Coding under Imperfect Synchronization

F.-C. Zheng, A. G. Burr, and S. Olafsson

Abstract—Significant performance gain can potentially be achieved by employing distributed space-time block coding (D-STBC) in ad hoc or mesh networks. So far, however, most research on D-STBC has assumed that cooperative relay nodes are perfectly synchronized. Considering the difficulty in meeting such an assumption in many practical systems, this paper proposes a simple and near-optimum detection scheme for the case of two relay nodes, which proves to be able to handle far greater timing misalignment than the conventional STBC detector.

Index Terms—Distributed space-time block coding, distributed transmit diversity, cooperative diversity, cooperative relay.

I. INTRODUCTION

SPACE-TIME block coding (STBC) has proved to be a very effective technique to achieve transmit diversity for wireless systems with co-located antennas at the transmitter, mainly due to its high diversity order and low decoding complexity [1]. In wireless systems such as ad hoc and sensor networks, however, it may only be possible to have a single antenna (rather than multiple co-located antennas) at the transmit/receive nodes due to cost and size constraints. As a result, there has over the past few years been a considerable research effort on creating and harnessing space diversity by applying STBC in a distributed fashion (termed distributed STBC or D-STBC) to single antenna systems [2]-[11].

So far, however, most research on D-STBC has assumed that cooperative relay nodes are perfectly synchronized so that the corresponding symbols from all the relay nodes arrive at the destination node at the same time. Such an assumption, unfortunately, is difficult or even impossible to achieve in many practical systems (e.g. ad hoc networks). With imperfect synchronization among the relay nodes (i.e. inter-relay-node synchronization), the channels may become dispersive even under flat fading conditions.

There has been only limited work reported in the literature addressing D-STBC under imperfect synchronization among the relay nodes, chiefly using block based equalization techniques at the destination node to mitigate the impact of asynchronous signals (e.g. [8][9]). Alternative techniques to D-STBC in the presence of asynchronism also exist and these include distributed space-time trellis coding [10] and delay diversity (which relies on equalization or sequence estimation) [11]. Compared with the original STBC schemes, however, these existing methods potentially incur a much higher computational complexity at the receiver.

This paper proposes a simple and near-optimum detection scheme for the case of two relay nodes under imperfect or rough synchronization. By cancelling the interference components in the received signal (caused by timing misalignment) in a decision feedback manner, a maximum likelihood (ML) detection scheme is realized on a symbol by symbol basis (assuming no feedback error), thus retaining the low computational complexity of the original STBC principle (i.e. achieving near-Alamouti simplicity).

For the rest of this paper, $[\cdot]^T$, $[\cdot]^*$ and $[\cdot]^H$ represent “transpose”, “conjugate”, and “transpose and conjugate”, respectively, while $\text{CN}(0, \sigma^2)$ denotes the set of Gaussian distributed complex numbers with the standard variance of $\sigma^2$ (i.e. 0.5$\sigma^2$ per dimension).

II. D-STBC UNDER IMPERFECT SYNCHRONIZATION

This paper assumes the 4-node model depicted in Fig. 1. As in almost all cooperative relay systems [3][4], there are two phases involved: Phase 1 for broadcasting and Phase 2 for relaying.

[Phase 1] The source node (S) transmits while relay nodes (R1 and R2) and destination node (D) receive. To prepare for the STBC operation in Phase 2, the data symbols at S are grouped into pairs. Denoting the $i$th pair of symbols in a data packet or frame transmitted by S as $s(i) = \{s(1,i), s(2,i)\}^T$, the corresponding signal received at D after direct transmission (DT) is

$$r_{sd}(i) = h_{sd} s(i) + n_{sd}(i),$$

(1)

where $h_{sd}$ is the channel gain between S and D, $r_{sd}(i) = [r_{sd}(1,i), r_{sd}(2,i)]^T$, $n_{sd}(i) = [n_{sd}(1,i), n_{sd}(2,i)]^T$, and $n_{sd}(j, i) \in \text{CN}(0, \sigma^2_d)$ is the additive noise.

[Phase 2] The data packet received at R1 and R2 are encoded using the Alamouti structure and transmitted to D. For clarity, this paper assumes that the channel gains $h_{sr1}$ and $h_{sr2}$ are such that R1 and R2 can always detect correctly [8][11]. Also, $h_{m}$ is the channel gain from Rm to D under perfect inter-relay-node synchronization.

Denoting the encoded symbol pair corresponding to $s(i)$ as $\mathbf{x}(i) = [x(1,i), x(2,i)]^T$ at R1 and $\mathbf{y}(i) = [y(1,i), y(2,i)]^T$...
always required and the fact that R received versions of these signals. There is normally a timing misalignment of accurate synchronization is dif-

\begin{align}
    r(2, i) &= h_1 x(2, i) + h_2(0) y(2, i) + h_2(-1) y(1, i)
    + h_2(-2) y(2, i - 1) + n_{rd}(2, i),
\end{align}

in this paper. The value of $\beta_m$ reflects the composite effect of timing delay $\tau$ and the particular pulse shaping waveform used. However, we normally have $\beta_m = 0$ for $\tau = 0$, and $\beta_1 = 1$ for $\tau = 0.5T$.

All the channel gains above are assumed to remain constant over the whole data packet, but to be subject to Rayleigh fading from packet to packet, i.e. $h_{sd} \in \mathbb{CN}(0, \sigma_d^2)$, $h_{1}$ and $h_{2} \in \mathbb{CN}(0, \sigma_e^2)$. For a fair comparison with the non-relay schemes, $R_1$ and $R_2$ transmit at half power, i.e. $\sigma_e^2 = 0.5\sigma_d^2$.

The above signal model may be viewed as a special case of some more general channel models (see [12] and the references therein). In [12], for example, an optimum scheme for STBC was proposed for co-located transmit antennas using zero-padding block transmission. The detection procedure in this paper, however, is based on symbol pairs only, and hence achieves a near-Alamouti simplicity (due to the special signal model in (3a) and (3b)).

Substituting (2) into (3a) and (3b), we have

\begin{equation}
    r(i) = H s(i) + I(i) + n_{rd}(i),
\end{equation}

where $\mathbf{r}(i) = [r(1, i), r^* (2, i)]^T$, $\mathbf{H} = \begin{bmatrix} h_1 & h_2(0) \\ h_2^*(0) & -h_1^* \end{bmatrix}$, $
\mathbf{I}(i) = [I(1, i), I(2, i)]^T = [h_2(-1)s^*(1, i - 1) + h_2(-2)s(2, i - 1), h_2^*(1, i - 1) + h_2^*(-2)s^*(2, i) + h_2^*(-2)s(1, i - 1)]^T$, and $n_{rd}(i) = [n_{rd}(1, i), n_{rd}^*(2, i)]^T$.

From (4), the conventional STBC detection can be carried out via the following standard 2-step procedure (assuming perfect channel state information at D).

\textbf{Step 1: Linear transform.}

\begin{equation}
    \mathbf{g}(i) \triangleq [g(1, i), g(2, i)]^T \triangleq \Theta \mathbf{r}(i) = \mathbf{D} s(i) + \Theta \mathbf{I}(i) + \nu(i),
\end{equation}

where $\Theta = H^H$, and $\nu(i) = [v(1, i), v(2, i)]^T = \Theta n_{rd}(i)$.

Also, “$\triangleq$" above means “is defined as".

Due to the Alamouti structure, $\mathbf{D} = \Theta \mathbf{H} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$, where

$$
\lambda = |h_1|^2 + |h_2(0)|^2.
$$
Also, note that
\[ \mathbf{v}(i) \mathbf{v}^H(i) = \sigma_n^2 \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}. \] (6)

**Step 2:** Least square (LS) detection.
\[ \hat{s}(j, i) = \arg\{ \min_{s_m \in S} |g(j, i) - \lambda s_m|^2 \}, \] (7)
where \( j = 1, 2 \), \( s_m \) is an arbitrary symbol in the corresponding symbol alphabet \( S \), and \( M \) is the number of elements or symbols within \( S \) (similarly hereinafter).

Due to the component of \( \Theta H \) in (5), however, the above procedure can suffer from significant detection errors, unless \( h_2(-m) = 0 \) (i.e. \( \tau = 0 \), the case of perfect synchronization).

### III. M A X I M U M L I K E L I H O O D D E T E C T I O N

Upon examining (3) and (4), we notice that \( s(m, i - 1), m = 1, 2 \), are in fact already known if the detection process has been initialized properly (e.g. through the use of pilot symbol(s) at the start of the packet). As such, \( I(1, i) = h_2(-1)s^*(1, i - 1) + h_2(-2)s(2, i - 1) \) and component \( h_2(-2)s(1, i - 1) \) can be removed (“decision feedback”) before applying the linear transform of \( \Theta = \mathbf{H}^H \) in (5). This leads to
\[ \mathbf{g}'(i) = [g'(1, i), g'(2, i)]^T = \Theta \mathbf{r}'(i) = \mathbf{D}s(i) + \mathbf{z}s^*(2, i) + \mathbf{v}(i), \] (8)
where
\[ \mathbf{z} = [z_1, z_2]^T = \Theta(0, h_2^*(-1))^T, \] (9)
and
\[ \mathbf{r}'(i) = \left[ r^*(1, i) - I(1, i) \right] \] (10)

Equation (8) can then be written as
\[ g'(1, i) = \lambda s(1, i) + z_1 s^*(2, i) + v(1, i), \] (11a)
and
\[ g'(2, i) = \lambda s(2, i) + z_2 s^*(2, i) + v(2, i). \] (11b)

In (11b), \( g'(2, i) \) is only related to \( s(2, i) \), and therefore \( s(2, i) \) can be detected as
\[ \hat{s}(2, i) = \arg\{ \min_{s_m \in S} |g'(2, i) - \lambda s_m - z_2 s^*_m|^2 \}. \] (12a)

Then, \( s(1, i) \) can be detected by substituting \( \hat{s}(2, i) \) back to (11a):
\[ \hat{s}(1, i) = \arg\{ \min_{s_m \in S} |g'(1, i) - \lambda s_m - z_1 s^*_m|^2 \}. \] (12b)

The above procedure totally eliminates the ISI caused by imperfect synchronization, but still retains the following two key properties of the original STBC principle.

- Optimality: If there is no decision feedback error for \( s(m, i - 1), m = 1, 2 \), then the above procedure is optimum in terms of maximum likelihood (ML). Since (6) holds, (12a) is an ML detector, and so is (13b). Our work shows that the impact of decision feedback error is normally very small (see Section V), hence the term “near-optimum detection” for the above procedure.

- Simplicity: The above ML search involves one symbol only and thus has a linear computational complexity. Compared with the conventional Alamouti detector, it only involves \( M/2 \) extra multiplications per symbol, hence achieving a near-Alamouti simplicity.

### IV. C O M B I N I N G W I T H D I R E C T T R A N S M I S S I O N

Even if the direct transmission (DT) fails, the received signal at D during Phase 1 (i.e. Eq.(1)) still contains valuable information and should therefore be retained and combined appropriately with its counterpart during Phase 2 wherever possible.

**A. The conventional STBC detector**

Applying maximum ratio combining (MRC) to (1) and (4), corresponding to (5) we now have
\[ \mathbf{g}(i) = [g(1, i), g(2, i)]^T = \Theta \mathbf{r}(i) + h^*_s \mathbf{r}_s(i) \]
\[ = \mathbf{D}s(i) + \Theta \mathbf{I}(i) + \mathbf{v}(i), \] (13)
where \( \mathbf{D} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \)
\[ \lambda = |h_s|^2 + |h_1|^2 + |h_2(0)|^2, \] (14)
and
\[ \mathbf{v}(i) = \Theta \mathbf{I}(i) + h^*_s \mathbf{r}_s(i). \]

Here, \( \Theta = \mathbf{H}^H \) as before. Note that we still have
\[ \mathbf{v}(i) \mathbf{v}^H(i) = \sigma_n^2 \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}, \]
where \( \lambda \) is now from (14). Step 2 is the same as (7) except for the new \( \lambda \) value in (14) above.

**B. The maximum likelihood detector**

The same maximum likelihood detection can now be applied in a similar manner as follows.

- Calculate \( \mathbf{g}'(i) = [g'(1, i), g'(2, i)]^T = \Theta \mathbf{r}'(i) + h^*_s \mathbf{r}_s(i). \)

- Carry out (12a) and (12b) using the above \( \mathbf{g}'(i) \) and the new \( \lambda \) value in (14).

Obviously, the two key properties of the STBC principle - simplicity and optimality (or near optimality in this case) - still hold. Also, as can be seen from (14), the utilisation of the DT signal has increased the diversity order of the system from 2 to 3. This increased diversity order, together with the fact that DT is at “full power”, will deliver a significant performance gain, as will be confirmed next.

### V. S I M U L A T I O N S

With an 8-PSK system and assuming that the relays R1 and R2 can always detect correctly, D-STBC is employed regardless of DT success or failure. Also, the signal to noise ratio (SNR) is defined as \( \text{SNR} = \sigma_n^2/\sigma_d^2 \) (dB). Also, the detection process is initialized only once, i.e., at the beginning of each symbol packet/stream.

**The impact of \( h_2(-1) \)** As \( h_2(-m) \) reflects the composite effect of both time delay \( \tau \) and the pulse shaping waveform (PSW) and \( h_2(-1) \) is the most dominant term, we set \( \beta_2 = 0 \) and \( \beta_1 = -10, 0, 5, 10 \) (dB) in this example. The bit error rates (BER’s) of the proposed ML scheme are displayed in Fig. 3 (without DT combining (DTC)) and Fig. 4 (with DTC). For comparison, the corresponding results of the conventional detector and those of “DT alone” and “STBC with perfect
synchronization" (i.e. \( \tau = 0 \)) are also shown. Fig. 3 indicates that the ML detector alone greatly outperforms the conventional STBC detector and DT with \( \beta_1 \) up to 5 dB, while Fig. 4 demonstrates that DTC has resulted in a significant improvement for the ML detector: it can now deal with much larger \( \beta_1 \) (or \( \tau \)) values.

[Propagation of decision feedback errors] To examine this critical issue, the ML detection is carried out (i) with error propagation (EP, i.e. as it is - with the natural propagation of any feedback errors), and (ii) with no EP (i.e. using the true symbols of the previous pairs for ISI removal). The BER results for \( \beta_1 = 5 \) dB (\( \beta_2 = 0 \)) are shown in Fig. 5. Clearly, the impact of error propagation is very minor indeed, showing the near-optimum nature of the detector in this paper.

[Practical pulse shaping waveforms] To gain some insight into the specific impact of time mismatch \( \tau \) alone, a raised-cosine pulse with the 3GPP roll-off factor of 0.22 is employed, and the results are presented in Fig. 6 (with DTC) for \( \tau = 0.2T, 0.4T, 0.6T, \) and \( 0.8T \), showing the effectiveness of the ML detector again.

VI. CONCLUDING REMARKS

It has been shown that the near-optimum detection scheme proposed in this paper is indeed very effective for 2-relay-node systems even under some relatively large \( \beta_n \) or \( \tau \) values while the conventional STBC detector fails even under small \( \beta_n \) or \( \tau \) values. DT combining can greatly improve the system performance due to the higher diversity order. Due to the reduced effective diversity order under imperfect synchronization, the proposed procedure does show gradual performance degradation with increasing \( \beta_n \) or \( \tau \) values. However, \( \tau \leq 0.5T \) (or \( \beta_1 = 0 \) dB) will guarantee an excellent performance. In practice, this (i) represents a much relaxed requirement for inter-relay-node synchronization, and (ii) can always be realized by simply instructing \( R_1 \) to delay its transmission by \( T \) if \( \tau \in [0.5T, T] \).
REFERENCES


