Knowledge-level analysis of belief base operations

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Abstract

Recent developments in philosophical studies of belief revision have led to the construction of models that are much closer to actual computing systems than the earlier models. An operation of contraction (revision) on a logically closed belief set can be obtained by assigning to it a finite belief base and an operation of contraction (revision) on that base. Exact characterizations of such base-generated operations have been obtained. In this way, computational tractability and other realistic properties of operations on finite bases can be combined with a precise analysis that is independent of the symbolic representation.

1. Introduction

The dominating formal model of belief revision and database updating is the AGM model, so called after the three authors, Carlos Alchourrón, Peter Gärdenfors and David Makinson, of the seminal paper in which this model was first presented [1]. In the AGM model, belief states are represented by logically closed sets of sentences, and operations of change are performed on these sets. A major alternative framework is that of belief bases. These are sets of sentences that—at least as a rough approximation—contain those sentences that represent the explicit information from which all other information is derived. Operations of change are performed on the belief base, and non-explicit information is changed only as the result of changes of the base. Belief bases have the advantages of computational tractability and of a greater expressive power that allows for realistic logical properties that are more difficult to obtain with belief set models.

One of the most important advantages of the AGM model is its representation theorems that exactly characterize the properties of its operations. Until recently,
no corresponding characterization was available of the properties, on the
knowledge level, of belief base operations. In this paper, such characterizations
are reported, and their significance for computer science are discussed.

2. The AGM model

In the AGM model, an individual's belief state (or the database state of an
artificial system) is represented by a set $K$ of sentences, the belief set. The belief
set is closed under logical consequence. In other words, you are supposed to
believe in all the logical consequences of what you believe.

To express logical closure in the formal language, a consequence operator $Cn$ is
used. For any set $A$ of sentences, $Cn(A)$ is the set of sentences that follow
logically from $A$. A set $A$ is closed under logical consequence if and only if
$A = Cn(A)$.

$Cn$ satisfies the standard conditions for a consequence operator, namely
inclusion ($A \subseteq Cn(A)$), monotony (if $A \subseteq B$, then $Cn(A) \subseteq Cn(B)$), and iteration
($Cn(A) = Cn(Cn(A))$). Furthermore, it satisfies the properties of supraclassicality
(if $\alpha$ follows by classical truth-functional logic from $A$, then $\alpha \in Cn(A)$),
deduction (if $\beta \in Cn(A \cup \{\alpha\})$, then $(\alpha \rightarrow \beta) \in Cn(A)$), and compactness (if
$\alpha \in Cn(A)$, then $\alpha \in Cn(A')$ for some finite set $A' \subseteq A$). $\vdash \alpha$ will be used as an
alternative notation for $\alpha \in Cn(A)$.

Logical closure is not satisfied by actual doxastic agents. We do not believe in
all the consequences of our beliefs, for the simple reason that we are unable to
make all the inferences and draw all the conclusions that this would require. Isaac
Levi has clarified this by pointing out that a belief set consists of the sentences
that someone is committed to believe in, not those that she actually believes in
[19,20]. According to Levi, you are doxastically committed to believe in all the
logical consequences of your beliefs, but typically your performance does not fulfil
this commitment. Belief sets (Levi's "corpora") represent your commitments.

Changes in belief are represented by operations on the belief set. There are
three types of change. By an expansion is meant that a new proposition, typically
consistent with a given belief set, is set-theoretically added to that belief set,
without anything else being retracted. In contraction, a proposition which was
earlier in a belief set is rejected from it. By revision is meant that a proposition is
added under the condition that the revised belief set be consistent and closed
under logical consequence [1, p. 510].

Operations on belief sets should preserve logical closure. This is the reason why
expansion is defined as $Cn(K \cup \{\alpha\})$ rather than the simpler $K \cup \{\alpha\}$.

Definition 1 (Isaac Levi [19]). Let $K$ be a belief set and $\alpha$ a sentence. $K + \alpha$ ("$K$
expanded by $\alpha"$), the (closing) expansion of $K$ by $\alpha$, is defined as follows:

$$K + \alpha = Cn(K \cup \{\alpha\}).$$
The standard AGM operator of contraction is \textit{partial meet contraction}. It is based on a selection among the maximal subsets of the belief set that do not imply the sentence to be discarded. The outcome of partial meet contraction by a sentence $\alpha$ is equal to the intersection of the set of selected maximal subsets of the original set that do not imply $\alpha$.

\textbf{Definition 2} [2]. Let $A$ be a set of sentences and $\alpha$ a sentence. The set $A \perp \alpha$ ("$A$ less alpha") is the set such that $B \in A \perp \alpha$ if and only if:

1. $B \subseteq A$,
2. $\alpha \notin Cn(B)$,
3. there is no set $B'$ such that $B \subseteq B' \subseteq A$ and $\alpha \notin Cn(B')$.

\textbf{Definition 3} [1]. Let $A$ be a set of sentences. A \textit{selection function} for $A$ is a function $\gamma$ such that for all sentences $\alpha$:

1. if $A \perp \alpha$ is non-empty, then $\gamma(A \perp \alpha)$ is a non-empty subset of $A \perp \alpha$, and
2. if $A \perp \alpha$ is empty, then $\gamma(A \perp \alpha) = \{ \alpha \}$.

\textbf{Definition 4} [1]. Let $A$ be a set of sentences and $\gamma$ a selection function for $A$. The \textit{partial meet contraction} on $A$ that is generated by $\gamma$ is the operation $\sim_\gamma$ such that for all sentences $\alpha$:

\[ A \sim_\gamma \alpha = \bigcap \gamma(A \perp \alpha) . \]

An operation $\div$ on $A$ is a partial meet contraction if and only if there is a selection function $\gamma$ such that for all sentences $\alpha$: $A \div \alpha = A \sim_\gamma \alpha$.

A variety of special cases of partial meet contraction are referred to in the AGM tradition. In particular, cases in which the selection function is based on a relation have turned out to have interesting formal properties.

\textbf{Definition 5}. A selection function $\gamma$ for a set $A$, and its associated operator $\sim_\gamma$ of partial meet contraction, are:

1. \textit{maxichoice} if and only if for all $\alpha$, $\gamma(A \perp \alpha)$ has exactly one element [3];
2. \textit{full meet} if and only if for all $\alpha$, $A \perp \alpha \subseteq \gamma(A \perp \alpha)$ [3];
3. \textit{relational} if and only if there is a relation $\sqsubseteq$ such that for all sentences $\alpha$, if $A \perp \alpha$ is non-empty, then [1]
\[ \gamma(A \perp \alpha) = \{ B \in A \perp \alpha \mid C \subseteq B \text{ for all } C \in A \perp \alpha \} ; \]
4. \textit{transitively relational} if and only if $\gamma$ is based in that manner on a relation $\sqsubseteq$ that is transitive [1];
5. \textit{transitively maximally relational} (TMR) if and only if $\gamma$ is based in that manner on a relation $\sqsubseteq$ that is transitive and also satisfies the maximizing property:

\[ \text{If } A \sqsubseteq B \text{, then } A \sqsubset B \]

(where $\sqsubset$ is the strict part of $\sqsubseteq$) [13].
It has been shown that an operator of partial meet contraction on a (logically closed) belief set is transitively maximally relational (TMR) if and only if it is transitively relational [14].

Revision is the consistency-preserving incorporation of a new belief. In the AGM model, the revision of a belief set $K$ by a sentence $\alpha$ takes place in two steps:

1. contract $K$ by $\neg \alpha$,
2. expand the resulting belief set by $\alpha$.

In the first step, enough of $K$ is removed to ensure that it no longer contradicts $\alpha$ (which a belief set does if and only if it contains $\neg \alpha$). After that, $\alpha$ can be added through expansion, which is done in the second step.

Letting * stand for revision, this procedure can be summarized in the form of an equation:

$$K * \alpha = (K \div \neg \alpha) + \alpha.$$

This is the Levi identity [3, 6]. In the AGM model, operators of revision are derived from operators of partial meet contraction via the Levi identity:

**Definition 6** [1]. Let $K$ be a belief set and $\gamma$ a selection function for $K$. The operator $\mp_{\gamma}$ such that

$$K_{\mp_{\gamma}} \alpha = (K_{\sim_{\gamma}} \neg \alpha) + \alpha$$

for all sentences $\alpha$ is the operator of partial meet revision on $K$ that is based on $\gamma$.

$\mp_{\gamma}$ stands in the same relation to * as $\sim_{\gamma}$ to $\div$. We use * ($\mp$) to denote operators of revision (contraction) in general, and $\mp_{\gamma}$ ($\sim_{\gamma}$) to denote the operator that is generated by a particular selection function $\gamma$.

Most logicians and philosophers who have studied belief revision have focused on operators of contraction, whereas computer scientists have been primarily interested in operators of revision. The Levi identity provides a firm connection between the two approaches.

The contraction and revision operators of AGM owe much of their attractiveness to a set of representation theorems. The following postulates are used to characterize contraction:

1. ($G \div 1$) $K \div \alpha$ is a theory if $K$ is a theory (closure).
2. ($G \div 2$) $K \div \alpha \subseteq K$ (inclusion).
3. ($G \div 3$) If $\alpha \not\subseteq \text{Cn}(K)$, then $K \div \alpha = K$ (vacuity).
4. ($G \div 4$) If $\alpha \not\subseteq \text{Cn}(\emptyset)$, then $\alpha \not\subseteq \text{Cn}(K \div \alpha)$ (success).
5. ($G \div 5$) If $\alpha \leftrightarrow \beta = \text{Cn}(\emptyset)$, then $K \div \alpha = K \div \beta$ (extensionality).
6. ($G \div 6$) $K \subseteq \text{Cn}((K \div \alpha) \cup \{\alpha\})$ whenever $K$ is a theory (recovery).
7. ($G \div 7$) $(K \div \alpha) \cap (K \div \beta) \subseteq K \div (\alpha \& \beta)$ (conjunctive overlap).
8. ($G \div 8$) If $\alpha \not\subseteq \text{Cn} \div (\alpha \& \beta)$, then $K \div (\alpha \& \beta) \subseteq K \div \alpha$ (conjunctive inclusion).

An operator $\div$ on a (logically closed) belief set $K$ is a partial meet contraction if and only if it satisfies ($G \div 1$)–($G \div 6$), the "basic Gärdenfors postulates". It is
transitively relational if and only if it also satisfies \((G \div 7)-(G \div 8)\), the "supplementary Gärdenfors postulates" [1].

The following postulates are used to characterize partial meet revision:

\[(G*1) \ K \star \alpha \text{ is a belief set (closure).} \]

\[(G*2) \ \alpha \in K \star \alpha \text{ (success).} \]

\[(G*3) \ K \star \alpha \subseteq Cn(K \cup \{\alpha\}) \text{ (inclusion).} \]

\[(G*4) \ \text{If } \neg \alpha \notin K, \text{ then } Cn(K \cup \{\alpha\}) \subseteq K \star \alpha \text{ (vacuity).} \]

\[(G*5) \ K \star \alpha \text{ is inconsistent if and only if } \alpha \text{ is inconsistent (consistency).} \]

\[(G*6) \ \text{If } (\alpha \leftrightarrow \beta) \in Cn(\emptyset), \text{ then } K \star \alpha = K \star \beta \text{ (extensionality).} \]

\[(G*7) \ K \star (\alpha \land \beta) \subseteq Cn((K \star \alpha) \cup \{\beta\}). \]

\[(G*8) \ \text{If } \neg \beta \notin K \star \alpha, \text{ then } Cn((K \star \alpha) \cup \{\beta\}) \subseteq K \star (\alpha \land \beta). \]

An operator \(*\) on a (logically closed) belief set \(K\) is a partial meet revision if and only if it satisfies \((G*1)-(G*6)\), the "basic Gärdenfors postulates". It is transitively relational if and only if it in addition satisfies \((G*7)-(G*8)\), the "supplementary Gärdenfors postulates".

3. Weaknesses of the AGM model

The identification of belief states with belief sets has non-negligible disadvantages, in particular from a computational point of view. A belief set is a very large entity. For any two sentences \(\alpha\) and \(\beta\), if \(\alpha\) is included in the belief set, then so are both \(\alpha \lor \beta\) and \(\alpha \lor \neg \beta\), since they are logical consequences of \(\alpha\). Therefore, if the language is sufficiently rich, then the belief set contains myriads of sentences that the believer has never thought of. In particular, if the language is infinite, then the belief set will contain an infinite number of sentences.

It seems unnatural for changes to be performed on such large entities as belief sets, that contain all kinds of irrelevant and never-thought-of sentences. This alone is a sufficient reason to search for alternatives to the AGM model. Another reason for this is that one of the characteristic properties of partial meet contraction, namely recovery, is "open to query from the point of view of acceptability under its intended reading" [21]. Several authors have argued against recovery as a general principle of belief contraction [5, 11, 14, 18, 20, 24]. The following two examples indicate that recovery does not always hold:

**Example** [11]. I have read in a book about Cleopatra that she had both a son and a daughter. My set of beliefs therefore contains both \(\phi\) and \(\psi\), where \(\phi\) denotes that Cleopatra had a son and \(\psi\) that she had a daughter. I then learn from a knowledgeable friend that the book is in fact a historical novel. After that I contract \(\phi \lor \psi\) from my set of beliefs, i.e., I do not any longer believe that Cleopatra had a child. Soon after that, however, I learn from a reliable source that Cleopatra had a child. It seems perfectly reasonable for me to then add \(\phi \lor \psi\) to my set of beliefs without also reintroducing either \(\phi\) or \(\psi\).

**Example.** I previously entertained the two beliefs "George is a criminal" \((\alpha)\) and
“George is a mass murderer” (β). When I received information that induced me to give up the first of these beliefs (α), the second (β) had to go as well (since α would otherwise follow from β).

I then received new information that made me accept the belief “George is a shoplifter” (δ). The resulting new belief set is the expansion of K = α by δ, (K + α) + δ. Since α follows from δ, (K + α) + α is a subset of (K + α) + δ. By recovery, (K + α) + α includes β, from which follows that (K + α) + δ includes β.

Thus, since I previously believed George to be a mass murderer, I cannot any longer believe him to be a shoplifter without believing him to be a mass murderer.

4. Belief bases

As an alternative to belief sets, belief states can be represented by sets of sentences that are not closed under logical consequence. Such sets are called belief bases.

**Definition 7.** Any set A of sentences is a belief base.

Let K be a belief set. Then a set A of sentences is a belief base for K if and only if K = Cn(A).

The criterion for a sentence α to be believed is that it is a consequence of the belief base, α ∈ Cn(A). The elements of the belief base are the basic beliefs, and the elements of its logical closure that are not elements of the belief base itself are the (merely) derived beliefs.

Belief bases are not required by definition to be finite, but in all realistic applications they will be so.

Changes are performed on the belief base. Although we (are committed to) believe the logical consequences of our basic beliefs, these consequences are subject only to exactly those changes that follow from changes of the basic beliefs. The underlying intuition is that the merely derived beliefs are not worth retaining for their own sake. If one of them loses the support that it had in basic beliefs, then it will automatically be discarded.

**Example.** Let α denote that I have money in my bank account, and β that I own a large fortune in stocks and shares. For good reasons, I believe that α is true and β is wrong. As a consequence of my belief in α, I also believe that α ∨ β is true. One day, my bank informs me that my account is empty. I therefore lose my belief in α. If I am a sensible person, I will then also lose my belief in α ∨ β.

- **Belief base approach:** α ∨ β is automatically lost when it is no longer implied by the belief base.
- **Belief set approach:** The retraction of α ∨ β has to be ensured by the use of a selection mechanism that determines (among other things) whether to retain α ∨ β or α ∨ ¬β.
Thus, when elements of a belief base are removed, this leads to disbelief in all sentences that depended on the removed elements. This is the process of "disbelief propagation" that has been discussed by computer scientists [22].

One and the same belief set can be represented by different belief bases. In this sense, belief bases have more expressive power than belief sets. As an example, the two belief bases \{α, β\} and \{α, α ↔ β\} have the same logical closure, since \(\text{Cn}\{\{α, β\}\} = \text{Cn}\{\{α, α ↔ β\}\}\). Nevertheless, these belief bases are not identical. They are statically equivalent, in the sense of representing the same beliefs. On the other hand, the following example shows that they are not dynamically equivalent in the sense of behaving in the same way under operations of change. They can therefore be taken to represent different ways of holding the same beliefs.

**Example.** Let α denote that the Liberal Party will support the proposal to subsidize the steel industry, and let β denote that Ms. Smith, who is a liberal MP, will vote in favour of that proposal.

Abe has the basic beliefs α and β, whereas Bob has the basic beliefs α and α ↔ β. Thus, their beliefs (on the belief set level) with respect to α and β are the same.

Both Abe and Bob receive and accept the information that α is wrong, and they both revise their belief states to include the new belief that ¬α. After that, Abe has the basic beliefs ¬α and β, whereas Bob has the basic beliefs ¬α and α ↔ β. Now, their belief sets are no longer the same. Abe believes that β whereas Bob believes that ¬β.

(In the AGM approach, cases like these are taken care of by assuming that although Abe's and Bob's belief states are represented by the same belief set, this belief set is associated with different selection mechanisms in the two cases [7]. Abe has a selection mechanism that gives priority to β over α ↔ β, whereas Bob's selection mechanism has the opposite priorities.)

There is only one logically closed inconsistent set, namely the whole language. Hence, there is only one inconsistent belief set. In other words, if \(K\) and \(K'\) are inconsistent belief sets, then \(K = K'\). The corresponding property does not hold for belief bases. The inconsistent belief bases \{p, ¬p, q\} and \{p, ¬p, ¬q\} are statically equivalent, but they are not dynamically equivalent since, by any reasonable operator of contraction, \{(p, ¬p, q) ÷ p = ¬p, q\} and \{(p, ¬p, ¬q) ÷ p = ¬p, ¬q\}.

This is a valuable property of belief bases as compared to belief sets. A database that contains inconsistent information does not have to be beyond repair. It should be possible to remove the inconsistency, while keeping the information that is not affected by contradiction. This can be done if the inconsistent belief state is represented by a belief base. On the other hand, once we have arrived at an inconsistent belief set, all distinctions are lost, and they cannot be so easily regained by operations performed directly on the belief set.
5. Operations on belief bases

Expansion on belief bases must be different from expansion on belief sets in order not to introduce logical closure.

Definition 8. Let $A$ be a belief base and $\alpha$ a sentence. $A + \alpha$, the (non-closing) expansion of $A$ by $\alpha$, is defined as follows:

$$A + \alpha = A \cup \{ \alpha \}.$$ 

In order to limit the number of symbols, the same symbol ("+") will be used for both closing and non-closing expansions.

Partial meet contraction, as outlined in Section 2, is equally applicable to belief bases. An important difference is that recovery $(A \subseteq \text{Cn}((A \div \alpha) \cup \{ \alpha \}))$ does not hold for partial meet contraction on belief bases. Another interesting difference is that some transitively relational partial meet contractions on bases are not transitively maximally relational (TMR). For belief sets, the two categories of contraction coincide [13].

Partial meet contraction is based on a selection among maximal subsets of $A$ that do not imply $\alpha$. Another possible approach is to select the sentences to be discarded. They should be chosen among those elements of $A$ that contribute to make it imply $\alpha$. An operator that does exactly this, kernel contraction, was introduced recently [15].

Definition 9 [15]. Let $A$ be a set of sentences and $\alpha$ a sentence. Then $A \ll \alpha$ is the set such that $X \in A \ll \alpha$ if and only if:

(1) $X \subseteq A$,
(2) $X \not\vdash \alpha$, and
(3) if $Y \subseteq X$, then $Y \not\vdash \alpha$.

$A \ll \alpha$ is a kernel set, and its elements are the $\alpha$-kernels of $A$.

The function that selects sentences to be removed is called an incision function since it makes an incision into every $\alpha$-kernel.

Definition 10 [15]. Let $A$ be a set of sentences. An incision function for $A$ is a function $\sigma$ such that for all sentences $\alpha$:

(1) $\sigma(A \ll \alpha) \subseteq \bigcup(A \ll \alpha)$,
(2) if $\emptyset \neq X \in A \ll \alpha$, then $X \cap \sigma(A \ll \alpha) \neq \emptyset$.

The outcome of a kernel contraction consists of all elements of the original set not selected for removal by the incision function:

Definition 11 [15]. Let $A$ be a set of sentences and $\sigma$ an incision function for $A$. The kernel contraction on $A$ that is generated by $\sigma$ is the operation $\approx_{\sigma}$ such that for all sentences $\alpha$:
An operation $\div$ on $A$ is a kernel contraction if and only if there is some incision function $\sigma$ for $A$ such for all sentences $\alpha$: $A \div \alpha = A \approx_{\sigma} \alpha$.

All partial meet contractions are kernel contractions. The converse relationship does not hold, or in other words: some kernel contractions are not partial meet contractions [15]. Thus, as applied to a belief base, kernel contraction is a more general type of operation than partial meet contraction. It turns out, however, that when we apply the two types of operation directly to belief sets, then the distinction disappears (unless we accept contraction-outcomes that are not logically closed, which is clearly not reasonable in contraction of belief sets.)

**Observation 12** [15]. Let $K$ be a belief set, and let $\div$ be an operation on $K$ that satisfies closure, i.e., such that for all $\alpha$, $K \div \alpha = \text{Cn}(K \div \alpha)$. Then the following two conditions are equivalent:
1. $\div$ is a kernel contraction on $K$,
2. $\div$ is a partial meet contraction on $K$.

Operators of revision on belief bases can be obtained in essentially the same way as for belief sets, i.e., through the Levi identity. The only difference is that here, of course, non-closing rather than closing expansion has to be applied.

**Definition 13.** Let $\gamma$ be a selection function for the belief base $A$. Then the operator $\div_{\gamma}$ of *internal partial meet revision* on $A$ is defined as follows:

$$A \div_{\gamma} \alpha = (A \sim_{\gamma} -\alpha) \cup \{\alpha\}.$$

The Levi identity is based on the following recipe for revision:
1. Contract by $-\alpha$.
2. Expand by $\alpha$.
Alternatively, the two suboperations may take place in reverse order [16]:
1. Expand by $\alpha$.
2. Contract by $-\alpha$.
In equational terms, this corresponds to the following identity:

$$A * \alpha = (A + \alpha) \div -\alpha \quad \text{(reversed Levi identity [16]).}$$

The composite operation $*$ obtained in this way is called *external revision*. The names "internal" and "external" indicate that in internal revision, the suboperation of contraction takes place inside the original belief base, whereas in external revision it takes place outside of the original set.

In typical cases, external revision has an *intermediate inconsistent state* in which both $\alpha$ and $-\alpha$ are believed, whereas internal revision has an *intermediate non-committed state* in which neither $\alpha$ nor $-\alpha$ is believed. The two types of operations have different logical properties [16].

(In external revision, when one and the same belief base $(A)$ is revised by two
different sentences α and β, the suboperation of contraction will be applied to
different belief bascs, namely $A \cup \{α\}$ and $A \cup \{β\}$, respectively. Therefore,
external revision requires a global contraction operator that can be applied to any
belief base. The global generalization of partial meet contraction is straight-
forward, but will not be performed here [12].

It does not make much sense to apply external revision to belief sets. Since $K + α = K' + α$
for all belief sets $K$ and $K'$ that contain $¬α$, the information
inherent in the previous belief set is completely lost in the first step, expansion, of
belief-contravening external revision, and it cannot be regained in the second
step.

6. Knowledge-level analysis

In an influential 1982 paper, Allen Newell postulated the existence in artificial
intelligence of a knowledge level [23]. In the traditional hierarchy of system levels,
beginning with the device level and the circuit level, the knowledge level is
positioned immediately above the symbol level (program level). The knowledge
level is specified entirely in terms of the contents of the knowledge (beliefs).
There is no distinction on this level between information that is explicitly
available and information that is implied by available information [4]. It should be
possible to predict and understand what an agent does on the knowledge level,
without referring to the symbol level, in much the same way as the symbol level
should allow for prediction and understanding without reference to the lower
levels of the system.

Belief sets are knowledge-level entities. More precisely put, changes on belief
sets represent on the knowledge level “what an ideal reasoner would or should do
when forced to reorganize his beliefs”, thus indicating what people and computers
should do “if they were not bounded by limited logical reasoning capabilities” [9].

We are thus faced with a dilemma. On the one hand, we wish to be able to
describe and analyze epistemic behaviour on the knowledge level. This would
lead us to use belief sets as models of belief states. On the other hand, we need
models that do not force us to accept the recovery postulate, that have a finite
representation and are suitable for actual computing. This leads us to use finite
belief bases, rather than belief sets, to represent belief states.

This dilemma can be solved. It is possible to exactly characterize the operations
on a belief set that can be generated by assigning to it a (finite) belief base and
operators of contraction and revision that are applied to that belief base. In other
words, we can perform knowledge-level analysis of belief base operations.

Let $B$ be a belief base, and let $K$ be the corresponding belief set, i.e., let
$K = \text{Cn}(B)$. Furthermore, let $−$ be an operator of contraction on $B$. Let $\div$ be an
operator on $K$ such that

$$\text{for all } α, \ K \div α = \text{Cn}(B − α).$$

Then $\div$ is an operator of contraction on $K$. It will be called the closure of the
operator $−$ on $B$. More generally:
Definition 14. Let $\circ$ be an operator on a set $B$. The closure of $\circ$ is the operator $\circ'$ on $\text{Cn}(B)$ such that for all sentences $\alpha$: 

\[(\text{Cn}(B)) \circ' \alpha = \text{Cn}(B \circ \alpha).\]

In this way, every operator of belief base contraction generates an operator of belief set contraction.

For simplicity, closures of various types of operations will be referred to as "base-generated" versions of these operations. (The operations that are applied directly to the belief set will be referred to as the "direct" versions.) Thus, the closure of an operator of partial meet contraction will be called a "base-generated partial meet contraction".

In addition to the postulates referred to in Section 2, the following contraction postulates have turned out to be useful for the characterization of base-generated contraction operators:

1. There is a finite set $A$ such that for every sentence $\alpha$, $K \div \alpha = \text{Cn}(A')$ for some $A' \subseteq A$ (finitude).
2. If it holds for all $\delta$ that $K \div \delta \vdash \alpha$ if and only if $K \div \delta \vdash \beta$, then $K \div \alpha = K \div \beta$ (symmetry).
3. If $K \div \beta \not\subseteq K \div \alpha$, then there is some $\delta$ such that $K \div \delta \not\vdash \alpha$ and $K \div \delta \cup K \div \beta \vdash \alpha$ (weak conservativity).
4. If $K \div \beta \not\subseteq K \div \alpha$, then there is some $\delta$ such that $K \div \alpha \subseteq K \div \delta \not\vdash \alpha$ and $K \div \delta \cup K \div \beta \vdash \alpha$ (conservativity).
5. If $K \div \alpha \rightarrow K$ and the set of $\beta$-removals that are also maximally preservative $\alpha$-removals is non-empty, then it coincides with the set of maximally preservative $\beta$-removals (regularity). (An $\alpha$-removal is a contraction-outcome $K \div \alpha$ such that $K \div \alpha \not\vdash \gamma$. A maximally preservative $\alpha$-removal is a contraction-outcome $K \div \beta$ such that $K \div \alpha \subseteq K \div \beta \not\vdash \gamma$ and that for all $\delta$, if $K \div \beta \subseteq K \div \delta$ then $K \div \delta \vdash \alpha$.)
6. If $\alpha \in K \div (\alpha \& \beta)$, then $\alpha \in K \div (\alpha \& \beta \& \delta)$ (conjunctive trisection).

The following axiomatic characterizations have been obtained for base-generated contractions.

Theorem 15. An operator $\div$ on a consistent belief set $K$ is generated

1. by an operator of kernel contraction on a finite base for $K$ if and only if $\div$ satisfies closure, inclusion, vacuity, success, extensionality, finitude, symmetry, and weak conservativity [15];
2. by an operator of partial meet contraction on a finite base for $K$ if and only if $\div$ satisfies closure, inclusion, vacuity, success, extensionality, finitude, symmetry, and conservativity [17];
3. by an operator of TMR partial meet contraction on a finite base for $K$ if and only if $\div$ satisfies closure, inclusion, vacuity, success, extensionality, finitude, symmetry, conservativity, and regularity [17];
4. by an operator of TMR partial meet contraction on a finite, disjunctively closed base for $K$ if and only if $\div$ satisfies closure, inclusion, vacuity,
success, extensionality, finitude, symmetry, conservativity, conjunctive overlap, conjunctive inclusion, and conjunctive trisection [14].

A belief base $B$ is disjunctively closed if and only if it holds for all sentences $\alpha$ and $\beta$ that if $\alpha \in B$ and $\beta \in B$, then $\alpha \lor \beta \in B$.

7. Comments on the representation results

The representation results of Theorem 15 refer to several new postulates, and also to disjunctive closure of belief bases. In this section, the intuitive plausibility of these new formal properties will be discussed.

7.1. Finitude

We have finite minds, and other belief-carrying systems such as computers and animals are finite as well. A realistic representation of a belief state should reflect the finiteness of actual belief systems. This was pointed out in a 1988 paper by two members of the AGM trio, Peter Gardenfors and David Makinson. They wrote:

In all applications, the knowledge sets [belief sets] will be finite in the sense that the consequence relation $\vdash$ partitions the elements of $K$ into a finite number of equivalence classes. [8]

The property referred to here can also be expressed as follows:

Strong finitude:
If $A$ is an infinite set such that $\text{Cn}({\alpha}) \neq \text{Cn}({\beta})$ for all $\alpha, \beta \in A$, then $\text{Cn}(A) \subseteq K$.

The following example brings out the force of strong finitude:

Example. For every positive integer $n$, let $\pi_n$ denote that the Roman Catholic Church has a present at least one and at most $n$ popes. I believe in every sentence in the infinite sequence $\pi_1, \pi_2, \pi_3, \ldots$.

The “infiniteness” of this example should be admissible even for a finite mind. Surely a finite mind, such as that of a human being, should be capable of holding all elements of this infinite set of beliefs. Then, however, its belief set will violate strong finitude, since no two sentences in the sequence $\pi_1, \pi_2, \pi_3, \ldots$ are logically equivalent.

More generally, strong finitude precludes the use of a language with infinitely many non-equivalent sentences. The finiteness of actual belief systems seems to be weaker than what is expressed by this postulate.

There are at least two weaker but much more plausible finiteness properties. The first of these follows from the condition that belief sets have finite representations:
Finite representability:
For every sentence \( \alpha \), there is some finite set \( A \) such that \( K \vdash \alpha = \text{Cn}(A) \).

Finite representability need not be violated in our example. \( \pi_1 \) implies all of \( \pi_2, \pi_3, \ldots \). Therefore, for the infinite sequence \( \pi_1, \pi_2, \pi_3, \ldots \) to be included in \( K \) (or \( K \vdash \alpha \)) it is sufficient that \( \pi_1 \) is included in the finite representation (base) of \( K \) (respectively \( K \vdash \alpha \)).

The second of the two finiteness properties states that although there may be infinitely many sentences by which the belief set can be contracted, there are only a finite number of belief sets that can be obtained through contraction.

Finite number of contractions:
\[ \{K' \mid K' = K \vdash \alpha \text{ for some } \alpha \} \text{ is finite.} \]

This postulate need not either be violated in our example. With reasonable background beliefs about the Roman Catholic Church, \( \pi_2, \pi_3, \ldots \) are believed only as a consequence of belief in \( \pi_1 \). They all stand or fall with \( \pi_1 \), so that if one of them is lost, then the rest of them will be lost as well. Thus, e.g., \( K \vdash \pi_8 = K \vdash \pi_9 \).

These two finiteness properties can be combined into the following:

Finitude [17]:
There is a finite set \( A \) such that for every sentence \( \alpha \), \( K \vdash \alpha = \text{Cn}(A') \) for some \( A' \subseteq A \).

The following observation follows directly from the definitions:

**Observation 16.** A contraction operator \( \div \) on \( K \) satisfies finitude if and only if it satisfies both finite representability and finite number of contractions.

Finitude is acceptable as a general requirement on rational belief change.

7.2. Symmetry

In actual belief states, some beliefs are so strongly connected that they stand or fall together:

**Example.** I believe that either Paris or Oslo is the capital of France (\( \alpha \)). I also believe that either Paris or Stockholm is the capital of France (\( \beta \)). Both these beliefs are entirely based on my belief that Paris is the capital of France. Therefore, a contraction by some sentence \( \delta \) removes \( \alpha \) if and only if it removes \( \beta \) (namely if and only if it removes the common justification of these two beliefs). There is no contraction by which I can retract \( \alpha \) without retracting \( \beta \) or vice versa.

This pattern can be generalized. If two beliefs stand or fall together, then this must be because their justifications are the same, so that you cannot remove the
justifications of one without removing those of the other. Contraction should be a process in which the belief state is changed so that the belief to be contracted is no longer justified. Therefore it can reasonably be expected that two beliefs that stand or fall together have the same contraction-outcome. This assumption is expressed in the symmetry postulate:

**Symmetry [17]:**
If it holds for all \( \delta \) that \( K + \delta \vdash \alpha \) if and only if \( K + \delta \vdash \beta \), then \( K + \alpha = K + \beta \).

7.3. Conservativity and weak conservativity

As we saw in Section 3, the recovery postulate seems to be too strong as a general requirement on belief contraction. To replace it we need some weaker postulate that prevents unmotivated deletions from the original belief set. The following is a first approximation:

(1) If \( \varepsilon \in K \) and \( \varepsilon \notin K + \alpha \), then \( \varepsilon \) contributes to the fact that \( K \) implies \( \alpha \).

In order to translate (1) into formal language, we need to interpret what it means to "contribute to the fact that \( K \) implies \( \alpha \)". The following seems to be a reasonable interpretation:

(2) If \( \varepsilon \in K \) and \( \varepsilon \notin K + \alpha \), then there is a set \( A \) such that \( A \subseteq K \), \( \alpha \notin \text{Cn}(A) \) and \( \alpha \in \text{Cn}(A \cup \{\varepsilon\}) \).

This postulate has been called "core-retainment". Unfortunately, it fails for our purposes, since any operation on a belief set that satisfies core-retainment also satisfies recovery [11]. The following example helps us to see why the application of core-retainment to belief sets goes wrong.

**Example.** Let \( \alpha \) denote that John's wife is faithful and \( \beta \) that the Earth rotates around the Sun. When John's wife tells him that she is having a love affair, he contracts his belief set by \( \gamma \). Presumably, his belief in \( \beta \) will remain after this contraction.

All that core-retainment requires for \( \beta \notin K + \alpha \) is that there is some \( A \subseteq K \) such that \( \alpha \notin \text{Cn}(A) \) and \( \alpha \in \text{Cn}(A \cup \{\beta\}) \). This is clearly satisfied in our example; just let \( A = \{\beta \rightarrow \alpha\} \).

However, \( \{\beta \rightarrow \alpha\} \) is a highly artificial subset of \( K \) (and the same applies to its closure \( \text{Cn}(\{\beta \rightarrow \alpha\}) \)). It does not seem reasonable for John to give up both \( \alpha \) and \( \beta \) but retain belief in \( \beta \rightarrow \alpha \), that he only believed in from the beginning as a consequence of his belief in \( \alpha \). Although \( \text{Cn}(\{\beta \rightarrow \alpha\}) \) is a subset of the original belief set, it is not itself a reasonable belief set. In order to exclude constructions of this type we should require of \( A \) in (2) that it be a reasonable belief set (from the perspective of \( K \)). This leads us to the following condition:

(3) If \( \varepsilon \in K \) and \( \varepsilon \notin K + \alpha \), then there is a reasonable belief set \( K' \) such that \( K' \subseteq K \), \( \alpha \notin \text{Cn}(K') \), and \( \alpha \in \text{Cn}(K' \cup \{\varepsilon\}) \).
The next step is to determine which of the subsets of $K$ are reasonable belief sets. It does not seem possible to do this in terms of the truth-functional relationships of the elements of $K$. The only other means that we have at our disposal is the operator of contraction. The following definition employs that operator in what seems to be the most intuitively reasonable way:

(A) A subset $K'$ of $K$ is a reasonable belief set from the viewpoint of $K$ if and only if there is some finite series of contractions, starting with $K$, that results in $K'$, i.e.

$$K' = K \div \alpha_1 \div \alpha_2 \div \cdots \div \alpha_n.$$

Since iterated contractions are often difficult to handle in a formal context, it is worth investigating whether our purposes can be served by the following approximation of (A):

(B) A subset $K'$ of $K$ is a reasonable belief set from the viewpoint of $K$ if and only if there is some sentence $\alpha$ such that $K' = K \div \alpha$.

If (B) is applied to (3), then we obtain the following:

(4) If $\varepsilon \in K$ and $\varepsilon \notin K \div \alpha$, then there is a sentence $\delta$ such that $K \div \delta \subseteq K$ and $\alpha \notin \text{Cn}(K \div \delta)$ and $\alpha \in \text{Cn}(K \div \delta \cup \{\varepsilon\})$.

Provided that the inclusion postulate is satisfied, (4) is equivalent to the following:

(5) If $\varepsilon \in K$ and $\varepsilon \notin K \div \alpha$, then there is a sentence $\delta$ such that $K \div \delta \neq \alpha$ and $K \div \delta \cup \{\varepsilon\} \uparrow \alpha$.

However, we are not quite finished. In (1), and consequently in (2)–(5), it was assumed that all elements $\varepsilon$ of $K$ are worth retaining, in the sense that if $\varepsilon \notin K \div \alpha$, then $\varepsilon$ contributes to the fact that $K$ implies $\alpha$. This is not a realistic feature. We should restrict (5) to hold only for those sentences $\varepsilon$ in $K$ that represent self-sustained beliefs:

(6) If $\varepsilon$ is a self-sustained belief in $K$ and $\varepsilon \notin K \div \alpha$, then there is a sentence $\delta$ such that $K \div \delta \neq \alpha$ and $K \div \delta \cup \{\varepsilon\} \uparrow \alpha$.

Our next task is to explicate in formal terms what it means for a belief to be self-sustained, or capable of standing on its own. For a belief to be self-sustained in $K$, it should be in principle possible to “peel off” all other beliefs in $K$, and retain only this one belief. In other words:

(C) $\varepsilon$ is a self-sustained belief in $K$ if and only if there are sentences $\beta_1, \beta_2, \ldots, \beta_n$ such that $\text{Cn}(\{\varepsilon\}) = K \div \beta_1 \div \beta_2 \div \cdots \div \beta_n$.

(C) has the same disadvantages as (A). By an analogous argument, it can be simplified as follows:

(D) $\varepsilon$ is a self-sustained belief in $K$ if and only if there is a sentence $\beta$ such that $\text{Cn}(\{\varepsilon\}) = K \div \beta$. 

By applying (D) to (6) we obtain:

(7) If there are $e$ and $\beta$ such that $Cn(\{e\}) = K \vdash \beta$, and $e \notin K$ and $e \not\subseteq K \vdash \alpha$, then there is a sentence $\delta$ such that $K \vdash \delta /\alpha$ and $K \vdash \delta \cup \{e\} \vdash \alpha$.

Under the assumption that closure holds, $e \not\subseteq K \vdash \alpha$ in (7) can be replaced by $K \vdash \beta \subseteq K \vdash \alpha$. Since $K$ is logically closed, $e \in K$ can be replaced by $K \vdash \beta \subseteq K$.

Under the assumption that inclusion holds, this is a redundant condition. We can therefore transform (7) to the following:

(8) If there are $e$ and $\beta$ such that $Cn(\{e\}) = K \vdash \beta$, and $K \vdash \beta \subseteq K \vdash \alpha$, then there is a sentence $\delta$ such that $K \vdash \delta \not\vdash \alpha$ and $K \vdash \delta \cup K \vdash \beta \vdash \alpha$.

If finite representability holds, then (8) is equivalent to the following:

(9) If $K \vdash \beta \subseteq K \vdash \alpha$, then there is some $\delta$ such that $K \vdash \delta \not\vdash \alpha$ and $K \vdash \delta \cup K \vdash \beta \vdash \alpha$ (weak conservativity).

Admittedly, the argument that led up to the postulate of weak conservativity was somewhat roundabout, as a result of compromises aiming at making the underlying intuitions accessible to formal treatment. Since this formulation has turned out to work, in the sense of corresponding to plausible operations of contraction (see Section 6), it can nevertheless be accepted, at least as a first approximation.

One more aspect needs attention: In (1), we required of an excluded sentence $e$ that it “contributes to the fact that $K$ implies $\alpha$”. A stronger but not implausible requirement is that $e$ “contributes to the fact that $K$ but not $K \vdash \alpha$ implies $\alpha$". The starting point for our derivation will then be:

(1') If $e \in K$ and $e \not\subseteq K \vdash \alpha$, then $e$ contributes to the fact that $K$ but not $K \vdash \alpha$ implies $\alpha$.

The same chain of arguments can be applied to (1') as to (1), leading us to the following postulate:

(9') If $K \vdash \beta \not\subseteq K \vdash \alpha$, then there is some $\delta$ such that $K \vdash \alpha \subseteq K \vdash \delta \not\vdash \alpha$ and $K \vdash \delta \cup K \vdash \beta \vdash \alpha$ (conservativity).

7.4. Conjunctive trisection

Roughly speaking, there are three ways to ensure that a set does not imply $\alpha \& \beta \& \delta$, namely to see to it that it does not imply $\alpha$, that it does not imply $\beta$, or that it does not imply $\delta$. The first two of these are also the two ways to make it not imply $\alpha \& \beta$. If $\alpha \in K \vdash (\alpha \& \beta)$, then it is better (more information-economical) to exclude $\beta$ than to exclude $\alpha$. Thus, there is at least one way to remove $\alpha \& \beta \& \delta$ that is better than to exclude $\alpha$. We can therefore reasonably expect that $\alpha \in K \vdash (\alpha \& \beta \& \delta)$. Hence, conjunctive trisection should hold.
7.5. Regularity

A contraction-outcome \( K \div \beta \) is an \( \alpha \)-removal if and only if \( \alpha \not\in K \div \beta \). An \( \alpha \)-removal \( K \div \beta \) will be called a preservative \( \alpha \)-removal if and only if \( K \div \alpha \subseteq K \div \beta \), and a strictly preservative \( \alpha \)-removal if and only if \( K \div \alpha \subseteq K \div \beta \). A strictly preservative \( \alpha \)-removal is an operation that removes \( \alpha \), and does this in a more economical way than what is done by the contraction by \( \alpha \).

Often, a belief can be removed more economically if more specified information can be used. As an example, I believe that Albert Schweitzer was a German missionary (\( \alpha \)). Let \( \alpha_1 \) denote that he was a German and \( \alpha_2 \) that he was a missionary, so that \( \alpha \leftrightarrow \alpha_1 \& \alpha_2 \). If I have to contract my belief set by \( \alpha \), then the contracted belief set will contain neither \( \alpha_1 \) nor \( \alpha_2 \). Admittedly it would be logically sufficient to withdraw one of them. However, they are both equally entrenched, so that I do not know which to choose in preference over the other. Therefore, both will have to go. On the other hand, if I have to contract my belief set by \( \alpha_1 \), then I have no reason to let go of \( \alpha_2 \). To contract by \( \alpha_1 \) is, given the structure of my belief state, a more specified way to remove \( \alpha \). Thus we may expect that \( K \div \alpha \subseteq K \div \alpha_1 \), so that \( K \div \alpha_1 \) is a strictly preservative \( \alpha \)-removal.

Let \( \delta \) denote that Albert Schweitzer was a Swede, and let us consider the contraction of \( K \) by \( \alpha_1 \lor \delta \). \"Albert Schweitzer was a German or a Swede\". Since I believe in \( \alpha_1 \lor \delta \) only as a consequence of my belief in \( \alpha_1 \), I can only retract \( \alpha_1 \lor \delta \) by retracting \( \alpha_1 \). Therefore, \( K \div (\alpha_1 \lor \delta) \) is not a proper superset of \( K \div \alpha_1 \), i.e., it is not a more conservative \( \alpha \)-withdrawal than \( K \div \alpha_1 \). Indeed, the way my beliefs are structured, \( \alpha_1 \) cannot be further subdivided in the way that \( \alpha \) was subdivided into \( \alpha_1 \) and \( \alpha_2 \). There is no part of \( \alpha_1 \) that stands on its own and can be retracted from \( K \) without the rest of \( \alpha_1 \) being lost as well. In this sense, no \( \alpha \)-removal can be more conservative than \( K \div \alpha_1 \).

More generally, \( K \div \beta \) is a maximally preservative \( \alpha \)-removal if and only if it is a preservative \( \alpha \)-removal and there is no \( \alpha \)-removal \( K \div \delta \) such that \( K \div \beta \subseteq K \div \delta \). Intuitively, to perform a maximally preservative \( \alpha \)-removal is to make the belief set not imply \( \alpha \), making use of information that is sufficiently specified to allow one to remove a part of \( \alpha \) so small that no smaller part of it can be removed alone.

Maximally preservative removals are information-maximal contraction-outcomes. In an orderly and coherent belief state, one would expect their relative epistemic value (entrenchment) to be independent of what sentence we are contracting by.

As an illustration, let us extend the Albert Schweitzer example. Let \( \alpha_1 \) denote that Schweitzer was a German, \( \alpha_2 \) that he was a missionary and \( \alpha_3 \) that he was a physician. Let us assume that \( K \div \alpha_1 \) is a maximally preservative \( (\alpha_1 \& \alpha_2 \& \alpha_3) \)-removal, i.e. a maximally economical way to remove \( \alpha_1 \& \alpha_2 \& \alpha_3 \) from the belief set. Since \( K \div \alpha_1 \) is also an \( (\alpha_1 \& \alpha_2) \)-removal, and since the \( (\alpha_1 \& \alpha_2) \)-removals are a subset of the \( (\alpha_1 \& \alpha_2 \& \alpha_3) \)-removals, it should also be maximally preservative among these. Furthermore, if \( K \div \alpha_2 \) is equally economical as \( K \div \alpha_1 \) in the context of removing \( \alpha_1 \& \alpha_2 \) (i.e., if it is also a maximally preservative \( (\alpha_1 \& \alpha_2) \)-
removal), then it should also be equally economical as $K \vdash \alpha$, in the context of removing $\alpha_1 \& \alpha_2 \& \alpha_3$ (i.e., it should also be a maximally preservative $(\alpha_1 \& \alpha_2 \& \alpha_3)$-removal). In general:

If $\vdash \alpha \rightarrow \beta$ and the set of $\beta$-removals that are also maximally preservative $\alpha$-removals is non-empty, then it coincides with the set of maximally preservative $\beta$-removals (regularity).

It may be convenient to divide regularity into two parts:

If $\vdash \alpha \rightarrow \beta$ and some maximally preservative $\alpha$-removal is also a $\beta$-removal, then all maximally preservative $\beta$-removals are maximally preservative $\alpha$-removals (regularity 1).

If $\vdash \alpha \rightarrow \beta$ and $K \vdash \delta$ is both a $\beta$-removal and a maximally preservative $\alpha$-removal, then it is a maximally preservative $\beta$-removal (regularity 2).

Regularity 1 is closely related to Amartya Sen's $\beta$ property for rational choice behaviour, and regularity 2 to his $\alpha$ property [25]. Clearly, regularity holds if and only if both regularity 1 and regularity 2 hold.

7.6. Disjunctive closure

The ultimate criterion for a sentence to be an element of the belief base is that it is “self-sustained”, i.e., worth retaining for its own sake (even if it is not implied by some other sentence that is worth retaining). Belief bases have often been taken to consist of the beliefs that have an independent justification. This is only a very rough approximation. Independently justified beliefs are worth retaining for their own sake, but they are not necessarily the only beliefs that are worth retaining for their own sake. This will be clear from the following example:

Example [10]. I originally believed, for good and independent reasons, both that Andy is the son of the mayor ($\alpha$) and that Bob is the son of the mayor ($\beta$). Then I hear the mayor say in a public speech: “I certainly have nothing against our youth studying abroad. My only son did it for three years”.

Upon hearing this, I contract my belief state by $\alpha \& \beta$. As a result of this I lose both my belief in $\alpha$ and my belief in $\beta$. However, I retain my belief that $\alpha \lor \beta$, i.e., that either Andy or Bob is the son of the mayor.

In this case, we may assume that $\alpha \lor \beta$ had no independent justification. It was believed only as a consequence of my beliefs in $\alpha$ and $\beta$. It seems reasonable, however, in this and many other cases, to retain belief in the disjunction of two independently justified beliefs, when they can no longer coexist and one cannot choose between them.

If we wish to make this into a general pattern of belief change, then we should postulate that if both $\alpha$ and $\beta$ are worth retaining for their own sake, then so is $\alpha \lor \beta$. This is another way of saying that if both $\alpha$ and $\beta$ satisfy the requirements
for being elements of the base, then so does $\alpha \vee \beta$. Such a belief base will be closed under disjunction.

Disjunctively closed belief bases do not share the immediate appeal of bases simpliciter. This should be seen as an interesting special case rather than as a mandatory property of plausible belief bases.

8. Base-generated revision

Operators of revision on a belief set can be base-generated in the same sense as operators of contraction. Base-generated internal partial meet revision and direct (internal) partial meet revision turn out to coincide in the following sense:

Observation 17 [14]. The following two statements about an operation $*$ on a belief set $K$ are equivalent:

1. There is some selection function $\gamma$ for $K$ such that for all $\alpha$, $K * \alpha = K \vdash 私\alpha$.
2. There is some base $B$ for $K$ and some selection function $\gamma$ for $B$ such that for all $\alpha$, $K * \alpha = Cn(B \vdash 私\alpha)$.

The concurrence of base-generated and direct revision comes to an end if further requirements are imposed on the revision operators. In particular, this will happen if the operators are required to be transitively maximisingly relational (TMR). The following two statements about an operation $*$ on a logically closed set $K$ are not equivalent [14]:

3. There is some TMR selection function $\gamma$ for $K$ such that for all $\alpha$, $K * \alpha = K \vdash 私\alpha$.
4. There is some base $B$ for $K$ and some TMR selection function $\gamma$ for $B$ such that for all $\alpha$, $K * \alpha = Cn(B \vdash 私\alpha)$.

However, if the belief base is also required to be disjunctively closed, then an equivalence similar to that of Observation 17 is obtained.

Observation 18 [14]. The following two statements about an operation $*$ on a belief set $K$ are equivalent:

3. There is some TMR selection function $\gamma$ for $K$ such that for all $\alpha$, $K * \alpha = K \vdash 私\alpha$.
5. There is some disjunctively closed base $B$ for $K$ and some TMR selection function $\gamma$ for $B$ such that for all $\alpha$, $K * \alpha = Cn(B \vdash 私\alpha)$.

9. Conclusion

By shifting the operations of contraction and revision from the belief set itself to some finite base for the belief set, we have obtained more plausible models of belief change; in particular the controversial recovery property was avoided in this way. Furthermore, since these models are finite, they are more realistic from a
computational point of view. These advantages have been obtained while still having exact characterizations on the knowledge level of the type that is one of the hallmarks of the AGM model.

Idealized models of updating, such as those introduced here, provide background values against which actual updating systems can be tested. Comparisons between theoretical and actual updating systems should largely be performed on the knowledge level, rather than on the symbol level or lower levels of the system. In order to make such knowledge-level comparisons possible, we need (1) knowledge-level characterizations of idealized models of updating, and (2) knowledge-level characterizations of actual updating systems. The results reported in this paper are a contribution to the first of these two preconditions for a convergence of theoretical and practical studies of database updating.

References


